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HEAT AND MASS TRANSFER EFFECTS ON STEADY MHD FLOW OVER AN EXPONENTIALLY STRETCHING SURFACE WITH VISCOUS DISSIPATION, HEAT GENERATION AND RADIATION

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Abstract: A numerical analysis has been carried out to study heat and mass transfer effects on steady two-dimensional flow of a viscous incompressible, electrically conducting dissipating fluid past an exponentially stretching surface in presence of magnetic field, heat generation and radiation. The governing partial differential equations are reduced to nonlinear ordinary differential equations by similarity transformation, before being solved numerically by fourth order Runge-Kutta method with shooting technique. A comparison with the previous results shows a very good agreement. The effects of various governing parameters on the velocity, temperature, concentration, skin-friction coefficient, Nusselt number and Sherwood number are computed and discussed in detail.

Keywords: MHD ; Boundary layer flow ; Exponentially stretching sheet; Radiation; viscous dissipation; heat generation parameter.

INTRODUCTION

The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. Many researchers inspired by Sakiadis [1,2] who initiated the boundary layer behavior studied the stretching flow problem in various aspects. Crane [3] was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane [4], under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. Magyari and Keller [5] investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution.

The study of Magnetohydrodynamics (MHD) boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as the aerodynamic extrusion of plastic sheets, liquid film, hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Liu [6] analyzed the hydromagnetic fluid flow past a stretching sheet in the presence of uniform transverse magnetic field. Chen [7] investigated the fluid flow and heat transfer on a stretching vertical sheet, and his work has been extended by Ishak et al. [8] to hydromagnetic flow.

In physics and engineering, the radiative effects have important applications. In space technology and high temperature processes, knowledge of radiation heat transfer becomes very important for the design of pertinent equipment [9]. Many researchers have considered the effect of thermal radiation on flows over stretching sheets. Studies by Raptis [10], Raptis and Perdikis [11] address the effect of radiation in various situations. Siddheshwar and Mahabaleswar [12] studied the effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet. Bidin and Nazar [13] studied the effects of numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. Ishak [14] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. Thermal radiation effects on hydro-magnetic flow due to an exponentially stretching sheet were studied by Reddy and Reddy [15].

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. The flow and heat transfer from an exponentially stretching surface was considered by Magyari and Keller [16]. Sanjayanad and Khan [17] studied the heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet. Kameswaran et al. [18] investigated the heat and mass transfer effects on MHD Newtonian liquid flow over an exponentially stretching sheet in presence of radiation. Seini and Makinde [19] found that radiation and chemical reaction effects on MHD boundary layer flow over an exponential stretching surface.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al [20] has investigated the Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Furthermore, Moalem [21] studied the effect of temperature dependent heat sources taking place in electrically heating on the heat transfer within a porous medium. Radiation and mass transfer effects on MHD free convection fluid flow embedded in a porous medium with heat generation/absorption was studied by Shankar et al [22].

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Vajravelu and Hadjinicalaou [23] analyzed the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. Convective boundary layer flow has wide applications in engineering as post accidental heat removal in nuclear reactors, solar collectors, drying processes, heat exchangers, geothermal and oil recovery, building construction, etc. The effect of viscous dissipation in natural convection processes has been studied by Gebhart [24] and Gebhart and Mollendorf [25]. Jat and Gopi Chad [26] proposed the effects of dissipation and radiation on MHD flow and heat transfer over an exponentially stretching sheet. Partha et al. [27] analyzed the effects of viscous dissipation on mixed convection heat transfer from an exponentially stretching surface. Mohammed Ibrahim and Bhaskar Reddy [28] noticed that effects of radiation and mass transfer effects on MHD flow along a stretching surface in presence of viscous dissipation and heat generation.

