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Some aspects on s- near-ring

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Abstract: In this paper we prove some results on a distributive generated s-near ring w.r.t. a set of right complete orthogonal idempotent. If A is an ideal of a s-near ring N in which left annihilators are distributive generated then $\frac{N}{A}$ is a s-near ring. Also we have that the classical near ring of left quotions of a s- near ring is also a s- near ring. Lastly we prove if N possesses strictly projective summand then $\frac{J^{(l,t)}e_i}{J^{(l,t+1)}e_i}$ is either zero or simple for each tame N-group $J^{(l,t)}e_i$.

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INTRODUCTION

In a near-ring N with a right identity e if $N=\sum_{i=1}^{t} L_i(L_i \lhd N)$ and $e=\sum_{i=1}^{t} e_i(e_i \in L_i)$ then $e_1, e_2, \ldots, e_t$ are orthogonal idempotents and each e_i is a right identity of each L_i such that $L_i = Ne_i$ [2]. We see more decompositions induced by idempotents in Fain [11] and Lyons [14]. The idea of near-rings in which N-subgroups form a chain gives rise to the notion of strictly nearly ordered near ring [6] which leads us to the concept of what may be called a s- near-ring .

N is said to be s-near ring if for a set $\{e_i, 1 \le i \le n\}$ of orthogonal idempotents N=$Ne_1 \oplus Ne_2 \oplus \ldots \oplus Ne_n$ where N-subgroups of each Ne_i are linearly ordered by set inclusion.

A set of orthogonal idempotents whose sum is a right identity seems to carry some important characteristics. This relate the s-character of near-rings together with linearly ordered principal N-subgroups in some special cases of projectivity.

Here we prove that if A is an ideal of a s- near-ring N w.r.t. a set of right complete orthogonal idempotents then ,$\frac{N}{A}$ is a s-near ring. Also classical near ring of left quotients of a s-near ring is also a s- near ring. Moreover if N possesses strictly projective N-group,in case of an invariant radical J(N) in the sense J(N)=J(N)N , then for each tame N-group $J^{(\ell,t)}e_i$ we find $\frac{J^{(l,t)}e_i}{J^{(l,t+1)}e_i}$ is either zero or simple.

Throughout our discussion, unless otherwise specified, N will denote a zero symmetric right near ring with 1ε N.

1. DEFINITIONS AND NOTATIONS

The basic concepts which are used in this paper can be found in Pliz [18]. We now begin our discussion with some preliminary definitions and examples..

Two subsets A and B of an N-group are said to be linearly ordered if either $A \subseteq B$ or B⊆**A.**

If N-subgroups of an N-group E are linearly ordered then E is called a weak slnr-group(wslnr) and if principal N-subgroups E are linearly ordered then E is called a principal weak slnr-group (pwslnr). Near ring N is called a pwslnr if it is pwslnr as an N-group.

For any two left subgroups A and B of N if we define AB as AB=$\{\sum_{finite} a_i b_i, a_i \epsilon A, b_i \epsilon B\}$ then AB is a left N-subgroup in a d.g.n.r N . If A=B then AA =$\{\sum_{finite} a_i b_i, a_i \epsilon A, b_i \epsilon A\}$. Clearly (AA)A ≠ A(AA) .In this sense we call A(AA) as left 3- power

of A, written as $A^{(l,3)}$. We consider$A^{(l,t)} = A.A^{(l,t-1)}$.
Consider the near-ring N = {0,a,b,c} with addition and multiplication defined by the following tables:

Table 1.1

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	o

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Here orthogonal idempotents are 0, a and b. Again No = 0, Na = {0, a} and Nb = {0,b}. Thus we see N= N0 ⊕ Na ⊕ Nb. Also we observe that Na and Nb are wslnr-N-groups. In this sense we call, N is a s- near-ring.
In other words we define,A near-ring N is a s- near-ring if for a set $\{e_1, e_2, \dots \dots e_k\}$ of orthogonal idempotents of N, we have N=Ne_1⊕Ne_2⊕….⊕Ne_k where Ne_i (i=1,2…..,k) is a wslnr-N-group.

A set $\{e_1, e_2, \dots\dots, e_n\}$ of orthogonal idempotents of N is called a right complete orthogonal idempotents if $e = \sum_{i=1}^{n} e_i$ is a right identity of N.
N = { 0 , a , b , c } is a near-ring under addition and multiplication defined by the following tables:

Table 1.2

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	o

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Here orthogonal idempotents are 0 and b. Again, 0.b =0, a.b = a, b.b.=b, c.b=c. But b.a.=b,b.c.=b. Therefore b=0+b is a right identity of N..Thus {0,b} is a set of right complete orthogonal idempotents of N.

