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STATISTICAL PROCESS CONTROL FOR CONSTRAINED PROBABILISTIC MISS SAFETY STOCK INVENTORY SYSTEM WITH CONTINUOUS INCREASING ORDER COST FUNCTION

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Abstract: This paper treated the statistical process control (SPC) for the constrained probabilistic multi- item, single source (MISS) inventory system with varying order cost and instantaneous replenishments of the ordered quantity. The expected maximum inventory level of each item is a function of the expected order quantity and no shortages are to be allowed. A geometric programming approach is used to drive the analytical solution of the optimal number of periods N_i^* and the optimal expected maximum inventory **level** M_i^* . A numerical example is added with R and X bar- charts to confirm that the process is under control.

Keywords: *Statistical process control, safety stock, MISS inventory system, varying order cost and geometric programming, R and X bar-charts*

INTRODUCTION

Statistical process control (SPC) is a powerful collection of problem-solving tools useful in achieving process stability and improving capability through the reduction of variability. One of these tools, the control chart is probably the most technically complicated. It was developed by Shewhart (1931). Control charts can be defined as broken-line graph illustrates how process or a point in process behaves over time. The charts can show how the specific measurement changes, how the variation in measurements changes, or how the proportion of defective pieces changes over time. An introduction to statistical quality control introduced by Chandra (2000) and Montgomery (1996). Chou (2011) studied the economic design of $\overline{X}$ charts for non-normally correlated data. Lipsitz et. al.(2000) used a Box-Cox transformation in the analysis of longitudinal data with incomplete responses.

Most probabilistic inventory models assumed that the demand rate is probabilistic since the probability distribution of the future demand rate rather than the exact value of demand rate itself, is known. Also, assumed that the units of cost are constant and independent of the number of periods. Unconstrained probabilistic inventory models with constant unit of costs have been treated by Gupta and Hira (1994), Hadley and Whitin (1963), and Taha (2007). Fabrycky and Banks (1967) studied the probabilistic single- item, single source (SISS) inventory system with zero lead-time, using the classical optimization. Fergany and Elwakeel (2006), applied several continuous distributions for constrained probabilistic lost sales inventory models with varying order cost using Lagrangian method. Other related inventory were written by Cheng (1989) and mandal et. al. (2006).

Recently, Elwakeel (2011) treated quality control for probabilistic (Q , r) inventory model with varying order cost and continuous lead time demand under the annual holding cost constraint using Lagrangian multipliers. Kotb et al (2011) studied quality control for constrained probabilistic single-item EOQ model with zero lead time demand using geometric programming approach.

This paper was investigated to treat the statistical process control (SPC) for probabilistic MISS inventory system with continuous increasing order cost and Shortage is not allowed. The model is constrained under linear and non- linear constraints, one of them is on the expected varying order cost and the other on the expected holding cost. The decision variables of this model are the number of periods N_i^* and the expected maximum inventory level M_i^* which minimize the expected annual total cost. Finally, a numerical example is added with R and X bar- charts to confirm that the process is in control.

BASIC NOTATIONS

The following notations are adopted for developing our model:

A_1 = The limitation on the expected order cost,

A_2 = The limitation on the expected holding cost,

CL = Control limit,

D_i = A random variable demand rate of the i^{th} item per period,

$\bar{D}_i$ = The expected value of the demand rate,

$f(D_i)$ = The probability density functions of the demand rate of the i^{th} item per period,

H_i = The holding cost of the i^{th} item per period,

$K_i(N_i)$ = The varying order cost of the i^{th} item per cycle, $K_i(N_i) = K_i\, N_i^{\beta}$, $0 < \beta < 1$

LCL = Lower control limit,

M_i = The expected maximum inventory level of the i^{th} item (a decision variable),

$$M_i = a_i\ \bar{Q}_i \quad , a_i > \frac{1}{2}$$

$\min \overline{TC}$ = The minimum expected total cost function.

N_i = The number of periods of the i^{th} item (a decision variable),

P_i = The purchase cost of the i^{th} item,

$\bar{Q}_i$ = The expected order quantity of the i^{th} item,

R_i = The range of the i^{th} subgroup

$\overline{TC}$ = The expected total cost function.

