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C-CLASS FUNCTION ON NEW CONTRACTIVE CONDITIONS OF INTEGRAL TYPE ON COMPLETE S-METRIC SPACES

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Abstract:In this paper, we generalised the concept of a new contractive conditions of integral type on complete S-metric spaces via C-class function.

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INTRODUCTION AND MATHEMATICAL PRELIMINARIES

In 2012 [5] Sedghi. S et. al introduced the concept of generalization of fixed point theorems in S-metric spaces. Rahman M.U and Sarwar M discussed about the fixed point results of Altman integral type mappings in S-metric spaces in [6]. Recently, Nihal Yilmaz Ozgur, Nihal Tas [4] are discussed new contractive conditions of integral type on complete S-metric spaces. In 1984, M.S. Khan, M. Swalech and S. Sessa [12] expanded the research of the metric fixed point theory to a new category by introducing a control function which they called an altering distance function. In 2002, Branciari in [16] introduced a general contractive condition of integral type. Farshid Khojasteh et.al,[13] discuss some fixed point theorems of integral type contraction in cone metric spaces.

In this paper we discuss generalised result on C-class function on new contractive conditions of integral type on complete S-metric spaces.

Definition 1.1 *Let* $X \neq \emptyset$ *be any set and* $S : X \times X \times X \to [0,\infty)$ *be a function satisfying the following conditions for all* $u, v, z, a \in X$.

1. $S(u,v,z) \geq 0$

2. $S(u,v,z)=0$ if and only if $u=v=z$.

3. $S(u,v,z) \le S(u,u,a)+S(v,v,a)+S(z,z,a)$

Then the function S is called an S-metric on X and the pair (X,S) is called an S -metric space simply SMS.

Example 1.2 *[3] Let X be a non empty set, d is ordinary metric space on X, then $S(x,y,z)=d(x,z)+d(y,z)$ is an S - metric on X.*

Lemma 1.3 *Let (X,S) be an S -metric space. Then we have $S(u,u,v)=S(v,v,u)$*

Definition 1.4 *Let (X,S) be an S -metric space .*

1. A sequence $\{u_n\}$ in X converges to u if and only if $S(u_n,u_n,u)\to 0$ as $n\to\infty$. That is, there exists $n_0 \in N$ such that for all $n \ge n_0, S(u_n,u_n,u)<\varepsilon$. We denote this by $\lim_{n\to\infty} u_n = u$ or $\lim_{n\to\infty} S(u_n,u_n,u)=0$.

2. A sequence $\{u_n\}$ in X is called a Cauchy sequence if $S(u_n,u_n,u_m)\to 0$ as $n,m\to\infty$. That is, there exists $n_0 \in N$ such that for all $n,m \ge n_0,$ $S(u_n,u_n,u_m)<\varepsilon$.

3. The S -metric space (X,S) is called complete if every Cauchy sequence is convergent.

In the following lemma we see the relationship between a metric and an S -metric.

Lemma 1.5 *Let (X,d) be a metric space. Then the following properties are satisfied:*

1. $S_d(u,v,z)=d(u,z)+d(v,z)$ for all $u,v,z\in X$ is an S -metric on X .

2. $u_n \to u$ in $\{X,d\}$ if and only if $u_n \to u$ in (X,S_d) :

3. $\{u_n\}$ is Cauchy in $\{X,d\}$ if and only if $\{u_n\}$ is Cauchy in (X,S_d) :

4. $\{X,d\}$ is complete if and only if (X,S_d) is complete .

Definition 1.6 *[1] A mapping* $F:[0,\infty)^2 \to [0,\infty)$ *is called* C*-class function if it is continuous and satisfies following axioms:*

(1) $F(s,t) \le s$;

(2) $F(s,t) = s$ implies that either $s=0$ or $t=0$; for all $s,t \in 0,\infty)$.

Note for some F we have that $F(0,0)=0$.

We denote C-class functions as $\mathcal{C}$.

Example 1.7 *[1]The following functions* $F:[0,\infty)^2 \to \mathbb{R}$ *are elements of* $\mathcal{C}$*, for all* $s,t \in 0,\infty)$*:*

(1) $F(s,t) = s-t$, $F(s,t)=s \Rightarrow t=0$;

(2) $F(s,t) = ms$, $0<m<1$, $F(s,t)=s \Rightarrow s=0$;

(3) $F(s,t) = \dfrac{s}{(1+t)^r}$; $r\in(0,\infty)$, $F(s,t)=s \Rightarrow s=0$ or $t=0$;

(4) $F(s,t) = \log(t+a^s)/(1+t)$, $a>1$, $F(s,t)=s \Rightarrow s=0$ or $t=0$;

(5) $F(s,t) = \ln(1+a^s)/2$, $a>e$, $F(s,1)=s \Rightarrow s=0$;

(6) $F(s,t) = (s+l)^{(1/(1+t)^r)} - l$, $l>1, r\in(0,\infty)$, $F(s,t)=s \Rightarrow t=0$;

(7) $F(s,t) = s\log_{t+a} a$, $a>1$, $F(s,t)=s \Rightarrow s=0$ or $t=0$;

(8) $F(s,t) = s-(\dfrac{1+s}{2+s})(\dfrac{t}{1+t})$, $F(s,t)=s \Rightarrow t=0$;

(9) $F(s,t) = s\beta(s)$, $\beta:[0,\infty)\to(0,1)$, and is continuous, $F(s,t)=s \Rightarrow s=0$;

(10) $F(s,t) = s-\dfrac{t}{k+t}, F(s,t)=s \Rightarrow t=0$;

(11) $F(s,t) = s-\varphi(s), F(s,t)=s \Rightarrow s=0,$ here $\varphi:[0,\infty)\to 0,\infty)$ is a continuous function such that $\varphi(t)=0 \Leftrightarrow t=0$;

(12) $F(s,t) = sh(s,t), F(s,t)=s \Rightarrow s=0,$ here $h:[0,\infty)\times 0,\infty)\to 0,\infty)$ is a continuous function such that $h(t,s)<1$ for all $t,s>0$;

(13) $F(s,t) = s-(\dfrac{2+t}{1+t})t$, $F(s,t)=s \Rightarrow t=0$.

