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PATH RELATED MEAN CORDIAL GRAPHS

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Abstract: Let G = (V, E) be a simple graph. G is said to be a mean cordial graph if f : V(G)→{0,1,2} such that for each edge uv the induced map f* defined by f*(uv) = $\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where $\lfloor x \rfloor$ denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is no.of edges with zero label. $e_f(1)$ is no.of edges with one label.

The graph that admits a mean cordial labeling is called a mean cordial graph(MCG).In this paper , we proved that $P_n \odot K_1$, (P_n+K_1) , $P_n \times P_n$,$(P_n :C_3)$,$(P_n: S_1)$, $P_n \times P_2$,$P_n +2K_1$

Z-(P_n) are mean cordial graphs.

Keywords : Mean cordial labeling, Mean cordial graph.

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1. INTRODUCTION:

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each e = {uv} of vertices in E is called an edge or a line of G. For graph theoretical Terminology we follow

2. PRELIMINARIES:

We define the concept of mean cordial labeling as follows. Let G = (V, E) be a simple graph. G is said to be a mean cordial graph if f : V(G)→{0,1,2}

Such that for each edge uv the induced map f* defined by f*(uv) = $\left\lfloor \frac{f(u)+f(v)}{2} \right\rfloor$ where $\lfloor x \rfloor$ denote the least integer which is ≤ x and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ is no.of edges with zero label. $e_f(1)$ is no.of edges with one label.

A graph that admits a mean cordial labeling is called a mean cordial graph. We proved that $P_n \odot K_1$,($P_n +K_1$), $P_n \times P_n$, (P_n :C_3), (P_n : S_1), $P_n \times P_2$, $P_n +2K_1$,Z-(P_n) are mean cordial graphs.

Definition 2.1(Comb)

The ***Corona*** $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a ***comb.***

Definition 2.2(Fan)

The ***join*** $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2 as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup [\, uv : u \in V(G_1)$ and $v \in V(G_2)\,]$. The graph $P_n + K_1$ is called a ***Fan.***

Definition 2.3(Grid)

To define the ***product*** $G_1 \text{ x } G_2$, consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 adj v_2) or ($u_2 = v_2$ and u_1 adj v_1). The product $P_m \times P_n$ is called ***planar grids.***

Definition 2.4($P_n : C_3$)

A vertex of cycle C_3 attached and every vertex of a path P_n is denoted by $[P_n : C_3]$.

Definition 2.5(Pn : S1)

Star of length one is joined with every vertex of a path P_n through an edge.

It is denoted by $[P_n : S_1]$.

Definition 2.6[(Z-(Pn)]

In a pair of path P_n i^{th} vertex of a path P_1 is joined with $i+1^{th}$ vertex of a path P_2. It is denoted by Z-(P_n).

Definition 2.7(Double fan)

The ***join*** $G_1 + G_2$ of G_1 and G_2 consists of $G_1 \cup G_2$ and all lines joining V_1 with V_2as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup [\, uv : u \in V(G_1)$ and $v \in V(G_2)\,]$. The graph $P_n + 2K_1$ is called the ***Doublefan.***

Definition 2.8(Ladder)

To define the ***product*** G_1 x G_2, consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 adj v_2) or ($u_2 = v_2$ and u_1 adj v_1). The product $K_2 \times P_n$ is called ***Laddar.***

3. Main Results on Path Related Mean Cordial Graphs

Theorem 3.1

Graph $P_n \odot K_1$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $P_n \odot K_1$

Let $V[P_n \odot K_1] = \{(u_i , v_i): 1\leq i\leq n\}$

Let $E[Pn \odot K1] = \{[(u_i\, u_{i+1}):1\leq i\leq n-1]$ □ $[(u_i\, v_i): 1\leq i\leq n]\}$