UCL = Upper control limit,

$\bar{X}_i$ = the average of the i^{th} subgroup

MATHEMATICAL MODEL

Our objective is to minimize the annual relevant expected total cost function (i.e., the sum of the expected purchase cost, the expected order cost, and the expected holding cost) which, according to the basic notations and assumptions of the model is:

$$\overline{TC} = \sum_{i=1}^{n} \left[\overline{PC} + \overline{OC} + \overline{HC}\right]$$
$$= \sum_{i=1}^{n} \left[P_i \, \overline{D}_i + K_i \, N_i^{\beta - 1} + \frac{H_i \, \overline{D}_i \, N_i (2a_i - 1)}{2} \right] \tag{1}$$

Under the constraints:

$$\left.\begin{aligned} &\sum_{i=1}^{n} K_i \, N_i^{\beta-1} \le A_1 \\ &\sum_{i=1}^{n} \frac{H_i \overline{D}_i N_i (2a_i - 1)}{2} \le A_2 \end{aligned}\right\} \tag{2}$$

Where A_1 and A_2 set the limits on the expected varying order cost and the expected holding cost respectively. Also, the term $\sum_{i=1}^{n} P_i \overline{D}_i$ is a constant and hence can be ignored without any effect.Thus the following simplified version can be obtained:

$$\min \overline{TC} = \sum_{r=1}^{n} \left[K_i N_i^{\beta-1} + \frac{H_i \overline{D}_i \, N_i \, \alpha_i}{2} \right] \quad , \alpha_i = (2a_i - 1) \tag{3}$$

Subject to:

$$\sum_{i=1}^{n} \frac{K_i \, N_i^{\beta-1}}{A_1} \le 1 \quad \textit{and} \quad \sum_{i=1}^{n} \frac{H_i \, \overline{D}_i \, N_i \, \alpha_i}{2\, A_2} \le 1 \tag{4}$$

Applying the geometric programming techniques to the equations (3) and (4), the enlarged predual function could be written as:

$$G(\underline{W}) = \prod_{i=1}^{n} \left(\frac{K_i N_i^{\beta-1}}{W_{1i}} \right)^{W_{1i}} \left(\frac{H_i \, \overline{D}_i \, N_i \, \alpha_i}{2W_{2i}} \right)^{W_{2i}} \left(\frac{K_i \, N_i^{\beta-1}}{A_1 W_{3i}} \right)^{W_{3i}} \left(\frac{H_i \overline{D}_i \, N_i \, \alpha_i}{2\, A_2 W_{4i}} \right)^{W_{4i}}$$

$$= \prod_{i=1}^{n} \left(\frac{K_i}{W_{1i}} \right)^{W_{1i}} \left(\frac{H_i \overline{D}_i \alpha_i}{2W_{2i}} \right)^{W_{2i}} \left(\frac{K_i}{A_1 W_{3i}} \right)^{W_{3i}} \left(\frac{H_i \overline{D}_i \, \alpha_i}{2\, A_2 W_{4i}} \right)^{W_{4i}} \times N_i^{(\beta-1)W_{1i} + W_{2i} + (\beta-1)W_{3i} + W_{4i}} \tag{5}$$

Where the dual variable vector $\underline{W} = W_{ji}$, $0 < W_{ji} < 1$, $j = 1,2,3,4$, $i = 1,2,^{*}$, n is arbitrary and can be chosen according to convenience subject to the normality condition:

$$W_{1i} + W_{2i} = 1 \tag{6}$$

We choose $\underline{W}$ such that the exponent of N_i is zero, which making the right hand of (5) is independent of the decision variable. This requires:

$$(\beta - 1)W_{1i} + W_{2i} + (\beta - 1)W_{3i} + W_{4i} = 0 \tag{7}$$

This is called the orthogonality condition, which together with (6) are two linear equations in four unknowns having infinite number of solutions. However the problem is to select the optimal solution of the weights W_{ji}^{*}

Solving the equations (6) and (7), we get:

$$W_{1i} = \frac{\left(1 + (\beta - 1)W_{3i} + W_{4i}\right)}{2 - \beta} \quad \textbf{and} \quad W_{2i} = \frac{(1-\beta)(1+W_{3i}) - W_{4i}}{2-\beta} \quad ,i = 1,2,* \quad ,n. \tag{8}$$

