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NEW SIMILARITY MEASURES OF INTERVAL VALUED FUZZY SOFT SET AND ITS APPLICATION

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Abstract: In this paper, we proposed the concept of reduced fuzzy soft set of interval valued fuzzy soft set and also defined different types of reduction along with it. Then based on these reduction, three types of similarity measures are done with example .Also we developed an algorithm which is a new approach in the medical diagnosis field by employing reduced fuzzy soft set of interval valued fuzzy soft set.

Keywords: Soft set, fuzzy soft set, interval valued fuzzy soft set, reduced fuzzy soft set.

1. Introduction

There are many uncertain problems in our real life. In literature, theories like probability, fuzzy sets[28],intuitionist fuzzy sets[2,3,4],rough sets, interval mathematics are dealing with uncertain data. But these theories have their own difficulties as pointed out in [17]. Soft set theory has received much attention since its introduction by Molodtsov[17]. The concept and basic properties of soft set theory are presented in [17,14]. Theory of fuzzy soft set, generalised fuzzy soft sets, intuitionistic fuzzy soft sets have been studied by many authors[13,15,16]. Yang et al[26] presented the concept of the interval valued fuzzy soft sets by combining the interval valued fuzzy sets and soft set.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties. Fuzzy matrices play a vital role in scientific Development. In [27], Yong et al initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. In[5] Borah et al and in[18,9] Neog et al extended fuzzy soft matrix theory and its application. In [7], Chetia et al proposed intuitionistic fuzzy soft matrix theory. Accordingly, Rajarajeswari et al proposed new definitions for intuitionistic fuzzy soft matrices, interval valued intuitionistic fuzzy soft matrices and its application in[6,19,22,25].

Similarity measure have extensive application in pattern recognition, region extraction, coding theory, image processing and in many other areas. Similarity of two sets have been studied by Majumder and Samantha in[10,11,12]. D.K.Sut et al [8] and Rajarajeswari et al [20,21] used the notion of similarity measure in [11] to make decision. Q.Feng et al [9] studied new similarity measures of fuzzy soft sets based on distance measure. Cagman et al [6] and Rajarajeswari et al [24] studied similarity measure of intuitionistic fuzzy soft sets. Similarity measures of interval valued fuzzy soft sets and their applications are dicussed in [1].

In this paper, we have introduced the concept of reduced fuzzy soft set of interval valued fuzzy soft set and defined different types of reduction along with it. Then based on these reduction, three types of similarity measures are done with example .Also we are developing an algorithm which is a new approach in the medical diagnosis field by employing reduced fuzzy soft set of interval valued fuzzy soft set.

2. PRELIMINARIES

In this section, we recall some basic notion of fuzzy soft set theory.

2.1.Soft set [17] Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F,E) is called a soft set over U where F is a mapping given by F: $E \rightarrow P(U)$. Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

2.2. Fuzzy soft set [15]

Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: $A \rightarrow I^{U}$, where I^{U} denotes the collection of all fuzzy subsets of U.

2.3.Interval valued Fuzzy soft set[26]

Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: $A \rightarrow I^{U}$, where I^{U} denotes the collection of all interval valued fuzzy subsets of U.

2.4.Intuitionistic Fuzzy soft set (IFSS)[16]

Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F,A) is called intuitionistic fuzzy soft set over U where F is a mapping given by F: $A \rightarrow I^{U}$, where I^{U} denotes the collection of all intuitionistic fuzzy subsets of U.

3.Reduced Fuzzy Soft set(RFSs) of Interval Valued Fuzzy Soft set(IVFSs)

In this section, we propose an adjustable approach to Interval valued Fuzzy Soft set as Reduced Fuzzy Soft set. Here, we present the idea to convert interval-valued fuzzy membership into one fuzzy value. For this we define Reduced fuzzy soft set.

Definition

Let $U=\{c_1,c_2,c_3...c_m\}$ be an Universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3...e_n\}$. Let $A \subseteq E$ and (F,A) be a interval valued fuzzy soft set over U, where F is a mapping given by F: $A \rightarrow I^U$, where I^U denotes the collection of all interval valued fuzzy subsets of U. Then the interval valued fuzzy soft set is

$$F(e_j) = c_i, \begin{bmatrix} \mu_{jL} & c_i \end{bmatrix}, \mu_{jU} & c_i \end{bmatrix}, \forall e_j \in A \text{ and } \forall c_i \in U$$

 $\begin{bmatrix} \mu_{jL} & c_i \end{bmatrix}$ Represents the membership of c_i in the interval valued fuzzy set F(e_j).

Let $w_1, w_2 \in [0,1], w_1 + w_2 = 1$.