Define f: $V(G)\rightarrow\{0,1,2\}$ by

$f(u_i) = 1$

$f(v_i) = 0$

The induced edge labeling are

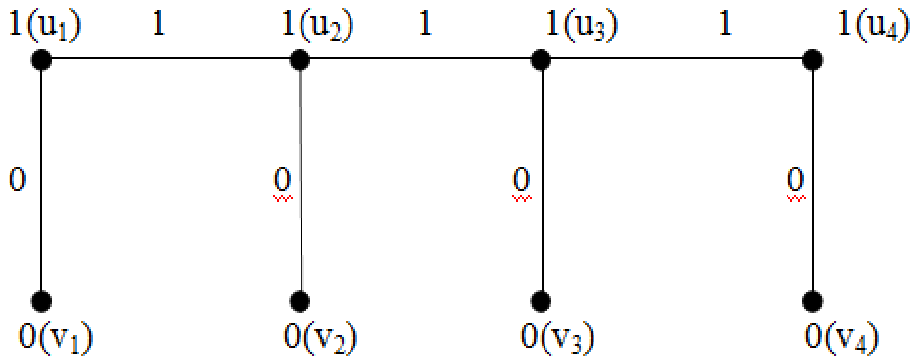
$f^*(u_i\, u_{i+1}) = 1$

$f^*(u_i\, v_i) = 0$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)|\leq 1$.

Therefore the graph $P_n \odot K_1$ is a mean cordial graph.

For example, the graph $P_4 \odot K_1$ is shown in the figure.



Theorem 3.2

Fan $(P_n + K_1)$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $(P_n + K_1)$

Let $V[P_n + K_1] = \{u , u_i : 1 \leq i \leq n\}$

Let $E[P_n + K_1] = \{[(u\, u_i) : 1 \leq i \leq n]$ □ $[(u_i\, u_{i+1}) : 1 \leq i \leq n-1]\}$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

$f(u) = 2$

$$f(u_i) = \begin{cases} 1 \ if \ i \equiv 1 mod 2 \\ 0 \ if \ i \equiv 0 mod 2 \end{cases}, \ 1 \leq i \leq n$$

The induced edge labeling are

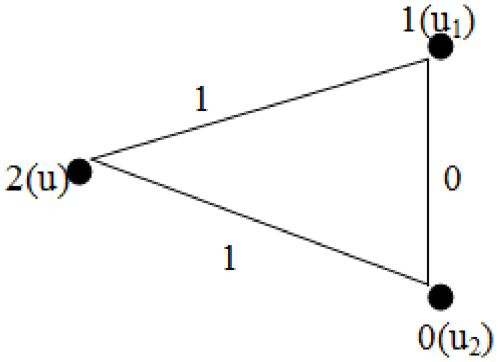
$f^*(u\, u_i) = 1$

$f^*(u_i\, u_{i+1}) = 0$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Therefore the graph $(P_n + K_1)$ is a mean cordial graph.

For example the graph $(P_2 + K_1)$ is shown in the figure



Theorem 3.3

Grid $P_n \times P_n$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $P_n \times P_n$

Let $V[G] = \{u_{ij} : 1 \le i \le n\}$

Let $E[G] = \{[(u_{ij}\, u_{i(j+1)}) : 1 \le i \le n, 1 \le j \le n-1]$ □ $[(u_{ij}\, u_{(i+1)j}) : 1 \le i \le n-1, 1 \le i \le n]$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

The vertex labeling are

When $i \equiv 1 mod 2$

$f(u_{ij}) = 1$, $1 \le i \le n$, $1 \le j \le n$

When $i \equiv 0 mod 2$

$$f(u_{ij}) = \begin{cases} 0 \text{ if } j \equiv 1 mod 2, 1 \le i \le n \\ 1 \text{ if } j \equiv 0 mod 2, 1 \le j \le n \end{cases}$$

The induced edge labelling are

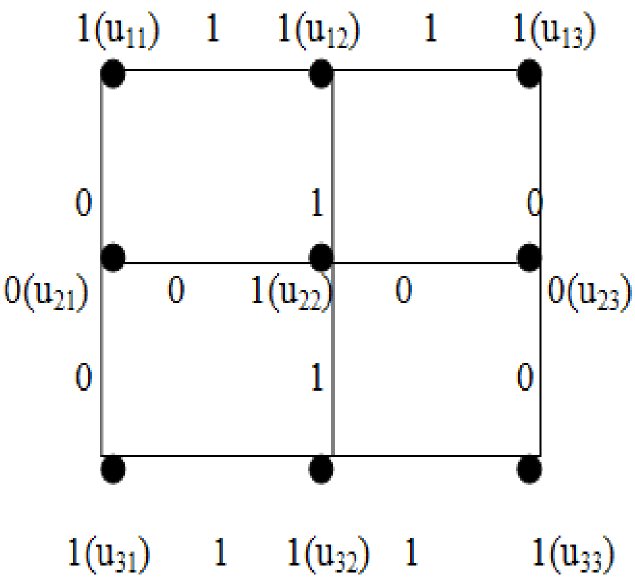
$$f^*(u_{ij}\, u_{i(j+1)}) = \begin{cases} 1 \text{ if } i \equiv 1 mod 2, 1 \le i \le n, 1 \le j \le n-1 \\ 0 \text{ if } i \equiv 0 mod 2, 1 \le i \le n, 1 \le j \le n-1 \end{cases}$$