Substituting W_{1i} and W_{2i} in equation (5), the dual function is given by:

$$g(W_{3i}, W_{4i}) = \prod_{i=1}^{n} \left(\frac{K_i}{W_{1i}}\right)^{W_{1i}} \left(\frac{H_i \bar{D}_i \alpha_i}{2W_{2i}}\right)^{W_{2i}} \left(\frac{K_i}{A_1 W_{3i}}\right)^{W_{3i}} \left(\frac{H_i \bar{D}_i \alpha_i}{2A_2 W_{4i}}\right)^{W_{4i}} \tag{9}$$

To calculate W_{3i}^* *and* W_{4i}^* *which maximize* $g(W_{3i}, W_{4i})$*, we have to take the logarithm of both sides of (9), then equate the first partial derivatives with respect to* W_{3i} *and* W_{4i} *respectively to zero and simplify, we obtain*

$$\left(\frac{2K_i\{(1-\beta)(1+W_{3i}) - W_{4i}\}}{H_i \bar{D}_i \alpha_i \{1 + (\beta - 1)W_{3i} + W_{4i}\}}\right)\left(\frac{K_i}{A_1 W_{3i}}\right)^{\frac{2-\beta}{\beta-1}} = e^{\frac{2-\beta}{\beta-1}} \tag{10}$$

and

$$\left(\frac{2K_i\{(1-\beta)(1+W_{3i}) - W_{4i}\}}{H_i \bar{D}_i \alpha_i \{1 + (\beta - 1)W_{3i} + W_{4i}\}}\right)\left(\frac{H_i \bar{D}_i \alpha_i}{2A_2 W_{4i}}\right)^{2-\beta} = e^{2-\beta} \tag{11}$$

Solving equations (10) and (11), we have:

$$W_{3i} W_{4i}^{1-\beta} = \left(\frac{K_i}{A_1 e}\right)\left(\frac{2A_2 e}{H_i \bar{D}_i \alpha_i}\right)^{\beta - 1}$$

Substituting W_{4i} **and** W_{3i} equations (10), (11) respectively, we get:

$$f_3(W_{3i}) = W_{3i}^{\frac{4-2\beta}{1-\beta}} + W_{3i}^{\frac{3-\beta}{1-\beta}} + \left(A - \frac{1}{1-\beta}\right) B_i C_{1i} W_{3i}^{\frac{2-\beta}{1-\beta}} - \frac{A}{1-\beta} B_i C_{1i} W_{3i}^{\frac{1}{1-\beta}} - \frac{A}{1-\beta} B_i^2 C_{1i}^2 = 0 \tag{12}$$

and

$$f_4(W_{4i}) = W_{4i}^{4-2\beta} + W_{4i}^{3-2\beta} + \left(\frac{1}{A} - (1-\beta)\right) B_i C_{2i} W_{4i}^{2-\beta} - \left(\frac{1-\beta}{A}\right) B_i C_{2i} W_{4i}^{1-\beta} - \left(\frac{1-\beta}{A}\right) B_i^2 C_{2i}^2 = 0 \tag{13}$$

Where:

$$B_i = \frac{K_i H_i \bar{D}_i \alpha_i}{2A_1 A_2 e^2}, \quad C_{1i} = \left(\frac{K_i}{A_1 e}\right)^{\frac{\beta}{1-\beta}}, \quad C_{2r} = \left(\frac{2K_2 e}{H_i \bar{D}_i \alpha_i}\right)^{\beta} \quad \textit{and} \quad A = \frac{A_2}{A_1}.$$

Referring to the left hand side of equations (12) and (13) as $f_j(W_{ji}), j = 3,4$ respectively. It could be easily proved that $f_j(0) < 0$ **and** $f_j(1) > 0 \ ,\forall j = 3,4$, and this is means that there exists a root $W_{ji} \in (0,1), j = 3,4$. Numerical methods such as the trial and error approach can be used to calculate these roots for each item. However, we verify that any root W_{3i}^* and W_{4i}^* calculated from equations (12) and (13) maximize the dual function $g(W_{3i}^*, W_{4i}^*)$ respectively. This will be confirming by the second derivatives to $\ln g(W_{3i}, W_{4i})$ with respect to W_{4i} **and** W_{3i}, respectively , which are always negative and yield the following condition:

$$\Delta = \left(\frac{\partial^2 \ln g(W_{3i}, W_{4i})}{\partial W_{3i} \partial W_{4i}} \right)^2 - \left(\frac{\partial^2 \ln g(W_{3i}, W_{4i})}{\partial W_{3i}^2} \right) \left(\frac{\partial^2 \ln g(W_{3i}, W_{4i})}{\partial W_{4i}^2} \right) < 0$$