(14) $F(s,t) = \sqrt[n]{\ln(1+s^n)}$, $F(s,t)=s \Rightarrow s=0$.

(15) $F(s,t) = \phi(s), F(s,t)=s \Rightarrow s=0,$ here $\phi:[0,\infty)\to 0,\infty)$ is a upper semi continuous function such that $\phi(0)=0,$ and $\phi(t)<t$ for $t>0,$

(16) $F(s,t) = \dfrac{s}{(1+s)^r}$, $r\in(0,\infty)$, $F(s,t)=s$. $\Rightarrow$ $s=0$;

Definition 1.8 () *A function* $\psi:[0,\infty)\to[0,\infty)$ *is called an altering distance function if the following properties are satisfied:*

(i) ψ is non-decreasing and continuous,

(ii) $\psi(t)=0$ if and only if $t=0$.

Definition 1.9 *[1] An ultra altering distance function is a continuous, nondecreasing mapping* $\varphi : P \to P$ *such that* $\varphi(t) > 0$, $t \in [0,\infty)$ *and* $\varphi(0) \geq 0$.

We denote this set with Φ_u

Definition 1.10 *[13] The set* $\{a = x_0, x_1.x_2 \cdots, x_n = b\}$ *is called a partition for* $[a,b]$ *if and only if the sets* $\{x_{t-1}, x_t\}_{t=1}^{n}$ *are pairwise disjoint and* $[a,b] = \{\bigcup_{t=1}^{n} [x_{i-1}, x_t) \cup \{b\}\}$

Definition 1.11 *[13] The function* $\zeta : [0,\infty) \to 0,\infty)$ *is called subadditive integrable function if and only if for all* $a,b \in P$,

$$\int_0^{a+b} \zeta(t)dt \leq \int_0^{a} \zeta(t)dt + \int_0^{b} \zeta(t)dt$$

Example 1.12 *Let* $E = X = R, d(x,y) = |x-y|, P = (0,\infty)$, *and* $\zeta(t) = \dfrac{1}{(t+1)}$ *for all* $t > 0$. *Then for all* $a,b \in P$,

$\int_0^{a+b} \dfrac{dt}{(t+1)} = \ln(a+b+1), \int_0^{a} \dfrac{dt}{(t+1)} = \ln(a+1), \int_0^{b} \dfrac{dt}{(t+1)} = \ln(b+1)$ Since $ab \geq 0$, then $a+b+1 \leq a+b+1+ab = (a+1)(b+1)$. Therefore

$$\ln(a+b+1) \leq \ln(a+1) \leq \ln(b+1)$$

This shows that ζ is an example of subadditive integrable function.

Theorem 1.13 *[4] Let* (X,S) *be a complete* S*-metric space,* $h \in (0,1)$, *the function* $\zeta : [0,\infty) \to 0,\infty)$ *be defined as for each* $\varepsilon > 0, \int_0^{\varepsilon} \zeta(t)dt > 0$ *and* $T : X \to X$ *be a self-mapping of* X *such that*

$$\int_0^{S(Tu,Tu,Tv)} \zeta(t)dt \leq h \int_0^{S(u,u,v)} \zeta(t)dt$$

for all $u,v \in X$. Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Theorem 1.14 *[4] Let* (X,S) *be a complete* S*-metric space, the function*

$\zeta:[0,\infty)\to 0,\infty)$ *be defined as for each* $\varepsilon>0,\int_0^{\varepsilon}\zeta(t)dt>0$ *and* $T:X\to X$ *be a self-mapping of* X *such that*

$$\begin{aligned}\int_0^{S(Tu,Tu,Tv)}\zeta(t)dt &\le h_1\int_0^{S(u,u,v)}\zeta(t)dt+h_2\int_0^{S(Tu,Tu,v)}\zeta(t)dt\\ &+h_3\int_0^{S(Tv,Tv,u)}\zeta(t)dt+h_4\int_0^{\max\{S(Tu,Tu,v),S(Tv,Tv,u)\}}\zeta(t)dt\end{aligned}$$

for all $u,v\in X$ with non-negative real numbers $h_i(i\in\{1,2,3,4\})$ satisfying $\max\{h_1+3h_3+2h_4,h_1+h_2+h_3\}<1$. Then T has a unique fixed point $w\in X$ and we have $\lim_{n\to\infty}T^n u=w$, for each $u\in X$.