$$f^*(u_{ij}\, u_{(i+1)j}) = \begin{cases} 0 \text{ if } i \equiv 1 mod 2, 1 \le i \le n-1, 1 \le j \le n \\ 1 \text{ if } i \equiv 0 mod 2, 1 \le i \le n-1, 1 \le j \le n \end{cases}$$

Here the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence , $P_n \times P_n$ is a mean cordial graph.

For example, the graph $P_4 \times P_4$ is shown in the figure.



Theorem 3.4

Graph (P_n **:**C_3**)** is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be (P_n :C_3)

Let V[G] = {u_i :$1\leq i \leq n$, u_{ij} :$1\leq i \leq n$, $1\leq j \leq 2$}

Let E[G] = {[(u_i u_{i+1}):$1\leq i\leq n-1$] □ [(u_i u_{i1}):$1\leq i\leq n$]

□ [(u_i u_{i2}):$1\leq i \leq n$] □ [(u_{i1} u_{i2}):$1\leq i \leq n$]

Define f : V(G)→{0,1,2} by

$$f(u_i) = \begin{cases} 1 \ if \ i \equiv 1 mod 2 \\ 0 \ if \ i \equiv 0 mod 2 \end{cases} ,1\leq i \leq n$$

$f(u_{i1}) = 2$, $1\leq i \leq n$

$f(u_{i2}) = 0$, $1\leq i \leq n$

The induced edge labelling are

$f^*(u_i \ u_{i1}) = 1$,$1 \leq i \leq n$

$f^*(u_i \ u_{i2}) = 0$,$1\leq i \leq n$

$f^*(u_{i1}\, u_{i2}) = 1, 1 \le i \le n$

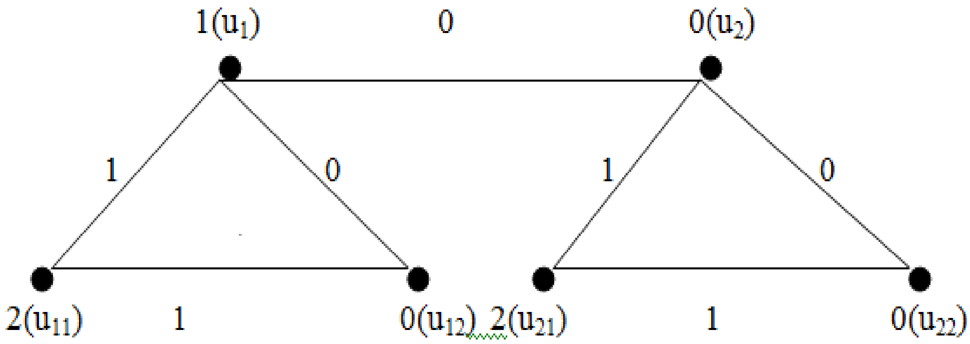
$f^*(u_i\, u_{i+1}) = 0, 1 \le i \le n$

Hence , $e_f(0) + 1 = e_f(1)$

It satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, the graph (P_n :C_3) is a mean cordial graph.

For example, the graph (P_2 :C_3) is shown in the figure.