Thus, the roots W_{3i}^* and W_{4i}^* calculated from equations (12) and (13) maximize the dual function $g(W_{3i}, W_{4i})$. Hence the optimal solution is $W_{ji}^*, j = 1,2,3,4,$ where W_{3i}^*, W_{4i}^* are the solution of (12), (13) and W_{1i}^*, W_{2i}^* are calculated by substituting the values of W_{3i}^*, W_{4i}^* in expression (8).

To find the optimal expected number of periods per cycle N_i^* and the expected maximum inventory level M_i^*, we applied the results of Duffin et. al. (1974) of geometric programming as:

$$K_i \ N_i^{\beta-1} = W_{1i}^* \ g(W_{3i}^*, W_{4i}^*)$$

and

$$\frac{H_i \overline{D}_i \ N_i \alpha_i}{2} = W_{2i}^* \ g(W_{3i}^*, W_{4i}^*)$$

Solving these equations, the optimal expected number of periods per cycle is given by:

$$N_i^* = \left(\frac{H_i \overline{D}_i \alpha_i \left\{ 1 + (\beta - 1) W_{3i}^* + W_{4i}^* \right\}}{2 K_i \left\{ (1 - \beta)(1 + W_{3i}^*) - W_{4i}^* \right\}} \right)^{\frac{1}{\beta - 2}}$$

Also, the optimal maximum inventory level M_i^* *is:*

$$M_i^* = a_i \ \overline{D}_i \ N_i^* = a_i \left(\frac{H_i \left(\overline{D}_i \right)^{\beta-1} \alpha_i \left\{ 1 + (\beta - 1) W_{3i}^* + W_{4i}^* \right\}}{2 K_i \left\{ (1 - \beta)(1 + W_{3i}^*) - W_{4i}^* \right\}} \right)^{\frac{1}{\beta - 2}}$$

Substituting the value of N_i^* *in equation (3) after adding the constant term, we get:*

$$\min \overline{TC} = \sum_{i=1}^{n} \left[P_i \overline{D}_i + \left(\frac{H_i \overline{D}_i K_i^{\frac{1}{1-\beta}} \alpha_i \left\{ 1 + (\beta - 1) W_{3i}^* + W_{4i}^* \right\}}{2 \left\{ (1 - \beta)(1 + W_{3i}^*) - W_{4i}^* \right\}} \right)^{\frac{\beta - 1}{\beta - 2}} + \left(\frac{H_i \overline{D}_i \alpha_i}{2} \right) \left(\frac{H_i \overline{D}_i \alpha_i \left\{ 1 + (\beta - 1) W_{3i}^* + W_{4i}^* \right\}}{2 K_i \left\{ (1 - \beta)(1 + W_{3i}^*) - W_{4i}^* \right\}} \right)^{\frac{1}{\beta - 2}} \right]$$

AS SPECIAL CASE:

Let $\beta = 0\ , i = 1$ **and** $A_1, A_2 \to \infty \Rightarrow K_i(N_i) = K =$ **constant**$, W_{3i}^*, W_{4i}^* = 0$, and $W_{1i}^* = W_{2i}^* = \frac{1}{2}$.

This is the classical probabilistic procurement model. By assuming that $a_i = 1\ ,\ \overline{D} = D$, then we get Harris's results.

STATISTICAL PROCESS CONTROL

Statistical process control is an effective method for improving a firm's quality and productivity. There has been an increased interest in their effective implementation in American industry, brought about by increased competition and improvements in quality in foreign-made products. Many tools may be utilized to gain the desired information on a firm's quality and productivity. Some of the more commonly used tools are control charts, which are useful in determining any changes in process performance. These include a variety of charts such as P charts, C charts, R and X bar charts. In this paper, I will be focusing on the latter two mentioned.

R Chart

An R Chart is a control chart that is used to monitor **process variation** when the variable of interest is a quantitative measure. Now, what does all this mean? These charts will allow us to see any deviations from desired limits within the quality process and, in effect, allow the firm to make necessary adjustments to improve quality.