MAIN RESULT

Theorem 2.1 *Let* (X,S) *be a complete* S*-metric space* $\psi:[0,\infty)\to[0,\infty)$ *is an altering distance function,* $\varphi\in\Phi_u$ *and* $F\in\mathcal{C}$*, the function* $\zeta:P\to P$ *be defined as for each* $\varepsilon>0,\int_0^{\varepsilon}\zeta(t)dt>0$ *and* $T:X\to X$ *be a self-mapping of* X *such that*

$$\psi\left(\int_0^{S(Tu,Tu,Tv)}\zeta(t)dt\right)\le F\left(\psi\left(\int_0^{S(u,u,v)}\zeta(t)dt\right),\varphi\left(\int_0^{S(u,u,v)}\zeta(t)dt\right)\right). \tag{2.1}$$

for all $u,v\in X$, Then T has a unique fixed point $w\in X$ and we have $\lim_{n\to\infty}T^n u=w$, for each $u\in X$

Proof. Let $u_0\in X$ and the sequence $\{u_n\}$ be defined as $T^n u_0=u_n$. Suppose that $u_n\ne u_{n+1}$ for all n. Using the inequality (2.1), we obtain

$$\begin{aligned}\psi\left(\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt\right) &\le F\left(\psi\left(\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt\right),\varphi\left(\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt\right)\right)\\ &\le\psi\left(\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt\right).\end{aligned} \tag{2.2}$$

$$\text{so}\quad\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt\le\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt \tag{2.3}$$

Since $\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt>0$, there exists $r\ge 0$ such that $\lim_{n\to\infty}\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt=r$.

If $r>0$, then take limit for $n\to\infty$, we get $\psi(r)\le F(\psi((r),\varphi(r))$

So $\psi(r)=0$ or $\varphi(r)=0$. Thus $r=0$, which is a contradiction. Thus, we conclude that $r=0$, that is,

$$\lim_{n\to\infty}\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt=0,$$

since for each $\varepsilon>0, \int_0^{\varepsilon}\zeta(t)dt>0,$ implies $\lim_{n\to\infty} S(u_n,u_n,u_{n+1})=0.$

Now we show that the sequence $\{u_n\}$ is a Cauchy sequence.

Assume that $\{u_n\}$ is not Cauchy. Then there exists an $\varepsilon>0$ and subsequences $\{m_k\}$ and $\{n_k\}$ such that $m_k<n_k<m_{k+1}$ with

$$S(u_{m_k},u_{m_k},u_{n_k})\geq\varepsilon \tag{2.4}$$

and

$$S(u_{m_k},u_{m_k},u_{n_{k-1}})<\varepsilon \tag{2.5}$$

Hence using Lemma (1.3), we have

$$\begin{aligned} S(u_{m_{k-1}},u_{m_{k-1}},u_{n_{k-1}}) &\leq 2S(u_{m_{k-1}},u_{m_{k-1}},u_{m_k})+S(u_{n_{k-1}},u_{n_{k-1}},u_{m_k}) \\ &< 2S(u_{m_{k-1}},u_{m_{k-1}},u_{m_k})+\varepsilon \end{aligned}$$

and

$$\lim_{k\to\infty}\int_0^{S(u_{m_{k-1}},u_{m_{k-1}},u_{m_{k-1}})}\zeta(t)dt\leq\int_0^{\varepsilon}\zeta(t)dt \tag{2.6}$$

Using the inequalities (2.1), (2.4) and (2.6) we obtain

$$\begin{aligned} \psi(\int_0^{\varepsilon}\zeta(t)dt) &\leq \psi(\int_0^{S(u_{m_k},u_{m_k},u_{n_k})}\zeta(t)dt) \\ &\leq F(\psi(\int_0^{S(u_{m_k-1},u_{m_k-1},u_{n_k-1})}\zeta(t)dt),\varphi(\int_0^{S(u_{m_k-1},u_{m_k-1},u_{n_k-1})}\zeta(t)dt)) \\ &\leq F(\psi(\int_0^{\varepsilon}\zeta(t)dt),\varphi(\int_0^{\varepsilon}\zeta(t)dt)) \end{aligned}$$

So $\psi(\int_0^{\varepsilon}\zeta(t)dt)=0$ or $\varphi(\int_0^{\varepsilon}\zeta(t)dt)=0$. Thus $\int_0^{\varepsilon}\zeta(t)dt=0$, which is a contradiction with our assumption. So the sequence $\{u_n\}$ is Cauchy. Using the completeness hypothesis, there exists $w\in X$ such that

$$\lim_{n\to\infty}T^n u_0=w.$$

From the inequality (2.1) we find

$$\psi(\int_0^{S(Tw,Tw,u_{n+1})}\zeta(t)dt)\leq F(\psi(\int_0^{S(w,w,u_n)}\zeta(t)dt),\varphi(\int_0^{S(w,w,u_n)}\zeta(t)dt))$$

If we take limit for $n\to\infty$, we get

$$\psi(\int_0^{S(Tw,Tw,w)} \zeta(t)dt) \le F(\psi(\int_0^{S(w,w,w)} \zeta(t)dt), \varphi(\int_0^{S(w,w,w)} \zeta(t)dt)).$$

So $\psi(\int_0^{S(Tw,Tw,w)} \zeta(t)dt) = 0$ or $\varphi(\int_0^{S(Tw,Tw,w)} \zeta(t)dt) = 0$.

Thus $\int_0^{S(Tw,Tw,w)} \zeta(t)dt = 0$, which implies that $S(Tw,Tw,w) = 0$. Thus $Tw = w$.