Theorem 3.5

Graph [P_n : S_1] is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be [P_n : S_1]

Let V[P_n : S_1] = $\{u_i : 1 \le i \le n ; (u_{ij}) : 1 \le i \le n , 1 \le j \le 2\}$

Let E[P_n : S_1] = $\{[(u_i\, u_{i+1}) : 1 \le i \le n-1]$ □ $[(u_i\, u_{i1})$ □ $(u_{i1}\, u_{i2}) : 1 \le i \le n]\}$

Define f : V(G)$\rightarrow\{0,1,2\}$ by

$f(u_i\, u_{i1}) = 1 , 1 \le i \le n$

$f(u_{i1}) = 0 , 1 \le i \le n$

$f(u_{i2}) = \begin{cases} 2\ if\ i \equiv 1 mod 2 \\ 0\ if\ i \equiv\ 0 mod 2 \end{cases}$, $1 \le i \le n$

The induced edge labeling are

$f^*(u_i\, u_{i+1}) = 1, 1 \le i \le n-1$

$f^*(u_i\, u_{i1}) = 0,\ 1 \le i \le n$

$f^*(u_{i1}\, u_{i2}) = \begin{cases} 1 \text{ if } i \equiv 1 mod 2 \\ 0 \text{ if } i \equiv 0 mod 2 \end{cases},\ 1 \le i \le n$

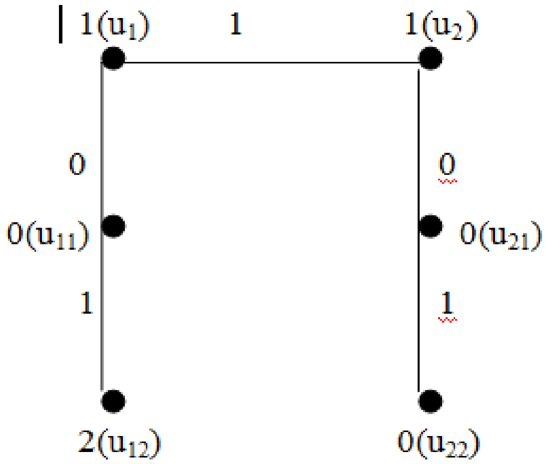
Here , $e_f(1) + 1 = e_f(0)$, if n is even

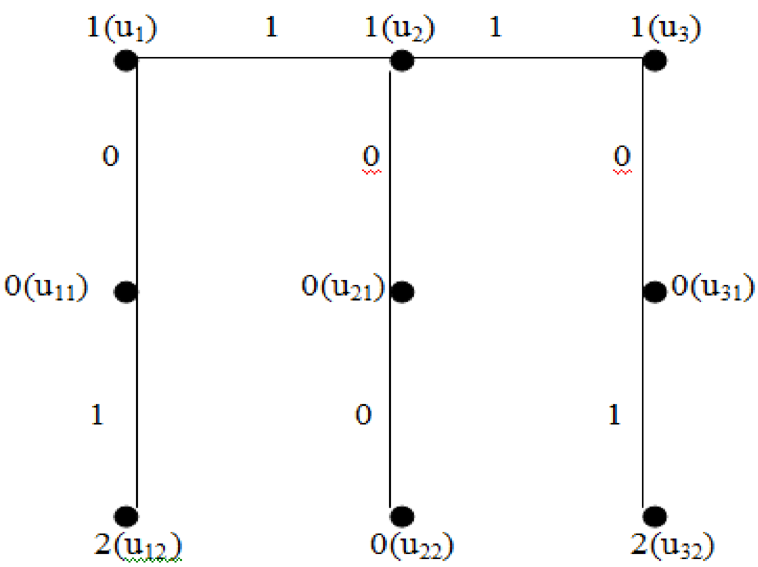
$e_f(1) = e_f(0)$, if n is odd

It satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, $[P_n : S_1]$ is a mean cordial graph.

For example $[P_2 : S_1]$ and $[P_3 : S_1]$ are mean cordial graphs as shown in the figure.





Theorem 3.6

Ladder $[P_n \times P_2]$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $[P_n \times P_2]$

Let $V[P_n \times P_2] = \{(u_{ij}) : 1 \le i \le n , 1 \le j \le 2\}$

Let $E[P_n \times P_2] = \{[(u_{i1}\ u_{i2}) : 1 \le i \le n]$ □ $[(u_{ij}\ u_{(i+1)j}): 1 \le i \le n-1, 1 \le j \le 2]\}$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

$$f(u_{i1}) = \begin{cases} 1 \ if \ i \equiv 1 mod 2 \\ 2 \ if \ i \equiv 0 mod 2 \end{cases}, 1 \le i \le n$$