In the view of the above discussions the aim of the present study is to analyze the heat and mass transfer effects on MHD steady flow past an exponentially sheet in presence of thermal radiation, heat generation and viscous dissipation. By suitable similarity transformation, the governing boundary layer equations are transformed to ordinary differential equations and solved numerically by using fourth order Runge-Kutta method with shooting technique. The effects of different physical parameters on the velocity, temperature and concentration profiles as well as skin-friction coefficient, Nusselt number and Sherwood numbers are presented. To verify the obtained results, I have compared the present numerical results with previous work by Kameswaran *et al.* [18]. The comparison results show a good agreement and I confident that our present numerical results are accurate.

FORMULATION OF THE PROBLEM

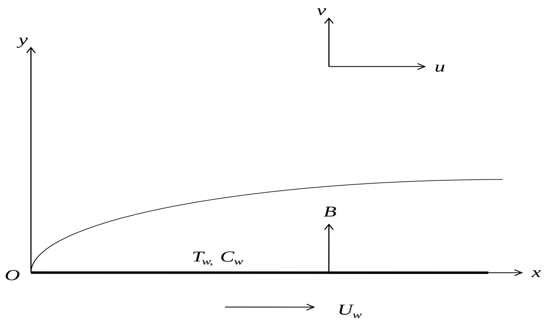


Figure 1 Schematics of the problem.

Consider a steady two dimensional laminar flow of a viscous incompressible electrically conducting fluid over a continuous exponentially stretching surface. The *x* – axis is taken along the stretching surface in the direction of motion and *y*- axis is perpendicular to it. The sheet velocity is assumed to vary as an exponential function of the distance *x* from the slit. The temperature and concentration far away from the fluid are assumed to be T_∞ and C_∞ respectively as shown in Figure 1. The sheet-ambient temperature and concentration differences are also assumed to be exponential functions of the distance *x* from the slit. A variable magnetic field of strength *B(x)* is applied normally to the sheet. Under the usual boundary layer approximation, subject to radiation, viscous dissipation and heat generation, the equations governing the momentum, heat and mass transports can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho c_p}u^2+\frac{Q_0}{\rho c_p}\;T-T_\infty \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where *u* and *v* are the velocity components in the *x, y* directions respectively, ν is the kinematic viscosity, ρ is the density, σ is the electrical conductivity of the fluid, *T* is the temperature, *C* is the concentration, *k* is the thermal conductivity, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, Q_0 is the heat generation coefficient, *D* is the species diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are

$$u=U_w=U_0e^{\frac{x}{L}},\quad v=0,\quad T=T_w=T_\infty+T_0e^{\frac{2x}{L}},\quad C=C_w=C_\infty+C_0e^{\frac{2x}{L}}\quad at\ \ y=0$$

$$u\to 0,\quad T\to T_\infty,\quad C\to C_\infty\quad as\ \ y\to\infty \tag{5}$$

Here the subscripts w, ∞ refer to the surface and ambient conditions respectively, T_0, C_0 are positive constants, U_0 is the characteristic velocity and *L* is the characteristic length.

To facilitate a similarity solution, the magnetic field *B(x)* is assumed to be of the form

$$B(x) = B_0 e^{\frac{x}{2L}} \tag{6}$$

where B_0 is a constant. It is also assumed that the fluid is weakly electrically conducting so that the induced magnetic field is negligible. Following Rosseland's approximation, the radiative heat flux q_r is modeled as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where σ^* is the Stefan-Boltzman constant, k^* is the mean absorption coefficient.

This approximation is valid at points optically far from the boundary surface and it is good for intensive absorption, which is for an optically thick boundary layer. It is assumed that the temperature difference within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 by Taylor series about T_∞ and neglecting higher-order terms gives:

$$T^4 \equiv 4T_\infty^3 T - 3T_\infty^4$$

We have $\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}$ (8)

Continuity equation (1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{9}$$

The following similarity variables are used:

$$u = U_0 e^{\frac{x}{L}} f'(\eta), \qquad v = -\sqrt{\frac{\nu U_0}{2L}} \quad e^{\frac{x}{2L}} \quad f(\eta) + \eta f'(\eta)$$

$$T = T_\infty + T_0 e^{\frac{2x}{L}} \theta(\eta), \qquad C = C_\infty + C_0 e^{\frac{2x}{L}} \phi(\eta), \quad \eta = \sqrt{\frac{U_0}{2\nu L}} \; y e^{\frac{x}{2L}} \tag{10}$$

Where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, and $\phi(\eta)$ is the dimensionless concentration.