Now we show the uniqueness of the fixed point. Suppose that w_1 is another fixed point of T. Using the inequality (2.1) we have

$$\psi(\int_0^{S(w,w,w_1)} \zeta(t)dt) = \psi(\int_0^{S(w,w,w_1)} \zeta(t)dt) \le F(\psi(\int_0^{S(w,w,w_1)} \zeta(t)dt), \varphi(\int_0^{S(w,w,w_1)} \zeta(t)dt))$$

So $\psi(\int_0^{S(w,w,w_1)} \zeta(t)dt) = 0$ or $\varphi(\int_0^{S(w,w,w_1)} \zeta(t)dt) = 0$. Thus $\int_0^{S(w,w,w_1)} \zeta(t)dt = 0.$

Using $\int_0^{\varepsilon} \zeta(t)dt > 0$ we get $w = w_1$. Consequently, the fixed point w is unique.

Choosing $F(s,t) = hs$, $0 < h < 1, \psi(t) = t,$ in theorem (2.1) we have

Corollary 2.2 *[4] Let (X,S) be a complete S-metric space $h\varepsilon(0,1)$, the function $\zeta : P \to P$ be defined as for each $\varepsilon > 0, \int_0^{\varepsilon} \zeta(t)dt > 0$ and $T : X \to X$ be a self-mapping of X such that*

$$\int_0^{S(Tu,Tu,Tv)} \zeta(t)dt \le h \int_0^{S(u,u,v)} \zeta(t)dt \tag{2.7}$$

for all $u,v \in X$. Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Example 2.3 *Let $X = R$, $k = 10$ be a fixed real number and function $S : X \times X \times X \to [0,\infty)$ be defined as*

$$S(u,v,z) = \frac{z}{k+1}(|v-z| + |v+z-2u|)$$

for all $u,v,z \in R$. It can be seen that the function S is an S-metric. Now we show that S-metric can not be generated by metric ρ. On the contrary, we assume that there exists a metric ρ such that

$$S(u,v,z) = \rho(u,z) + \rho(v,z) \tag{2.8}$$

for all $u,v,z \in R$.

$$\rho(u,z)=\frac{10}{11}|u-z| \tag{2.9}$$

Similarly, we have

$S(v,v,z)=2\rho(v,z)=\frac{20}{11}(|v-z|+|v+z-2u|)$ and

$$\rho(v,z)=\frac{10}{11}|v-z| \tag{2.10}$$

Using the equalities above equation (2.8), (2.9) and (2.10) we obtain

$$\frac{10}{11}(|v-z|+|v+z-2u|)=\frac{10}{11}|u-z|+\frac{10}{11}|v-u|$$

which is a contradiction, S is not generated by any metric and (R,S) is a complete S-metric space. $T:R\to R$ and $Tu=\frac{u}{4}$ for all $u\in R$ $\zeta:P\to P$ where $P=(0,\infty)$ as $\zeta(t)=2t$

Let $F(s,t)=s-t$ for all $s,t\in[0,\infty)$. Also define $\varphi,\psi:[0,\infty)\to[0,\infty)$ by $\psi(t)=t$ and $\varphi(t)=\frac{t}{2}$.

$$F(\psi(\int_0^{S(u,u,v)}\zeta(t)dt),\varphi(\int_0^{S(u,u,v)}\zeta(t)dt)) =\psi(\int_0^{S(u,u,v)}\zeta(t)dt)-\varphi(\int_0^{S(u,u,v)}\zeta(t)dt) \tag{2.11}$$

From equation (2.11), we have

$$\begin{aligned}F(\psi(\int_0^{\varepsilon}\zeta(t)dt),\varphi(\int_0^{\varepsilon}\zeta(t)dt)) &=\psi(\int_0^{\varepsilon}\zeta(t)dt)-\varphi(\int_0^{\varepsilon}\zeta(t))\\ &=\psi(\int_0^{\varepsilon}2tdt)-\varphi(\int_0^{\varepsilon}2tdt)\\ &=\varepsilon^2-\frac{\varepsilon^2}{2}>0\end{aligned}$$

for all $\varepsilon>0$, T satisfies the inequalities (2.1).

$$\frac{100}{4(121)}|u-v|^2\le\frac{4\times100}{121}|u-v|^2 \quad \forall u,v\in R$$

T has a unique fixed point $u=0$.

Theorem 2.4 *Let* (X,S) *be a complete* S*-metric space* $\psi:[0,\infty)\to[0,\infty)$ *is an altering distance function,* $\varphi\in\Phi_u$ *and* $F\in\mathcal{C}$*, the function* $\zeta:P\to P$ *be defined as for each* $\varepsilon?0,\int_0^{\varepsilon}\zeta(t)dt?0$ *and* $T:X\to X$ *be a self-mapping of* X *such that*

$$\begin{aligned}\psi\left(\int_0^{S(Tu,Tu,Tv)}\zeta(t)dt\right) \leq F\Big(&\psi\Big(h_1\int_0^{S(u,u,v)}\zeta(t)dt+h_2\int_0^{S(Tu,Tu,v)}\zeta(t)dt+h_3\int_0^{S(Tv,Tv,u)}\zeta(t)dt\\ &+h_4\int_0^{\max\{S(Tu,Tu,v),S(Tv,Tv,u)\}}\zeta(t)dt\Big),\varphi\Big(h_1\int_0^{S(u,u,v)}\zeta(t)dt+h_2\\ &\int_0^{S(Tu,Tu,v)}\zeta(t)dt+h_3\int_0^{S(Tv,Tv,u)}\zeta(t)dt+h_4\int_0^{\max\{S(Tu,Tu,v),S(Tv,Tv,u)\}}\zeta(t)dt\Big)\Big)\end{aligned} \tag{2.12}$$

for all $u,v\in X$ with non negative real numbers $h_i(i\in\{1,2,3,4\})$ satisfying $\max\{h_1+3h_3+2h_4,h_1+h_2+h_3\}=1$. Then T has a unique fixed point $w\in X$ and we have $\lim_{n\to\infty}T^n u=w$, for each $u\in X$.