Left annihilator of a ε N is defined as l(a)={n ε N|na = 0}.

An N-groupE is irreducible (simple) if it has no proper N-subgroup (ideal) of it [16].

An N-group E is called semi simple if it is the direct sum (or sum) of simple ideals [16]. The near ring N is semi simple if N is semi simple as an N-group.

An N-group E is called tame N-group [18] if any N-subgroup of E is an ideal of E.

The radical $J_2\ (N)$ is defined to be the intersection of all the annihilators of irreducible N-groups and J(N) is the intersection of all the maximal left ideals of N which are also maximal as left N-subgroups [16].If 1ε N,it is seen that $j_2(N) = J(N)(=J)$**[17].** Radical J is called D-regular if for each a ε J, we have ax εJ such that a=ax . The radical of E is defined as $J' = \cap \{\ M\ \ M\ is\ an\ ideal\ maximal\ as\ a\ N - subgroup\ of\ N\}$.

An N-group E is called semi-simple if it is a direct sum of simple ideals.[16]

The near-ring N is semi-simple if ${}_{N}N$ is semi-simple.

N is called strictly semi-simple if N is direct sum of irreducible left ideals[16]. Clearly strictly semi-simple near- rings are semi-simple.

N-group E is projective if for any N-groups A, B and for any epimorphism α: A→B and homomorphism β: A→B, there exists a homomorphism γ : E→A such that αγ=β.

A short exact sequence (s.e.s)$0\to A\to B\xrightarrow{f} C\to 0$ almost splits if there is a splitting homomorphism g : C→B such that fg = I_C (identity). The s.e.s splits if there is a normal splitting homomorphism. N-group E is strictly projective if every s.e.s splits.Clearly, a strictly projective N-group is projective.

A right near-ring $C_{c\ell}$ containing N as a subnear-ring is called a classical near-ring of left quotients of N w.r.t. a semigroup S of distributive non-zero divisors of N if and only if

(i) $1\in C_{c\ell}$, (ii) elements of S are invertible in $C_{c\ell}$ and (iii) for each $x\in C_{c\ell}$ there exists $s\in S$ such that $sx\in N$.

2. PRELIMINARIES

Lemma 2.1: If left annihilators of N are distributively generated then for any set $\{e_1,\ e_2, \ldots.., e_k\}$ of orthogonal idempotents, $l\left(\sum_{i=1}^{k} e_i\right) = l(e_1) \cap l(e_2) \cap \ldots .\, l(e_k)$.

Proof: Let x ε $l(e_1) \cap l(e_2) \cap \ldots..\, l(e_k)$

Then $xe_1 = xe_2 = \cdots = xe_k = 0$

Now $x \in l(e_1) \Rightarrow x = \sum_{finite} \pm s_{1_i}$where $s_{1_i} \in S_1$ and $\ell(e_1) = < S_1 >, S_1$ is a set of distributive elements of N.

As each $s_{1_i} \in l(e_1)$, we have $s_{1_i} e_1 = 0$, for all i.

Then we have $x(\sum_{i=1}^{k} e_i) = 0 \Rightarrow x \in l\left(\sum_{i=1}^{k} e_i\right)$

Thus $l(e_1) \cap l(e_2) \cap \dots \cap l(e_k) \subseteq l(\sum_{i=1}^{k} e_i)$................(i)

Conversely, suppose $y \, \varepsilon \, l \, (\sum_{i=1}^{k} e_i)$

Then y = $\sum_{finite} \pm d_i$ where $d_i \in$T and $l(\sum_{i=1}^{k} e_i) = < T >$,T is a set of distributive elements of N.

Now $d_i \in l(\sum_{i=1}^{k} e_i)$ for all i.

$\Rightarrow d_i \, l\left(\sum_{i=1}^{k} e_i\right) = 0 \Rightarrow \mathrm{d_i} \in \ell(\mathrm{e_1}),$ for all i

Similarly, we get $d_i \epsilon l(e_2), \dots, d_i \epsilon l(e_k)$,. for all i.

Hence $y \epsilon l(e_1) \cap l(e_2) \dots . \cap l(e_k)$.

Thus $l\left(\sum_{i=1}^{k} e_i\right) \sqsubseteq l(e_1) \cap l(e_2) \dots . \cap l(e_k)$ ....(ii).

From (i) and (ii) we get the result.

Lemma 2.2 : If left annihilators of N are distributively generated and $\{\mathrm{e_j} \mid 1 \le \mathrm{j} \le \mathrm{k}\}$ is a set of right complete orthogonal idempotents of N then an ideal I of N can be imbedded to $Ie_1 \oplus \dots\dots \oplus Ie_k$.