On using equations (6), (8) and (10), equations (2) – (5) are transformed to:

$$f''' - 2f'^2 + ff'' - Mf' = 0 \tag{11}$$

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left[f\theta' - f'\theta + Ec(f'')^2 + MEc(f')^2 + Q\theta\right] = 0 \tag{12}$$

$$\phi'' + Scf\phi' - Scf'\phi = 0 \quad (13)$$

$$f(0) = 0, \qquad f'(0) = 1, \quad f'(\infty) \to 0, \quad (14)$$

$$\theta(0) = 1, \qquad \theta(\infty) \to 0, \quad (15)$$

$$\phi(0) = 1, \qquad \phi(\infty) \to 0. \quad (16)$$

The non-dimensional constants appearing in equations (11) – (13) are the magnetic parameter *M*, the radiation parameter *R*, the Prandtl number *Pr*, the Eckert number *Ec*, *Q* is the heat generation parameter, *Sc* is the Schmidth number and *Kr* is the chemical reaction parameter respectively defined as

$$M = \frac{2\sigma B_0^2 L}{\rho U_0} \qquad R = \frac{4\sigma^* T_\infty^3}{k^* k}, \qquad \Pr = \frac{\rho \nu c_p}{k} \qquad Ec = \frac{U_0^2}{c_p T_0}$$

$$Q = \frac{2Q_0 L}{U_0 e^{\frac{x}{L}}}, \qquad Sc = \frac{\nu}{D},$$

SKIN FRICTION, HEAT AND MASS TRANSFER COEFFICIENTS

The parameters of engineering interest in heat and mass transport problems are the skin friction- coefficient C_f, the local Nusselt number *Nu*, and the local Sherwood number *Sh*. These parameters respectively characterize the surface drag, wall heat and mass transfer rates.

The shearing stress at the surface of the wall τ_w is given by

$$\tau_w = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{\mu U_0}{L}\sqrt{\frac{\text{Re}}{2}}\; e^{\frac{3x}{2L}} f''(0), \quad (17)$$

where μ is the coefficient of viscosity and $\text{Re} = \frac{U_0 L}{\nu}$ is the Reynolds number. The skin friction coefficient is defined as

$$C_f = \frac{2\tau_w}{\rho U_w^2} \quad (18)$$

and using equation (17) in equation (18), we obtain

$$\frac{C_f\sqrt{\text{Re}/2}}{\sqrt{x/L}} = -f''(0). \quad (19)$$

The heat transfer rate at the surface flux at the wall is given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{-k(T_w - T_\infty)}{L}\sqrt{\frac{\text{Re}}{2}}\; e^{\frac{x}{2L}}\theta'(0), \quad (20)$$

where *k* is the thermal conductivity of the fluid. The Nusselt number is defined as

$$Nu = \frac{x}{k}\frac{q_w}{T_w - T_\infty}. \tag{21}$$

Using Equation (20) in Equation (21), the dimensionless wall heat transfer rate is obtained as follows:

$$\frac{Nu}{\sqrt{x/L}\sqrt{\mathrm{Re}/2}} = -\theta'(0). \tag{22}$$

The mass flux at the surface of the wall is given by

$$J_w = -D\left(\frac{\partial C}{\partial y}\right)_{y=0} = \frac{-D(C_w - C_\infty)}{L}\sqrt{\frac{\mathrm{Re}}{2}}\;e^{\frac{x}{2L}}\phi'(0) \tag{23}$$