Proof. Let $u_0\in X$ and the sequence $\{u_n\}$ be defined as $\lim_{n\to\infty}T^n u_0=u_n$ Suppose that $u_n\neq u_{n+1}$ for all n. Using the inequality (2.12), the condition (S2) and Lemma 1.3 we get

$$\psi\left(\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt\right)=\psi\left(\int_0^{S(Tu_{n-1},Tu_{n-1},Tu_n)}\zeta(t)dt\right)$$

$$\begin{aligned}&\leq F\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt+h_2\int_0^{S(u_n,u_n,u_n)}\zeta(t)dt\\ &+h_3\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big),\end{aligned}$$

$$\begin{aligned}&\varphi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt+h_2\int_0^{S(u_n,u_n,u_n)}\zeta(t)dt\\ &+h_3\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big)\Big)\end{aligned}$$

$$\begin{aligned}&=F\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt+h_3\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt\\ &+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big),\varphi\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt\\ &+h_3\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big)\Big)\end{aligned}$$

$$\leq F(\psi(h_1 \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt + h_3 \int_0^{2S(u_{n+1},u_{n+1},u_n)} \zeta(t)dt$$
$$+ h_3 \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt + h_4 \int_0^{S(u_{n+1},u_{n+1},u_{n-1})} \zeta(t)dt$$

$$+ h_4 \int_0^{S(u_{n+1},u_{n+1},u_n)} \zeta(t)dt), \varphi(h_1 \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt$$
$$+ h_3 \int_0^{2S(u_{n+1},u_{n+1},u_n)} \zeta(t)dt + h_3 \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt$$
$$+ h_4 \int_0^{S(u_{n+1},u_{n+1},u_{n-1})} \zeta(t)dt + h_4 \int_0^{S(u_{n+1},u_{n+1},u_n)} \zeta(t)dt))$$

$$= F(\psi((h_1 + h_3 + h_4) \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt + (2h_3 + h_4) \int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt),$$
$$\varphi(h_1 + h_3 + h_4) \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt + (2h_3 + h_4) \int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt))$$

$$\text{whichimplies} \int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt \leq \frac{h_1 + h_3 + h_4}{1 - 2h_3 - h_4} \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt = \int_0^{S(u_{n-1},u_{n-1},u_n)} \zeta(t)dt \qquad (2.13)$$

Since $\int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt > 0$,

so there exists $r \geq 0$ such that $\lim_{n\to+\infty} \int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt = r$.

If $r > 0$, then take limit for $n \to \infty$, we get

$$\psi(r) \leq F(\psi(r), \varphi(r))$$

So $\psi(r) = 0$ or $\varphi(r) = 0$. Thus $r = 0$, which is a contradiction. Thus, we conclude that $r = 0$, that is, $\lim_{n\to\infty} \int_0^{S(u_n,u_n,u_{n+1})} \zeta(t)dt = 0,$ since for each $\varepsilon > 0, \int_0^{\varepsilon} \zeta(t)dt > 0,$ implies

$$\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0.$$

By the similar arguments used in the proof of Theorem (2.1), we see that the sequence $\{u_n\}$ is Cauchy. Then there exists $w \in X$ such that $\lim_{n\to\infty} T^n u_0 = w,$ since (X, S) is a complete S -metric space. From the inequality (2.12) we find

$$\begin{aligned}\psi\left(\int_0^{S(u_n,u_n,Tw)}\zeta(t)dt\right) &= \psi\left(\int_0^{S(Tu_{n-1},Tu_{n-1},Tw)}\zeta(t)dt\right)\\ &\le F\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},Tw)}\zeta(t)dt+h_2\int_0^{S(u_n,u_n,w)}\zeta(t)dt\\ &\quad +h_3\int_0^{S(Tw,Tw,u_{n-1})}\zeta(t)dt+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(Tw,Tw,w)\}}\zeta(t)dt\Big),\end{aligned}$$

$$\varphi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},Tw)}\zeta(t)dt+h_2\int_0^{S(u_n,u_n,w)}\zeta(t)dt+h_3\int_0^{S(Tw,Tw,u_{n-1})}\zeta(t)dt$$

$$+h_4\int_0^{\max\{S(u_n,u_n,u_{n-1}),S(Tw,Tw,w)\}}\zeta(t)dt\Big)\Big)$$

Taking limit for $n\to\infty$ and using Lemma 1.3 we get

$$\begin{aligned}\psi\left(\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right) &\le F\left(\psi\left((h_3+h_4)\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right),\varphi\left((h_3+h_4)\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right)\right)\\ &\le \psi\left((h_3+h_4)\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right)\le\psi\left(\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right)\end{aligned}$$

So $\psi\left((h_3+h_4)\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right)=0$ or $\varphi\left((h_3+h_4)\int_0^{S(Tw,Tw,w)}\zeta(t)dt\right)=0$.