$f(u_{i2}) = 0,\ 1 \le i \le n$

The induced edge labeling are

$$f^*(u_{i1}\ u_{i2}) = \begin{cases} 0 \ if \ i \equiv 1 mod 2 \\ 1 \ if \ i \equiv 0 mod 2 \end{cases}, 1 \le i \le n$$

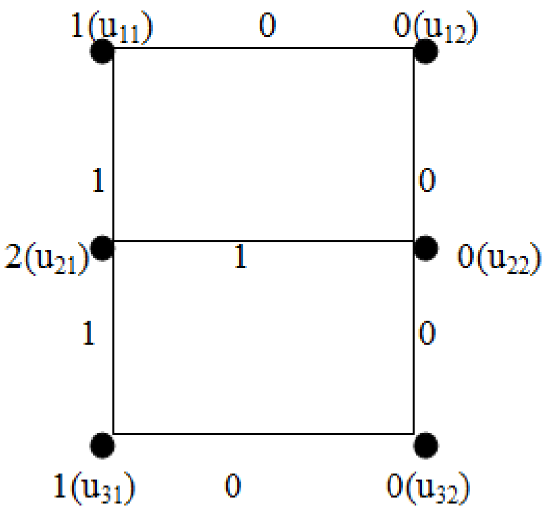
$f^*(u_{i1}\ u_{(i+1)1}) = 1,\ 1 \le i \le n-1$

$f^*(u_{i2}\ u_{(i+1)2}) = 0,\ 1 \le i \le n-1$

Hence , the graph satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Therefore , $[P_n \times P_2]$ is a mean cordial graph.

For example , the graph $[P_3 \times P_2]$ is shown in the figure.



Theorem 3.7

$Z - (P_n)$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $Z - P_n$

Let $V[Z - P_n] = \{u_i, v_i : 1 \leq i \leq n\}$

Let $E[Z - P_n] = \{[(u_i u_{i+1}) \square (v_i v_{i+1}) \square (v_i v_{i+1}) : 1 \leq i \leq n-1]\}$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

$$f(u_i) = \begin{cases} 1 \ if \ i \equiv 1 mod 2 \\ 0 \ if \ i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 2 \ if \ i \equiv 1 mod 2 \\ 0 \ if \ i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n$$

The induced edge labeling are

$f^*(u_i u_{i+1}) = 0$, $1 \leq i \leq n-1$

$f^*(v_i v_{i+1}) = 1$, $1 \leq i \leq n-1$

$$f^*(v_i u_{i+1}) = \begin{cases} 1 \ if \ i \equiv 1 mod 2 \\ 0 \ if \ i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n-1$$

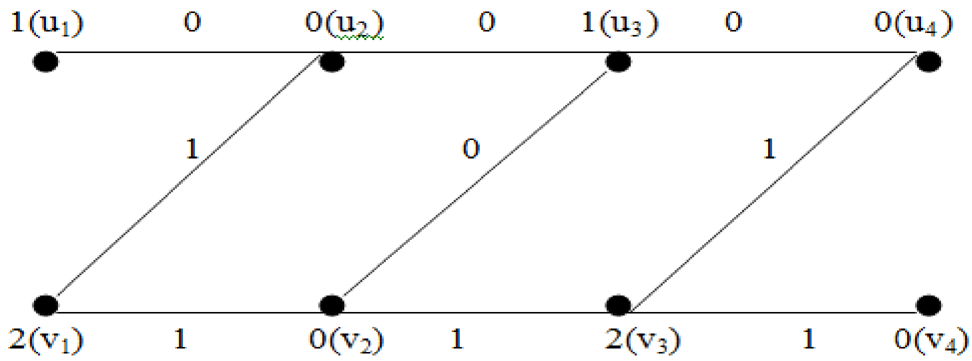
Here , $e_f(0) = e_f(1)$ when n is odd.