The Sherwood number is defined as

$$Sh = \frac{x}{D}\frac{J_w}{C_w - C_\infty}. \tag{24}$$

Using (23) in (24), the dimensionless wall mass transfer rate is obtained as

$$\frac{Sh}{\sqrt{x/L}\sqrt{\mathrm{Re}/2}} = -\phi'(0) \tag{25}$$

In Equations (19), (22) and (25), *Re* represents the local Reynolds number and it is defined as

$$\mathrm{Re} = \frac{xU_w}{\nu}.$$

NUMERICAL PROCEDURE

The set of nonlinear ordinary differential equations (11), (12), and (13) with boundary conditions (14) - (16) were solved numerically using Runge – Kutta fourth order algorithm with a systematic guessing of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta$ = 0.001 is used while obtaining the numerical solution and accuracy up to the fifth decimal place i.e. 1×10^{-5}, which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta\to\infty$, say η_∞, which depend on the values of the parameter used. The computations were done by a program which uses a symbolic and computational computer language in Mathematica.

RESULTS AND DISCUSSION

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various in governing parameters, namely, magnetic field parameter *M*, Prandlt number *Pr*, radiation parameter *R*, heat generation parameter *Q*, Eckert number *Ec*, Schmidt number *Sc*. In the present study following default parameter values are adopted for computations: *M = 1.0, Pr = 0.71, R = 0.5, Q = 0.1, Ec = 0.1, Sc = 0.6.* All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figure 2 shows the variation of the velocity profile against the magnetic parameter. We notice that the effect of the magnetic parameter is to reduce the velocity of the fluid in the boundary layer region. This is due to an increase in the Lorentz force, similar to Darcy's drag observed in the case of flow through a porous medium. This adverse force is responsible for slowing down the motion of the fluid in the boundary layer region.

The variation of the temperature distribution with the magnetic parameter is shown in Figure 3. The thermal boundary layer thickness increases with increasing values of the magnetic parameter. The opposing force introduced in the form of the Lorentz drag contributes in increasing the frictional heating between the fluid layers, and hence energy is released in the form of heat. This results in thickening of the thermal boundary layer.

The effect of the magnetic parameter on the concentration profile is shown in Figure 4. It is observed that increases in the values in *M* result in thickening of the species boundary layer.

The influence of the Prandtl number *Pr* on temperature field is shown in Fig.5 it is noticed that the temperature profiles decrease with the increase of Prandtl number *Pr*.

The influence of the thermal radiation parameter *R* on temperature is shown in Figure 6. It is clear that thermal radiation enhances the temperature in the boundary layer region. Thus radiation should be kept at its minimum in order to facilitate better cooling environment. The radiation parameter *R* defines the relative contribution of conduction heat transfer to thermal radiation transfer.

The effect of the Eckert number *Ec* on heat transfer is shown in Figure 7. It is clear that the temperature in the boundary layer region increases with an increase in the viscous dissipation parameter.

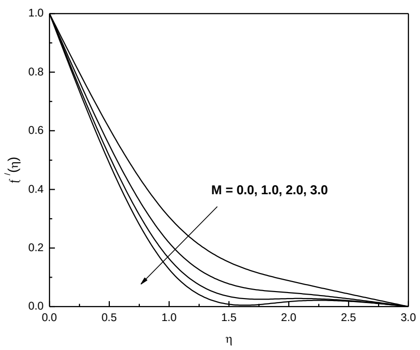
Figure 8 shows the influence of the heat generation parameter *Q* on the temperature profile within the thermal boundary layer. From the Figure 8 it is observed that the temperature increases with an increase in the heat generation parameter.

Figures 9 depict chemical species concentration profiles against co-ordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition. From these figures, it is noteworthy that the concentration boundary layer thickness decreases with an increase in Schmidt number.

We also note that since the energy equation is partially decoupled from the momentum and species conservation equations, the parameters affecting the energy equation, namely, the Prandtl number, the thermal radiation parameter, heat generation parameter and the Eckert number, do not alter velocity and concentration profiles.

Table 1 shows the comparison of Kameswaran *et al.* [18] work with the present work for $Pr = R = Ec = Q = Sc = 0$ and it note worthy that there is a good agreement.