The vector $W = (w_1, w_2)$ is called weighted vector.

The fuzzy soft set (F_W, A) over U such that

$$F_{w}(e_{j}) = (c_{i}, w_{1}\mu_{jL} \ c_{i} + w_{2}\mu_{jU} \ c_{i}), \forall c_{i} \in U, \forall e_{j} \in A$$

is called Reduced Fuzzy Soft set(RFSs) of the interval valued fuzzy soft set with respect to the weighted vector.

By adjusting the value of w_1 and w_2 an interval valued fuzzy soft set can be converted into reduced fuzzy soft set.

By adjusting the value of

 w_1 and w_2 an interval valued fuzzy soft set can be converted into any reduced fuzzy soft set.

Particularly, let $w_1 = 1$, $w_2 = 0$, $w_1 = 0$, $w_2 = 1$, and $w_1 = w_2 = 0.5$, respectively will have three reduced fuzzy soft set i.e. pessimistic reduced fuzzy soft set, optimistic reduced fuzzy soft set, neutral reduced fuzzy soft set. They are defined as

$$F_P(e_j) = (c_i, \mu_{jL} \ c_i), \forall c_i \in U, \forall e_j \in A$$

$$F_{O}(e_{j}) = (c_{i}, \mu_{jU} \ c_{i}), \forall c_{i} \in U, \forall e_{j} \in A$$

$$F_N(e_j) = \left\{ (c_i, \frac{\mu_{jL} \ c_i + \mu_{jU} \ c_i}{2}), \forall c_i \in U, \forall e_j \in A \right\}$$

Similarity Measures of Interval valued Fuzzy Soft sets

In this section we introduce similarity measure of IVFS sets based on matching function, distance and set theoretic approach.

Similarity Measures of Reduced Fuzzy Soft sets based on matching function.

Definition 3.1. Let (F,E) & (G,E) be two IVFSs over U, where F and G is a mapping given by F: $E \rightarrow I^{U}$, G: $E \rightarrow I^{U}$, I^{U} denotes the collection of all interval valued fuzzy subsets of U.

Then define similarity measure between (F,E) & (G,E) as

$$SIM_{O}(F,G) = \frac{\sum_{i=1}^{n} F_{O}(e_{j}) \bullet G_{O}(e_{j})}{\sum_{i=1}^{n} \max(F_{O}(e_{j})^{2}, G_{O}(e_{j})^{2})}, j = 1, 2, \dots, m$$

$$SIM_{N}(F,G) = \frac{\sum_{i=1}^{n} F_{N}(e_{j}) \bullet G_{N}(e_{j})}{\sum_{i=1}^{n} \max(F_{N}(e_{j})^{2}, G_{N}(e_{j})^{2})}, j = 1, 2, \dots, m$$

$$SIM_{P}(F,G) = \frac{\sum_{i=1}^{n} F_{P}(e_{j}) \bullet G_{P}(e_{j})}{\sum_{i=1}^{n} \max(F_{P}(e_{j})^{2}, G_{P}(e_{j})^{2})}, j = 1, 2, \dots, m$$

Example 3.1. Let $U = \{c_1, c_2, c_3, c_4\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3\}$. We consider two IVFS sets (F,E) & (G,E) such that their corresponding matrices are

 $\begin{array}{cccc} e_1 & e_2 & e_3 \\ c_1 \begin{pmatrix} [0.7, 0.9] & [0.6, 0.7] & [0.5, 0.8] \\ [0.6, 0.8] & [0.2, 0.5] & [0.6, 0.9] \\ c_3 \begin{pmatrix} [0.5, 0.6] & [0.0, 0.7] & [0.2, 1.0] \end{pmatrix} \\ e_1 & e_2 & e_3 \\ c_1 \begin{pmatrix} [0.2, 0.8] & [0.4, 0.9] & [0.3, 0.6] \\ [0.4, 0.7] & [0.4, 0.5] & [0.8, 0.9] \\ c_3 \begin{pmatrix} [0.0, 1.0] & [0.2, 0.5] & [0.8, 1.0] \end{pmatrix} \end{array}$

Then similarity measure between (F,E) & (G,E) is given by

$$SIM_{P}(F,G) = \frac{1.45}{2.87} = 0.5052$$
$$SIM_{O}(F,G) = \frac{5.4}{5.49} = 0.9168$$
$$SIM_{N}(F,G) = \frac{3.23}{3.91} = 0.8257$$

Proposition 3.1. Let (F,E), (G,E) be IVFSs over U, where F and G are mapping given by $F: E \rightarrow I^U$, G: $E \rightarrow I^U$, I^U denotes the collection of all interval valued fuzzy subsets of U. Similarity between (F,E) & (G,E) denoted by SIM(F,G), then the following holds.