X bar Charts

An x bar chart is used to monitor the **average (or mean**) value of the process over time. For each subgroup, the x bar value is plotted. The upper and lower control limits define the range of inherent variation in the subgroup means when the process is in control.

THE CHART CONSTRUCTION PROCESS

In order to construct x bar and R charts, we must first find our upper- and lower-control limits. This is done by utilizing the formula: and

While theoretically possible, since we do not know either the population process mean or standard deviation, these formulas cannot be used directly and both must be estimated from the process itself. First, the R chart is constructed. If the R chart validates that the process variation is in statistical control, the x bar chart is constructed.

AN APPLICANT EXAMPLE

Initially, we have to find the decision variables $N_i^* and\ M_i^*\ , i = 1,2,3,4$ whose values are determined to minimize the expected annual total cost $\min \overline{TC}$, and then we can confirm if the process is in control or not for the four items at different values of and .The parameters of the system are shown in TABLE1:

	$\overline{D}_i$	P_i	H_i	K_i
1	*3*	*250*	*0.35*	*320*
2	*1.8*	*100*	*0.20*	*150*
3	*3*	*200*	*0.8*	*200*
4	*5*	*400*	*0.50*	*500*

TABLE 1: The Parameters of The System

Assuming that the limitations on the total expected varying order cost and the total expected holding cost are $A_1 = 1500\$$ and $A_2 = 1200\ \$$ respectively. Using the data in TABLE1 to solve equations (12) and (13),we obtain the values of W_{3i}^* and W_{4i}^*, then $N_i^*\ and, M_i^*\ \ i = 1,2,3,4.$ can be calculated at different values of as given in the following four tables:

TABLE 2: The Expected Maximum Inventory Levels for Item 1

	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	83.135	93.639	105.97	120.24	136.16	152.45	165.26	164.6	126.12
3	93.26	101.72	111.061	121.03	130.92	139.11	141.96	131.783	92.914
5	106.911	114.89	123.354	131.92	139.699	144.87	143.75	99.5891	87.598
7	119.414	127.06	134.938	142.54	148.838	151.89	147.96	130.089	86.018
15	159.91	166.41	172.37	177.04	179.095	176.26	164.65	137.862	86.043
31	218.26	222.36	224.926	224.94	220.766	209.89	188.41	150.574	88.948
49	268.006	269.4	268.471	264.01	254.212	236.47	206.99	160.647	91.663
59	291.659	291.59	288.825	282.09	269.53	248.51	215.33	165.136	92.908
69	313.354	311.85	307.315	298.42	283.284	259.25	222.71	169.092	94.013
79	333.496	330.58	324.339	313.39	295.821	268.99	229.36	172.637	95.004
89	352.371	348.08	340.176	327.26	307.378	277.92	235.43	175.853	95.904

TABLE 3: The Expected Maximum Inventory Levels for Item 2

	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	58.676	66.698	76.258	87.524	100.42	114.11	125.85	127.88	100.3
3	65.823	72.456	79.919	88.096	96.459	104.13	108.11	102.39	73.92
5	75.458	81.831	88.766	96.027	103.02	108.44	109.47	100.34	69.69
7	84.282	90.505	97.102	103.75	109.76	113.7	112.68	101.07	68.43
15	112.864	118.53	124.038	128.87	132.08	131.93	125.39	107.11	68.45
31	154.048	158.38	161.857	163.74	162.81	157.11	143.47	116.99	70.76
49	189.158	191.89	193.192	192.18	187.47	177.01	157.63	124.82	72.93
59	205.853	207.7	207.838	205.34	198.77	186.02	163.97	128.3	73.92
69	221.164	222.12	221.144	217.23	208.92	194.06	169.59	131.37	74.79
79	235.381	235.47	233.395	228.12	218.16	201.35	174.66	134.13	75.58
89	248.703	247.93	244.79	238.22	226.68	208.03	179.28	136.63	76.3