Thus $\int_0^{S(Tw,Tw,w)}\zeta(t)dt=0$, which implies that $S(Tw,Tw,w)=0$. Thus $Tw=w$. Now we show the uniqueness of the fixed point. Let w_1 be another fixed point of T. Using the inequality (2.12) and Lemma 1.3, we get

$$\begin{aligned}\psi\left(\int_0^{S(w,w,w_1)}\zeta(t)dt\right) &= \psi\left(\int_0^{S(Tw,Tw,w_1)}\zeta(t)dt\right.\\ &\le F\Big(\psi\Big(h_1\int_0^{S(w,w,w_1)}\zeta(t)dt+h_2\int_0^{S(w,w,w_1)}\zeta(t)dt+h_3\int_0^{S(w_1,w_1,w)}\zeta(t)dt\\ &\quad +h_4\int_0^{\max\{S(w,w,w),S(w_1,w_1,w_1)\}}\zeta(t)dt\Big),\varphi\Big(h_1\int_0^{S(w,w,w_1)}\zeta(t)dt\\ &\quad +h_2\int_0^{S(w,w,w_1)}\zeta(t)dt+h_3\int_0^{S(w_1,w_1,w)}\zeta(t)dt\\ &\quad +h_4\int_0^{\max\{S(w,w,w),S(w_1,w_1,w_1)\}}\zeta(t)dt\Big)\Big)\end{aligned}$$

which implies

$$\begin{aligned}\psi(\int_0^{S(w,w,w_1)}\zeta(t)dt) &\le F(\psi((h_1+h_2+h_3)\int_0^{S(w,w,w_1)}\zeta(t)dt),\varphi((h_1+h_2+h_3)\int_0^{S(w,w,w_1)}\zeta(t)dt))\\ &\le \psi((h_1+h_2+h_3)\int_0^{S(w,w,w_1)}\zeta(t)dt)\\ &\le \psi(\int_0^{S(w,w,w_1)}\zeta(t)dt)\end{aligned}$$

So $\psi((h_1+h_2+h_3)\int_0^{S(w,w,w_1)}\zeta(t)dt)=0$ or $\varphi((h_1+h_2+h_3)\int_0^{S(w,w,w_1)}\zeta(t)dt)=0$. Then we obtain

$$\int_0^{S(w,w,w_1)}\zeta(t)dt=0$$

that is, $w=w_1$ since $h_1+h_2+h_3<1$. Consequently, T has a unique fixed point $w\in X$.

Choosing $F(s,t)=hs$, $0<h<1$, $\psi(t)=t$, (replace h_i with hh_i) in Theorem (2.4) we have

Corollary 2.5 *[4]Let (X,S) be a complete S-metric space $h\varepsilon(0,1)$, the function $\zeta:P\to P$ be defined as for each $\varepsilon?0,\int_0^{\varepsilon}\zeta(t)dt?0$ and $T:X\to X$ be a self-mapping of X such that*

$$\begin{aligned}\int_0^{S(Tu,Tu,Tv)}\zeta(t)dt &\le h_1\int_0^{S(u,u,v)}\zeta(t)dt+h_2\int_0^{S(Tu,Tu,v)}\zeta(t)dt\\ &+h_3\int_0^{S(Tv,Tv,u)}\zeta(t)dt+h_4\int_0^{\max\{S(Tu,Tu,v),S(Tv,Tv,u)\}}\zeta(t)dt\end{aligned}$$

for all $u,v\in X$ with non negative real numbers $h_i(i\in\{1,2,3,4\})$ satisfying $\max\{h_1+3h_3+2h_4,h_1+h_2+h_3\}<1$. Then T has a unique fixed point $w\in X$ and we have $\lim_{n\to\infty}T^nu=w$, for each $u\in X$.

Example 2.6 *Let $X=R$ be the complete S-metric space with S-metric space defined in example (2.3). Let us define the self mapping $T:R\to R$ as*

$$Tu=\begin{cases}2u+39 & u\in(0,3)\\ 90 & \text{otherwise}\end{cases}$$

for all $u\in R$ and define a function $\zeta:P\to P$ where $P=(0,\infty)$ as $\zeta(t)=2t$

$$\int_0^{\varepsilon}\zeta(t)dt=\int_0^{\varepsilon}2tdt=\varepsilon^2>0\ \ \varepsilon>0.$$

T satisfy the inequality (2.12) in theorem (2.4) for $h_1=h_2=h_3=0,h_4=\frac{1}{2}$ and the inequality (??) in theorem (2.7) for $h_1=h_3=h_5=0,h_2=\frac{1}{3}$. Hence T has a unique fixed point 90. But

T does not satisfy the inequality (2.1) in theorem (2.1). Indeed, if we take $u=0$ and $v=1$, then we obtain

$$\psi(\int_0^{10} 2tdt) = 100 \le F(\psi(h\int_0^{3} 2tdt), \varphi(h\int_0^{3} 2tdt)) \le \psi(h\int_0^{3} 2tdt) \le 9h$$

which is a contradiction since $h \in (0,1)$

Theorem 2.7 *Let* (X,S) *be a complete* S*-metric space* $\psi:[0,\infty)\to[0,\infty)$ *is an altering distance function,* $\varphi \in \Phi_u$ *and* $F \in C$*, the function* $\zeta : P \to P$ *be defined as for each* $\varepsilon ?0, \int_0^{\varepsilon} \zeta(t)dt ?0$ *and* $T : X \to X$ *be a self-mapping of* X *such that*