(i) SIM(F,G) = SIM(G,F) (ii) $0 \le SIM(F,G) \le 1$ (iii) SIM(F,F) = 1

Proof: Trivial

Similarity measure of Reduced Fuzzy Soft sets based on set theoretic approach .

Definition 3.2. Let $U = \{c_1, c_2, c_3...c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3...e_n\}$. A pair (F,E) and (G,E) is called IVFS set over U where F and G are mapping given by F: $E \rightarrow I^U$, G: $E \rightarrow I^U$ where I^U denotes the collection of all interval valued fuzzy subsets of U. Let $ST_i(F, G)$ denote the similarity between the two e_i approximations $F(e_i)$ and $G(e_i)$. Define

$$ST_{P_{i}}(F,G) = \frac{\sum_{j=1}^{n} \min(F_{P}(e_{i}), G_{P}(e_{i}))}{\sum_{j=1}^{n} \max(F_{P}(e_{i}), G_{P}(e_{i}))}$$
$$ST_{O_{i}}(F,G) = \frac{\sum_{j=1}^{n} \min(F_{O}(e_{i}), G_{O}(e_{i}))}{\sum_{j=1}^{n} \max(F_{O}(e_{i}), G_{O}(e_{i}))}$$

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$$ST_{N_i}(F,G) = \frac{\sum_{j=1}^{n} \min(F_N(e_i), G_N(e_i))}{\sum_{j=1}^{n} \max(F_N(e_i), G_N(e_i))}$$

If ST(F,G) indicates the similarity between (F,E) and (G,E) then

$$ST_{P}(F,G) = \max ST_{P_{i}}(F,G) ST_{O}(F,G) = \max ST_{O_{i}}(F,G) ST_{N}(F,G) = \max ST_{N_{i}}(F,G)$$

Example 3.2.

Consider the example (3.1) Then similarity measure between (F,E) & (G,E) is given by $ST_{P}(F,G) = \max ST_{P}(F,G)$

Proposition 3.1. Let (F,E), (G,E) be IVFSs over U, where F and G are mapping given by F: $E \rightarrow I^{U}$, G: $E \rightarrow I^{U}$, I^U denotes the collection of all interval valued fuzzy subsets of U. Similarity between (F,E) & (G,E) denoted by ST(F,G), then the following holds.

(i) ST(F,G) = ST(G,F) (ii) $0 \le ST(F,G) \le 1$ (iii) ST(F,F) = 1

Proof: Trivial

Similarity measure of Reduced Fuzzy Soft sets based on distance measure.

Definition 3.3. Let $U = \{c_1, c_2, c_3...c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3...e_n\}$. A pair (F,E) and (G,E) is called IVFS set over U where F and G are mapping given by F: $E \rightarrow I^U$, G: $E \rightarrow I^U$ where I^U denotes the collection of all interval valued fuzzy subsets of U.

a)Hamming distance

$$d_{H_{P}}(F,G) = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} \left| F_{P}(e_{i}) - G_{P}(e_{i}) \right|$$

b)Normalized Hamming distance

$$d_{NH_{P}}(F,G) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} |F_{P}(e_{i}) - G_{P}(e_{i})|$$

c)Euclidean distance

$$d_{E_{P}}(F,G) = \frac{1}{m} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| F_{P}(e_{i}) - G_{P}(e_{i}) \right|^{2} \right)^{\frac{1}{2}}$$

d)Normalized Euclidean distance

$$d_{NE_{P}}(F,G) = \frac{1}{mn} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| F_{P}(e_{i}) - G_{P}(e_{i}) \right|^{2} \right)^{\frac{1}{2}}$$

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e)Generalised Normalised distance

$$d(F,G) = \frac{1}{mn} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| F_{P}(e_{i}) - G_{P}(e_{i}) \right|^{q} \right)^{\frac{1}{q}}$$

NOTE:If q=1then e) reduces to Normalized Hamming distance

and if q=2 then e) reduces to Normalized Euclidean distance

Then the similarity measure between (F,E) and (G,E) denoted by D(F,G) is defined as D(F,G) = 1 - d(F,G)

Example 3.3.

Consider the example (3.1), Hamming distance between (F,E) and (G,E)

$$d_{H_{P}}(F,G) = \frac{1}{3} \ 0.5 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.5 + 0.2 + 0.6$$

=0.93

Then the similarity measure between (F,E) and (G,E) as D(F,G) = 1 - d(F,G) = 1 - 0.93 = 0.07

Application of Similarity Measures of Reduced Fuzzy soft sets in Medical Diagnosis.