TABLE 4: The Expected Maximum Inventory Levels for Item 3

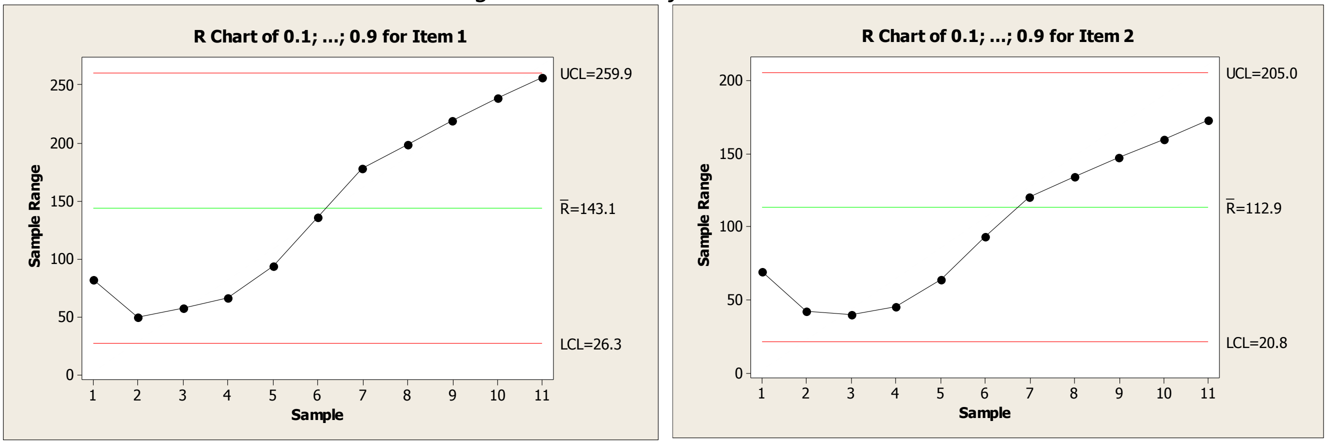
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	41.921	45.612	49.905	53.35	57.236	60.244	60.817	55.743	38.716
3	47.027	49.386	51.682	53.697	55.032	54.972	52.244	44.629	28.522
5	53.911	55.78	57.403	58.531	58.723	57.249	52.902	43.735	26.889
7	60.215	61.688	62.794	63.241	62.565	60.025	54.451	44.055	26.405
15	80.636	80.788	80.213	78.551	75.283	69.653	60.593	46.688	26.413
31	110.059	107.951	104.67	99.802	92.799	82.942	69.333	50.992	27.304
49	135.144	130.793	124.934	117.14	106.86	93.448	76.175	54.404	28.138
59	147.071	141.565	134.405	125.158	113.298	98.207	79.241	55.924	28.52
69	158.01	151.401	143.01	132.406	119.08	102.451	81.958	57.264	28.859
79	168.167	160.496	150.932	139.048	124.35	106.298	84.405	58.464	29.163
89	177.685	168.99	158.301	145.2	129.21	109.83	86.637	59.554	29.44

TABLE 5: The Expected Maximum Inventory Levels for Item 4

	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
1	110.8	123.2	137.6	153.7	170.998	187.65	198.77	192.7	143.02
3	124.3	133.9	144.16	154.7	164.41	171.23	170.75	154.28	105.36
5	142.4	151.2	160.12	168.6	175.44	178.32	172.9	151.19	99.33
7	159.1	167.22	175.15	182.17	186.92	186.97	177.96	152.3	97.54
15	213.1	218.99	223.74	226.27	224.92	216.95	198.04	161.4	97.57
31	290.8	292.6	291.96	287.5	277.25	258.35	226.6	176.28	100.86
49	357.07	354.5	348.5	337.42	319.25	291.07	248.96	188.07	103.94
59	388.59	383.7	374.9	360.53	338.5	305.89	258.98	193.33	105.35
69	417.49	410.4	398.9	381.41	355.76	319.11	267.86	197.96	106.6
79	444.33	435.1	421	400.54	371.51	331.09	275.86	202.11	107.73
89	469.47	458.1	441.6	418.26	386.02	342.08	283.15	205.87	108.75