$$\begin{aligned}
&\psi(\int_0^{S(Tu,Tu,Tv)} \zeta(t)dt) \le F(\psi(h_1 \int_0^{S(u,u,v)} \zeta(t)dt + h_2 \int_0^{S(Tu,Tu,u)} \zeta(t)dt + h_3 \\
&\int_0^{S(Tu,Tu,v)} \zeta(t)dt + h_4 \int_0^{S(Tv,Tv,u)} \zeta(t)dt + h_5 \int_0^{S(Tv,Tv,v)} \zeta(t)dt \\
&+ h_6 \int_0^{\max\{S(u,u,v),S(Tu,Tu,u),S(Tu,Tu,v),S(Tv,Tv,u),S(Tv,Tv,v)\}} \zeta(t)dt), \\
&\varphi(h_1 \int_0^{S(u,u,v)} \zeta(t)dt + h_2 \int_0^{S(Tu,Tu,u)} \zeta(t)dt + h_3 \int_0^{S(Tu,Tu,v)} \zeta(t)dt \\
&+ h_4 \int_0^{S(Tv,Tv,u)} \zeta(t)dt + h_5 \int_0^{S(Tv,Tv,v)} \zeta(t)dt + h_6 \\
&\int_0^{\max\{S(u,u,v),S(Tu,Tu,u),S(Tu,Tu,v),S(Tv,Tv,u),S(Tv,Tv,v)\}} \zeta(t)dt))
\end{aligned} \tag{2.14}$$

for all $u,v \in X$ with non negative real numbers $h_i (i \in \{1,2,3,4,5,6\})$ satisfying $h_1 + h_2 + 3h_4 + h_5 + 3h_6, h_1 + h_3 + h_4 + h_6 = 1$, Then T has a unique fixed point $w \in X$ and we have $\lim_{n\to\infty} T^n u = w$, for each $u \in X$.

Proof. Let $u_0 \in X$ and the sequence $\{u_n\}$ be defined as $\lim_{n\to\infty} T^n u_0 = u_n$ Suppose that $u_n \ne u_{n+1}$ for all n. Using the inequality (2.12), the condition (S2) and Lemma 1.3 we get

$$\begin{aligned}\psi\Big(\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt\Big) &= \psi\Big(\int_0^{S(Tu_{n-1},Tu_{n-1},Tu_n)}\zeta(t)dt\Big)\\ &\le F\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt + h_2\int_0^{S(u_n,u_n,u_{n-1})}\zeta(t)dt\\ &+ h_3\int_0^{S(u_n,u_n,u_n)}\zeta(t)dt + h_4\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt + h_5\int_0^{S(u_{n+1},u_{n+1},u_n)}\zeta(t)dt + h_6\\ &\int_0^{\max\{S(u_{n-1},u_{n-1},u_n),S(u_n,u_n,u_{n-1}),S(u_n,u_n,u_n),S(u_{n+1},u_{n+1},u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big),\end{aligned}$$

$$\begin{aligned}&\varphi\Big(\psi\Big(h_1\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt + h_2\int_0^{S(u_n,u_n,u_{n-1})}\zeta(t)dt + h_3\int_0^{S(u_n,u_n,u_n)}\zeta(t)dt\\ &+ h_4\int_0^{S(u_{n+1},u_{n+1},u_{n-1})}\zeta(t)dt + h_5\int_0^{S(u_{n+1},u_{n+1},u_n)}\zeta(t)dt + h_6\\ &\int_0^{\max\{S(u_{n-1},u_{n-1},u_n),S(u_n,u_n,u_{n-1}),S(u_n,u_n,u_n),S(u_{n+1},u_{n+1},u_{n-1}),S(u_{n+1},u_{n+1},u_n)\}}\zeta(t)dt\Big)\Big)\Big)\end{aligned}$$

$$\begin{aligned}&\le F\Big(\psi\Big((h_1+h_2+h_4+h_6)\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt + (2h_4+h_5+2h_6)\\ &\int_0^{S(u_{n+1},u_{n+1},u_n)}\zeta(t)dt\Big), \varphi\Big((h_1+h_2+h_4+h_6)\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt\\ &+(2h_4+h_5+2h_6)\int_0^{S(u_{n+1},u_{n+1},u_n)}\zeta(t)dt\Big)\Big)\end{aligned}$$

which implies

$$\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt \le \Big(\frac{h_1+h_2+h_4+h_6}{1-2h_4-h_5-2h_6}\Big)\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt = h\int_0^{S(u_{n-1},u_{n-1},u_n)}\zeta(t)dt \qquad (2.15)$$

Since $\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt > 0$, so there exists $r \ge 0$ such that

$$\lim_{n\to+\infty}\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt = r.$$

If $r > 0$, then taking limit for $n \to \infty$, we get $\psi(r) \le F(\psi(r),\varphi(r))$

So $\psi(r) = 0$ or $\varphi(r) = 0$. Thus $r = 0$, which is a contradiction. Thus, we conclude that $r = 0$, that is,

$$\lim_{n\to\infty}\int_0^{S(u_n,u_n,u_{n+1})}\zeta(t)dt = 0,$$

since for each $\varepsilon > 0, \int_0^{\varepsilon} \zeta(t)dt > 0,$ implies $\lim_{n\to\infty} S(u_n, u_n, u_{n+1}) = 0.$