Algorithm.

Step1:Input the INFSs (F,E) over the universe U based on the medical knowledge.

Step2: Compute the IVFSs (G,E) over the universe U based on a responsible person for the illness. **Step3:**Input the weighted vector $W = (w_1, w_2)$ and convert the IVFSs into any one of the reduced FSs.

Step4:Calculate the similarity measure of (F,E) & (G,E) based on matching function ordistance measure or set theoretic approach as explained in above section.

Step5:Conclude the result by using similarity measure of (F,E) & (G,E).

Now we are giving an example for application of similarity measure of IVFS sets in medical diagnosis by using set theoretic approach. We would say the IFS sets (F,E) and (G,E) in the IVFS class (U,E) to be significantly similar if $ST(\hat{A}, \hat{B}) > 0.75$.

Suppose that there two patients p_1, p_2 in a hospital with symptoms temperature, headache, vomiting, joint pain, cough and stomach problem. Let the universal set contain only two elements 'yes'(y), 'no'(n), U={y,n}. Here the set of parameters E is the set of certain approximations determined by the Hospital. Let $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ where e_1 =temperature, e_2 =headache, e_3 =vomiting, e_4 =joint pain, e_5 =cough, e_6 =stomach problem.

We construct the IVFS set (F,E) for malaria fever, from medical knowledge as given in Table1.

(F,E)	e ₁	e ₂	e ₃	e_4	e ₅	e ₆
у	[0.2,0.7]	[0.3,0.8]	[0.7,1.0]	[0.4,0.8]	[0.5,0.7]	[0.2,0.6]

n	[0.1,0.3]	[0.2,0.5]	[0.4,0.6]	[0.3,0.4]	[0.5,0.6]	[0.3,0.5]

Table1: IVFS set (F,E) for malaria fever.

Similarly, we construct the IVFS sets for two patients under consideration as given in Table2,3

(P ₁ ,E)	e ₁	e ₂	e ₃	e_4	e ₅	e ₆
у	[0.8,1.0]	[0,0.2]	[0.1,0.3]	[0,0.2]	[0.1,0.2]	[0.7,1.0]
n	[0.8,0.9]	[0.7,1]	[0,0.1]	[0.9,1.0]	[0.9,1.0]	[0.4,1.0]

Table2: IVFS set (P_1,E) for the first patient.

(F,E)	e ₁	e ₂	e ₃	e_4	e ₅	e ₆
У	[0.8,1.0]	[0,0.2]	[0.1,0.3]	[0,0.2]	[0.1,0.2]	[0.7,1.0]
n	[0.8,0.9]	[0.7,1]	[0,0.1]	[0.9,1.0]	[0.9,1.0]	[0.4,1.0]

Table3: IVFS set (P_2,E) for the second patient.

This IVFS set can be converted into Neutral Reduced Fuzzy soft set as

(F,E) _N	e ₁	e ₂	e ₃	e ₄	e ₅	e ₆
У	0.45	0.55	0.85	0.6	0.6	0.4
n	0.2	0.35	0.5	0.35	0.55	0.4

(P ₁ ,E) _N	e ₁	e ₂	e ₃	e_4	e ₅	e ₆
у	0.9	0.1	0.2	0.1	0.15	0.85
n	0.85	0.85	0.05	0.95	0.95	0.7

$(P_2,E)_N$	e ₁	e ₂	e ₃	e_4	e ₅	e ₆
у	0.35	0.25	0.6	0.55	0.5	0.5
n	0.35	0.55	0.55	0.35	0.6	0.5

	ST_1	ST_2	ST ₃	ST_4	ST_5	ST ₆	Maxi
$ST(F, P_1)$	0.371	0.375	0.185	0.290	0.451	0.51	0.51
$ST(F, P_2)$	0.688	0.545	0.786	0.947	0.878	0.8	0.947

Here, two sets of symptoms (F,E), (P_1,E) are not significantly similar. So we conclude that FIRST patient is not possibly suffering from malaria. But two sets of symptoms (F,E), (P_2,E) are significantly similar. So we conclude that SECOND patient is possibly suffering from malaria.

CONCLUSION

In this paper ,we have defined the concept of reduced fuzzy soft set of interval valued fuzzy soft set and defined different types of reduction along with it. Then based on these reductions, three types of similarity measures are discussed. Moreover, an example is given to illustrate the application of similarity measure of IVSsets in medical diagnosis problem .Thus the method can be used to solve the problem which contains uncertainties.

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