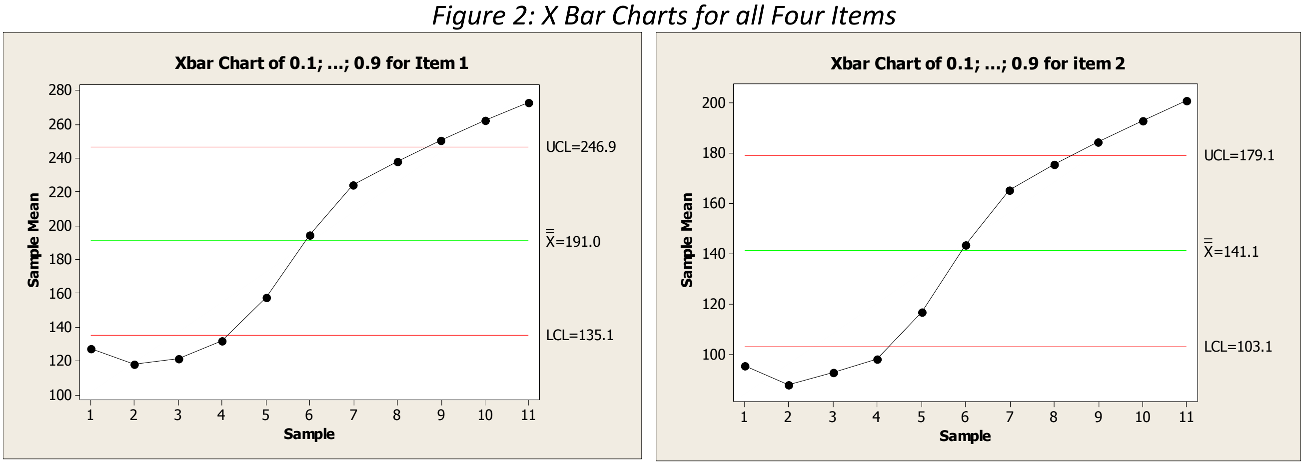
Utilizing the data given in TABLE 2 to TABLE 5, **R charts** can be obtained as illustrated in Figure 1 for each item for all values of which are assumed to be our subgroups to test the quality control of the system. We can see that the process is in control for all items at all values of

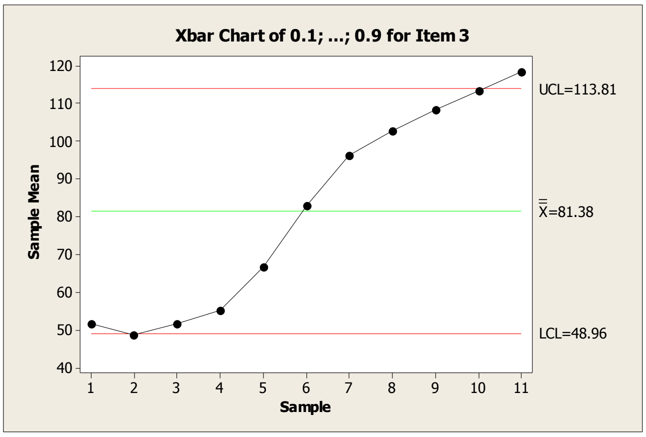
Figure 1: R Charts for all Four Items

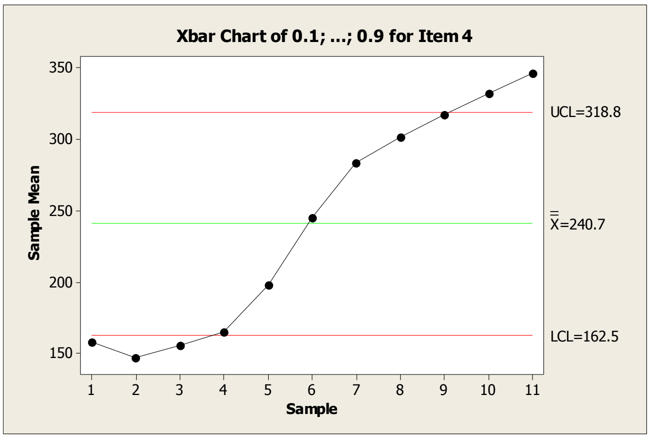


*Since the process is in control for all items, consequently we will construct **X bar charts** for all items as shown in Figure 2.*