By the similar arguments used in the proof of Theorem (2.1), we see that the sequence $\{u_n\}$ is Cauchy. Then there exists $w \in X$ such that $\lim_{n\to\infty} T^n u_0 = w,$ since (X,S) is a complete S-metric space. From the inequality (??) we find

$$\begin{aligned}
\psi\left(\int_0^{S(u_n,u_n,Tw)} \zeta(t)dt\right) &= \psi\left(\int_0^{S(Tu_{n-1},Tu_{n-1},Tw)} \zeta(t)dt\right. \\
&\leq F(\psi(h_1 \int_0^{S(u_{n-1},u_{n-1},w)} \zeta(t)dt + h_2 \int_0^{S(u_n,u_n,u_{n-1})} \zeta(t)dt \\
&+ h_3 \int_0^{S(u_n,u_n,w)} \zeta(t)dt + h_4 \int_0^{S(Tw,Tw,u_{n-1})} \zeta(t)dt + h_5 \int_0^{S(Tw,Tw,w)} \zeta(t)dt \\
&+ h_6 \int_0^{\max\{S(u_{n-1},u_{n-1},w), S(u_n,u_n,u_{n-1}), S(u_n,u_n,w), S(Tw,Tw,u_{n-1}), S(Tw,Tw,w)\}} \zeta(t)dt),
\end{aligned}$$

$$\begin{aligned}
&\varphi(h_1 \int_0^{S(u_{n-1},u_{n-1},w)} \zeta(t)dt + h_2 \int_0^{S(u_n,u_n,u_{n-1})} \zeta(t)dt \\
&+ h_3 \int_0^{S(u_n,u_n,w)} \zeta(t)dt + h_4 \int_0^{S(Tw,Tw,u_{n-1})} \zeta(t)dt + h_5 \int_0^{S(Tw,Tw,w)} \zeta(t)dt \\
&+ h_6 \int_0^{\max\{S(u_{n-1},u_{n-1},w), S(u_n,u_n,u_{n-1}), S(u_n,u_n,w), S(Tw,Tw,u_{n-1}), S(Tw,Tw,w)\}} \zeta(t)dt))
\end{aligned}$$

If we take limit for $n \to \infty$, using Lemma 1.3 we get

$$\begin{aligned}
\psi\left(\int_0^{S(Tw,Tw,w)} \zeta(t)dt\right) &\leq F\left(\psi((h_4+h_5+h_6)\int_0^{S(Tw,Tw,w)} \zeta(t)dt), \varphi((h_4+h_5+h_6)\int_0^{S(Tw,Tw,w)} \zeta(t)dt)\right) \\
&\leq \psi\left((h_4+h_5+h_6)\int_0^{S(Tw,Tw,w)} \zeta(t)dt\right) \\
&\leq \psi\left(\int_0^{S(Tw,Tw,w)} \zeta(t)dt\right)
\end{aligned}$$

So $\psi((h_4+h_5+h_6)\int_0^{S(Tw,Tw,w)} \zeta(t)dt) = 0$ or $\varphi((h_4+h_5+h_6)\int_0^{S(Tw,Tw,w)} \zeta(t)dt) = 0$. Thus $\int_0^{S(Tw,Tw,w)} \zeta(t)dt = 0$, which implies that $S(Tw,Tw,w) = 0$. Thus $Tw = w$ Now we show the uniqueness of the fixed point. Let w_1 be another fixed point of T. Using the inequality (??) and Lemma 1.3, we get

$$\begin{aligned}
\psi\left(\int_0^{S(w,w,w_1)} \zeta(t)dt\right) &= \psi\left(\int_0^{S(Tw,Tw,Tw_1)} \zeta(t)dt\right) \\
&\leq F\left(\psi\left(h_1 \int_0^{S(w,w,w_1)} \zeta(t)dt + h_2 \int_0^{S(w,w,w)} \zeta(t)dt\right.\right. \\
&\quad + h_3 \int_0^{S(w,w,w_1)} \zeta(t)dt + h_4 \int_0^{S(w_1,w_1,w)} \zeta(t)dt + h_5 \int_0^{S(w_1,w_1,w_1)} \zeta(t)dt \\
&\quad \left. + h_6 \int_0^{\max\{S(w,w,w_1),S(w,w,w),S(w,w,w_1),S(w_1,w_1,w),S(w_1,w_1,w_1)\}} \zeta(t)dt\right), \\
&\quad \varphi\left(h_1 \int_0^{S(w,w,w_1)} \zeta(t)dt + h_2 \int_0^{S(w,w,w)} \zeta(t)dt\right. \\
&\quad + h_3 \int_0^{S(w,w,w_1)} \zeta(t)dt + h_4 \int_0^{S(w_1,w_1,w)} \zeta(t)dt + h_5 \int_0^{S(w_1,w_1,w_1)} \zeta(t)dt \\
&\quad \left.\left. + h_6 \int_0^{\max\{S(w,w,w_1),S(w,w,w),S(w,w,w_1),S(w_1,w_1,w),S(w_1,w_1,w_1)\}} \zeta(t)dt\right)\right)
\end{aligned}$$

which implies

$$\begin{aligned}
\psi\left(\int_0^{S(w,w,w_1)} \zeta(t)dt\right) &\leq F\left(\psi\left((h_1+h_3+h_4+h_6) \int_0^{S(w,w,w_1)} \zeta(t)dt\right), \varphi\left((h_1+h_3+h_4+h_6) \int_0^{S(w,w,w_1)} \zeta(t)dt\right)\right) \\
&\leq \psi\left((h_1+h_3+h_4+h_6) \int_0^{S(w,w,w_1)} \zeta(t)dt\right) \leq \psi\left(\int_0^{S(w,w,w_1)} \zeta(t)dt\right)
\end{aligned}$$

So $\psi\left((h_1+h_3+h_4+h_6) \int_0^{S(w,w,w_1)} \zeta(t)dt\right) = 0$ or $\varphi\left((h_1+h_3+h_4+h_6) \int_0^{S(w,w,w_1)} \zeta(t)dt\right) = 0$. Then we obtain

$$\int_0^{S(w,w,w_1)} \zeta(t)dt = 0$$

that is, $w = w_1$ since $h_1+h_3+h_4+h_6 < 1$. Consequently, T has a unique fixed point $w \in X$.

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