Table 2 indicates the values of skin-friction coefficient, the wall temperature gradient and the wall concentration gradient in terms of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ respectively for various values embedded flow parameter. From Table 2, it is understood that, as increasing values of magnetic field parameter *(M)* results in considerable opposition to the flow in the form of a Lorenz drag which enhances the values of skin-friction coefficient, but there is a decrease in the wall temperature gradient and the wall concentration gradient. The wall temperature gradient reduces as increase the values of radiation parameter *R* or dissipation *Ec* or heat generation parameter *Q*, while it is increases for increasing value of Prandtl number *Pr*. It is also observed that the increase in Schmidt number *Sc* parameter lead to the increase in the dimensionless wall concentration gradient.



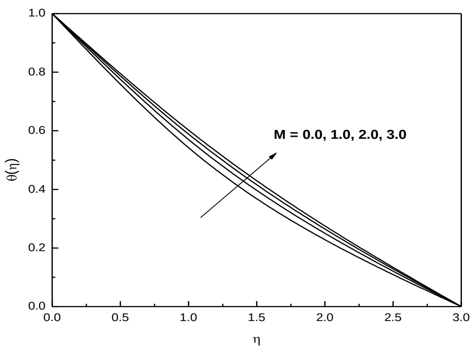
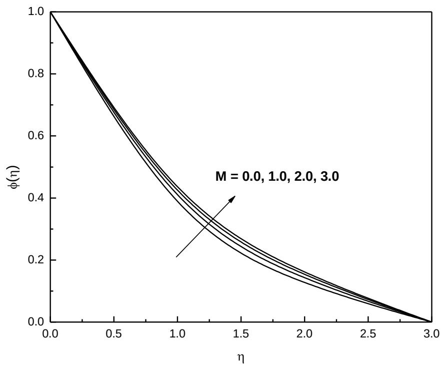


Fig. 2. Velocity profiles for varying values of magnetic parameter *(M)*

Fig. 3. Temperature profiles for varying values of magnetic parameter *(M)*



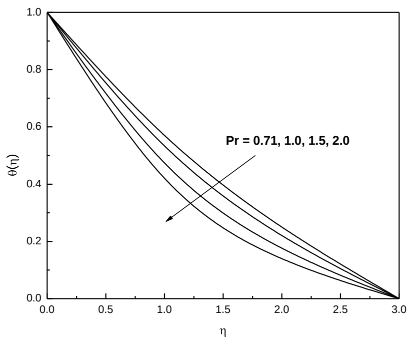
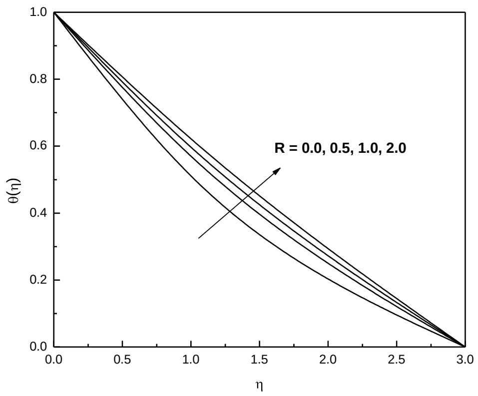


Fig. 4. Concentration profiles for varying values of magnetic parameter *(M)*

Fig. 5. Temperature profiles for varying values of Prandtl parameter (Pr)



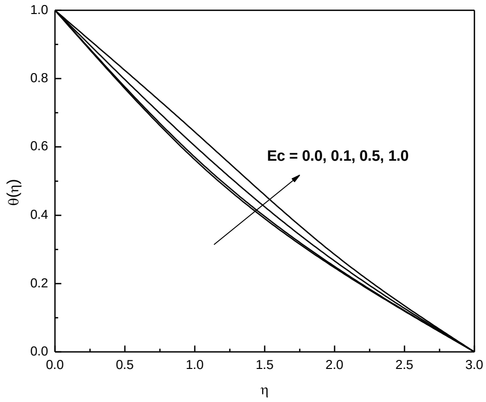
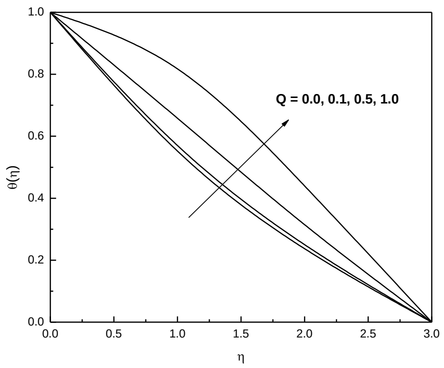