Figure 2: X Bar Charts for all Four Items







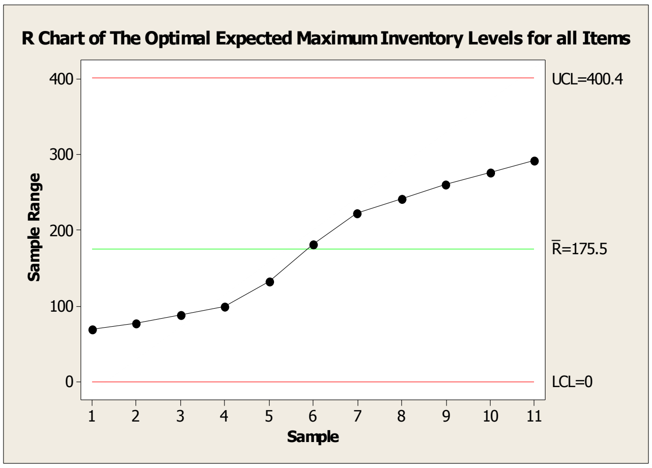
We can note in Figure 2 that the process for some items is not controlled at some points. The point is out of control if it is above the line UCL or under the LUL. Therefore we must calculate the expected total cost $\overline{TC}$ for each item at all values of , then it will be easy to find the optimal values of $M_i^*, \forall i = 1,2,3,4$ that corresponding to the $\min \overline{TC}$ for all values of at particular value of as presented in TABLE 6.

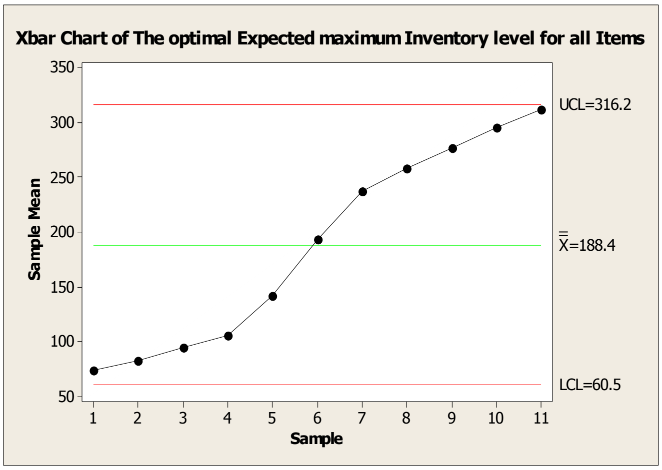
TABLE 6: The Optimal Expected Inventory Levels for All Items

	Item 1	Item 2	Item 3	Item 4		
1	*83.1346*	*58.676*	*41.921*	*110.8*	***68.879***	***73.6329***
3	*93.26*	*65.823*	*47.027*	*124.3*	***77.273***	***82.6025***
5	*106.911*	*75.458*	*53.911*	*142.4*	***88.489***	***94.67***
7	*119.414*	*84.282*	*60.215*	*159.1*	***98.885***	***105.75275***
15	*159.91*	*112.864*	*80.636*	*213.1*	***132.464***	***141.6275***
31	*218.26*	*154.048*	*110.059*	*290.8*	***180.741***	***193.29175***
49	*268.006*	*189.158*	*135.144*	*357.07*	***221.926***	***237.3445***
59	*291.659*	*205.853*	*147.071*	*388.59*	***241.519***	***258.29325***
69	*313.354*	*221.164*	*158.01*	*417.49*	***259.48***	***277.5045***
79	*333.496*	*235.381*	*168.167*	*444.33*	***276.163***	***295.3435***
89	*352.371*	*248.703*	*177.685*	*469.47*	***291.785***	***312.05725***

From TABLE 6, we calculate the range of all subgroups in column that used for R chart. Similarly, the sample mean of each sub-group is calculated in column to draw X bar chart as shown in Figure 3, which indicates that all subgroups of all items are under control by both R chart and X bar chart, means that the process is in control.

Figure 3: R and X Bar Charts for all Four Items





CONCLUSION

We deduced the optimal expected number of periods $N_i^*,\ i = 1,2,\ldots,n,$ consequently, the expected maximum inventory level M_i^* obtained. Using the previous values of M_i^* to get the R charts for all items that indicate that the system is in control, so we constructed the X bar charts for all items that indicated that the mean values of all subgroups in not in control. Finally, we used the minimum expected total cost $\min \overline{TC}$ to decide the optimal values of M_i^* for all items and then we made both the R and X bar charts to confirm that the values from our optimization for the process are under control. Therefore, the constrained probabilistic MISS safety stock Inventory system with continuous increasing order cost function is in control.

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