Proof: Define a mapping $f : I \to Ie_1 \oplus \dots\dots \oplus Ie_k$ by $f(x) = (xe_1, xe_2, \dots\dots, xe_k)$

For x, y$\in$ I and n$\in$N, f(x+y)=f(x)+f(y) and f(nx) =nf(x)

As $e = \sum_{j=1}^{k} e_j$ is a right identity, f is 1-1.

Thus f is an N-monomorphism and so I is embedded in $Ie_1 \oplus .....\oplus Ie_k$.

Note1 : As I is an ideal , $Ie_1 \oplus Ie_2 \oplus .....\oplus Ie_k \subseteq I$. Thus $I \cong Ie_1 \oplus ....\oplus Ie_k$.

3. Main results

Theorem 3.1: If N is a s- near-ring w.r.t. a set of right complete orthogonal idempotents such that left annihilators of N are distributively generated then for an ideal A of N, $\frac{N}{A}$ is a s-near ring.

Proof: Let S=$\{e_i, 1 \leq i \leq n\}$ be a set of right complete orthogonal idempotents of N. Then N=$Ne_1 \oplus Ne_2 \oplus \dots . \oplus Ne_n$ where each Ne_i is a wslnr N-group.

As A is an ideal of N, Ae_i is an ideal of the N-group Ne_i for all i = 1, 2,…..,n.

Define a mapping

$$f : Ne_1 \oplus Ne_2 \oplus \dots \dots \oplus Ne_n \to \frac{Ne_1}{Ae_1} \oplus \frac{Ne_2}{Ae_2} \oplus \dots \dots \oplus \frac{Ne_n}{Ae_n} \text{ by}$$

$f(n_1e_1, \dots \dots, n_ne_n) = (\overline{n_1e_1}, \overline{n_2e_2}, \dots \dots \dots \dots \overline{n_ne_n})$ where $\overline{n_ie_i} = n_ie_i + Ae_i$ Then clearly f is an N-epimorphism.

Again, Ker f = $\{(n_1e_1, \dots\dots, n_ne_n) \mid f(n_1e_1, \dots\dots, n_ne_n) = (\bar{0}, \dots . \bar{0})\}$

$\cong$ A , by Lemma 2.2

Thus $\frac{N}{A} \cong \frac{Ne_1}{Ae_1} \oplus \dots\dots \oplus \frac{Ne_n}{Ae_n}$.

Now define another mapping

$$\frac{N}{A} \times \frac{Ne_i}{Ae_i} \to \frac{Ne_i}{Ae_i} \text{ by}$$

$$(n + A, \alpha e_i + Ae_i) \mapsto n\alpha e_i + Ae_i$$

,this mapping is welldefined as Ae_i is an ideal of Ne_i

Also, $((n + A) + (n_1 + A))(\alpha e_i + Ae_i) = (n + A)(\alpha e_i + Ae_i) + (n_1 + A)(n_I \alpha e_i + Ae_i)$

and $((n+A)\,((n'+A)(\alpha e_i + Ae_i)) == ((n+A)(n'+A))(\alpha e_i + Ae_i)$

Thus $\dfrac{Ne_i}{Ae_i}$ is an $\frac{N}{A}$ -group for all $1 \le i \le n$.

Suppose $\overline{K}_1$ and $\overline{K}_2$ be two $\overline{N}$-subgroups of $\overline{Ne_i}$. Then K_1 and K_2 are N-subgroups of Ne_i . As each Ne_i is a wslnr, N-group, therefore $K_1 \subseteq K_2$ (say) $\Rightarrow \overline{K}_1 \subseteq \overline{K}_2$

Thus $\overline{Ne_i}$ is a wslnr $\frac{N}{A}$ −group, for all $1 \le i \le n$ and hence $\frac{N}{A}$ is a s near-ring.

Consider $N = Ne_1 \oplus Ne_2 \oplus \ldots . \oplus Ne_n$ where each Ne_i is a wslnr N-group and e_i's are orthogonal idempotents . If Q is the classical near ring of left quotients, then clearly $Q = Qe_1 \oplus Qe_2 \oplus \ldots . \oplus Qe_n$. Also as each Ne_i is a wslnr N-group and for any ideal K of Q we have, S^{-1} (K∩ N) = K,[ ] so each Qe_i *is a wslnr* $Q-$*group*. Thus we have

Result 3.2: The classical near ring of left quotients of a s-near ring is also a s-near ring.