Fig. 6. Temperature profiles for varying values of radiation parameter (R)

Fig. 7. Temperature profiles for varying values of viscous dissipation parameter (Ec)



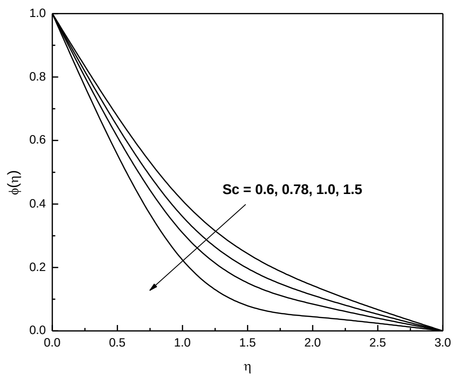


Fig. 8. Temperature profiles for varying values of heat generation parameter (Q)

Fig. 9. Concentration profiles for varying values of Schmidt parameter (Sc)

Table 1: A comparison of skin-friction coefficient $-f''(0)$ for different values of M for fixed values of Pr = R = Q = Ec = Sc = Kr = 0.

	$-f''(0)$	
M	Kameswaran *et al.* [27]	*Present*
0.0	1.28181	1.29038
1.0	1.62918	1.63038
2.0	1.91262	1.91285
3.0	2.15874	2.15879
4.0	2.37937	2.37938

Table 2: Computation showing $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ for different embedded flow parameter values.

M	*Pr*	*R*	*Ec*	*Q*	*Sc*	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1.0	0.71	0.5	0.1	0.1	0.6	1.63038	0.518152	0.695827
2.0	0.71	0.5	0.1	0.1	0.6	1.91285	0.47744	0.66308
3.0	0.71	0.5	0.1	0.1	0.6	2.15879	0.445763	0.638324
1.0	1.0	0.5	0.1	0.1	0.6	1.63038	0.591577	0.695827
1.0	2.0	0.5	0.1	0.1	0.6	1.63038	0.832569	0.695827
1.0	0.71	1.0	0.1	0.1	0.6	1.63038	0.465892	0.695827
1.0	0.71	2.0	0.1	0.1	0.6	1.63038	0.417967	0.695827
1.0	0.71	0.5	0.5	0.1	0.6	1.63038	0.355639	0.695827
1.0	0.71	0.5	1.0	0.1	0.6	1.63038	0.152498	0.695827
1.0	0.71	0.5	0.1	0.5	0.6	1.63038	0.354218	0.695827
1.0	0.71	0.5	0.1	1.0	0.6	1.63038	0.0846552	0.695827
1.0	0.71	0.5	0.1	0.1	0.78	1.63038	0.518152	0.796236
1.0	0.71	0.5	0.1	0.1	1.0	1.63038	0.518152	0.913902

CONCLUSIONS

In this article I have studied the effects of radiation and viscous dissipation on heat and mass transfer from an exponentially stretching surface in the presence of heat generation. The governing equations were solved numerically using the Runge-Kutta fourth order along shooting method. This has been shown to give accurate results. The effects of various physical parameters on the fluid properties, the skin-friction coefficient and the heat and mass transfer rates have been determined. We found that the effect of the magnetic parameter is to reduce the velocity of the fluid in the boundary layer region. It was also observed that the increase in values of *M* results in thickening of the species boundary layer. The chemical concentration boundary layer was found to decrease near the boundary with increasing the Schmidt parameter. The heat transfer rates increases with an increasing of individual effects of the magnetic parameter *M*, the radiation parameter *R*, heat generation parameter *Q* and Eckert number *Ec*. The mass transfer rate increases with an increasing of Schmidt parameter *Sc.* Also the numerical results obtained are agrees with previously reported case available in the literature Kameswaran *et al.* [27].

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