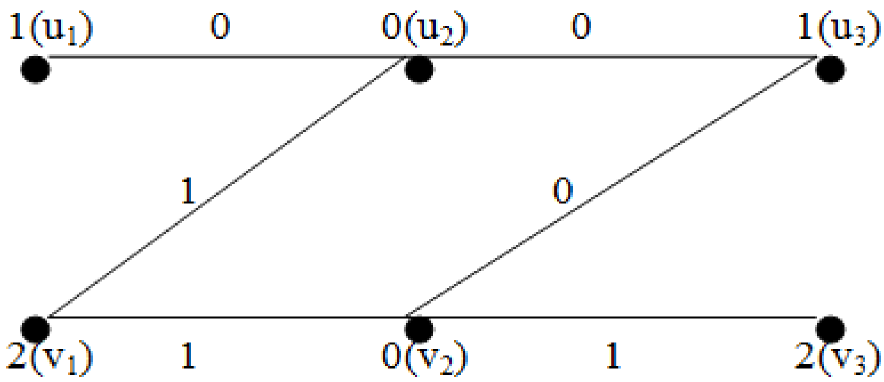
$e_f(0) + 1 = e_f(1)$ when n is even.

It satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence , $Z - (P_n)$ is a mean cordial graph.

For example the graphs $Z - (P_4)$ and $Z - (P_3)$ are shown in the figure.





Theorem 3.8

Double Fan $P_n + 2K_1$ is a Mean Cordial Graph.

Proof:

Let G = (V, E)

Let G be $P_n + 2K_1$

Let $V[P_n + 2K_1] = \{u, v, u_i : 1 \leq i \leq n\}$

Let $E[P_n + 2K_1] = \{[(u\ u_i)$ □ $(v\ u_i) : 1 \leq i \leq n]$ □ $[(u_i\ u_{i+1}) : 1 \leq i \leq n-1]$

Define $f : V(G) \rightarrow \{0,1,2\}$ by

$f(u) = 1$

$f(v) = 2$

$$f(u_i) = \begin{cases} 0 \; if \; i \equiv 1 mod 2 \\ 1 \; if \; i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n$$

The induced edge labeling are

$$f^*(u\, u_i) = \begin{cases} 0 \; if \; i \equiv 1 mod 2 \\ 1 \; if \; i \equiv 0 mod 2 \end{cases}, 1 \leq i \leq n$$

$f^*(v\, u_i) = 1$,$1 \leq i \leq n$

$f^*(u_i\, u_{i+1}) = 0$,$1 \leq i \leq n-1$

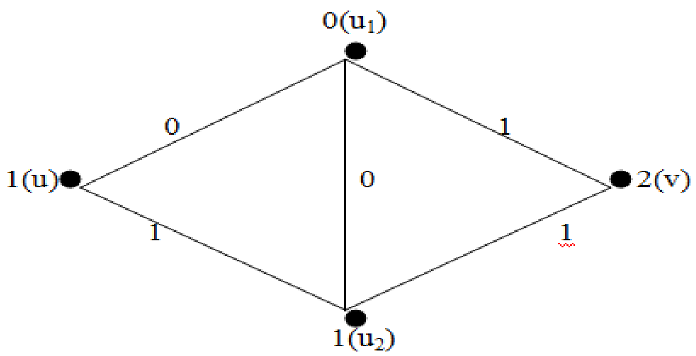
Here $e_f(0) = e_f(1)$ when n is odd.

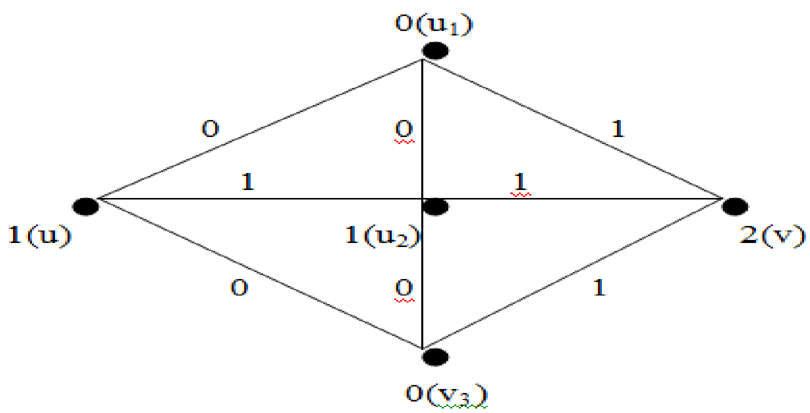
$e_f(0) + 1 = e_f(1)$ when n is even.

Hence it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Therefore the graph $P_n + 2K_1$ is a mean cordial graph.

For example $P_2 + 2K_1$ and $P_3 + 3K_1$ are shown in the figure.





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