In the following result N is a pwslnr and left annihilators of N are distributively generated. Also N satisfies the acc on left N-subgroups and each summand of N is a strictly projective N-group with invariant radical J(N) such that J(N) = J(N)N.

Result 3.3: If N is a s- near-ring w.r.t. a set $\{e_i, 1 \le i \le k\}$ of right complete orthogonal idempotents then $\frac{J^{(l,t)}e_i}{J^{(l,t+1)}e_i}$ is either zero or simple for each tame N-group $J^{(\ell,t)}e_i$ $t \ge 0$.

Proof: Here $N = Ne_1 \oplus Ne_2 \oplus \ldots . \oplus Ne_k$ where each $Ne_i, 1 \le i \le k$ is a wslnr N-group.

Now $J(Ne_i) = J(N)\ Ne_i$ [13]

$= (J(N)N)\ e_i = J(N)e_i = Je_i$, where $J(N) = J$

Then Je_i is the unique ideal maximal as N-subgroup and so each $\dfrac{Ne_i}{Je_i}$, is an irreducible N-group for all i= 1,2,…..,k.

Define a mapping

$$\phi : Ne_1 \oplus \ldots . \oplus Ne_k \rightarrow \frac{Ne_1}{Je_1} \oplus \ldots . \oplus \frac{Ne_k}{Je_k} \text{ by}$$

$$\phi(n_1e_1, \ldots\ldots, n_ne_n) = (\overline{n_1e_1}, \ldots\ldots \overline{n_ne_n}), \text{where } \overline{n_ie_i} = n_ie_i + Je_i$$

Clearly ϕ is an N-epimorphism.

Also $\ker\ \phi = \{(n_1e_1,.......,n_ke_k) \mid \phi(n_1e_1,.......,n_ke_k) = (\bar{0},.....\bar{0})\}$

$= \{(n_1e_1,.......,n_ke_k) \mid n_ie_i \in Je_i, 1 \le i \le k\} = Je_1 \oplus .....\oplus Je_k \cong J$, by Lemma 2.2

So, $\frac{N}{J} \cong \frac{Ne_1}{Je_1} \oplus ....\oplus \frac{Ne_k}{Je_k}$.

J being invariant, $\frac{N}{J}$ is a near-ring.

Define another mapping

$\alpha : \frac{N}{J} \times \frac{Ne_i}{Je_i} \to \frac{Ne_i}{Je_i}$ by

$\alpha(\overline{n}, \overline{n_ie_i}) = \overline{nn_ie_i}$

It can be easily seen that α is well defined

Suppose $\frac{Ke_i}{Je_i}$ be a proper $\frac{N}{J}$ sub-group of $\frac{Ne_i}{Je_i}$.

Then Ke_i is a proper N-subgroup of Ne_i .hence $\frac{Ke_i}{Je_i}$ is an N-subgroup of $\frac{Ne_i}{Je_i}$ which is a contradiction.

Thus $\frac{Ne_i}{Je_i}$ is an irreducible $\frac{N}{J}$ -group.

By lemma 3.5 [6] , $J^{(\ell, t+1)}$ is an ideal of $J^{(\ell, t)}$.

Therefore, $J^{(\ell, t+1)}e_i$ is an ideal of $J^{(\ell, t)}e_i$.

Now suppose $\frac{Me_i}{J^{(\ell, t+1)}e_i}$ be an $\frac{N}{J}$-subgroup of $\frac{J^{(\ell, t)}e_i}{J^{(\ell, t+1)}e_i}$.

Then Me_i is an N-subgroup of $J^{(\ell, t)}e_i$ ($\subseteq$N).

As for each $t \geq 0$, $J^{(\ell, t)}e_i$ is a tame N-group, therefore Me_i is an ideal of $J^{(\ell, t)}e_i$.

$\Rightarrow \frac{Me_i}{J^{(\ell, t+1)}e_i}$ is $\frac{N}{J}$ ideal of $\frac{J^{(\ell, t)}e_i}{J^{(\ell, t+1)}e_i}$

Thus $\frac{N}{J}$ -subgroups of $\frac{J^{(\ell, t)}e_i}{J^{(\ell, t+1)}e_i}$ are ideals, $t \geq 0$.

Hence $\frac{N}{J}$ is the direct sum of irreducible left ideals. Therefore $\frac{N}{J}$ is semi-simple and has the dcc on left ideals[15].

Therefore $\frac{J^{(\ell, t)}e_i}{J^{(\ell, t+1)}e_i}$ is semi-simple as $\frac{N}{J}$-group [15]. As each Ne_i are wslnr, therefore $\frac{J^{(\ell, t)}e_i}{J^{(\ell, t+1)}e_i}$ is either zero or simple.

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