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SOME NEW FIFTH MULTIPLICATIVE ZAGREB INDICES OF PAMAM DENDRIMERS

V.R.Kulli
Department of Mathematics
Gulbarga University, Gulbarga 585106, India
E-mail: vrkulli@gmail.com

Abstract: In this paper, we introduce the general fifth multiplicative M-Zagreb indices, fifth multiplicative product connectivity index, fifth multiplicative sum connectivity index of a molecular graph and determine these multiplicative indices for PAMAM dendrimers. Also we compute the fourth multiplicative atom bond connectivity index and fifth multiplicative geometric-arithmetic index for PAMAM dendrimers.

Keywords: molecular graph, fifth multiplicative indices, PAMAM dendrimer.
Mathematics Subject Classification: 05C05, 05C07, 05C35.

1. INTRODUCTION

In this paper, we consider only a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of a molecular graph represents an atom of the molecule and its edges to the bonds between atoms. Chemical Graph Theory has an important effect on the development of Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have been considered in Theoretical Chemistry and have found some applications, especially in *QSPRlQSAR* study, see [1].

Let $S_G(u)$ denote the sum of the degrees of all vertices adjacent to a vertex u. For all further notation and terminology we refer the reader to [2].

In [3], Kulli introduced the fifth multiplicative Zagreb indices as

$$M_1G_5II(G) = \prod_{uv\in E(G)} \left[S_G(u)+S_G(v)\right], \qquad M_2G_5II(G) = \prod_{uv\in E(G)} S_G(u)S_G(v).$$

Motivated by the definitions of the multiplicative Zagreb indices, we define other multiplicative indices of a graph as follows:

The fifth multiplicative hyper-Zagreb indices of a graph G are defined as

$$HM_1G_5II(G) = \prod_{uv\in E(G)} \left[S_G(u)+S_G(v)\right]^2, \qquad HM_2G_5II(G) = \prod_{uv\in E(G)} \left[S_G(u)S_G(v)\right]^2.$$

The general fifth multiplicative Zagreb indices of a graph G are defined as

$$M_1^aG_5II(G) = \prod_{uv\in E(G)} \left[S_G(u)+S_G(v)\right]^a, \qquad M_2^aG_5II(G) = \prod_{uv\in E(G)} \left[S_G(u)S_G(v)\right]^a \tag{1}$$

where a is a real number.

The fifth multiplicative product connectivity index of a graph G is defined as

$$P_5II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{S_G(u)S_G(v)}}.$$

The fifth multiplicative sum connectivity index of a graph G is defined as

$$S_5II(G)=\prod_{uv\in E(G)}\frac{1}{\sqrt{S_G(u)+S_G(v)}}.$$

In [4], Kulli proposed the fourth multiplicative atom bond connectivity index of a graph G and it is defined as

$$ABC_4II(G)=\prod_{uv\in E(G)}\sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}}. \qquad (2)$$

In [3], Kulli defined the fifth multiplicative geometric-arithmetic index of a graph G and it is defined as

$$GA_5II(G)=\prod_{uv\in E(G)}\frac{2\sqrt{S_G(u)S_G(v)}}{S_G(u)+S_G(v)}. \qquad (3)$$

Recently many other multiplicative indices were studied, for example, in [5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18].

In this paper, the general fifth multiplicative M-Zagreb indices, fifth multiplicative M-Zagreb indices, fifth multiplicative product connectivity index, fifth multiplicative sum connectivity index, fourth multiplicative atom bond connectivity index and fifth multiplicative geometric-arithmetic index of a PAMAM dendrimer $PD_1[n]$ are computed. For more information about PAMAM dendrimers, see [19, 20].

2. PAMAM DENDRIMER

In this section, we consider an important chemical structure $PD_1[n]$, where n is the steps of growth in this type of PAMAM dendrimers for $n\geq 0$. Figure 1 shows the molecular structure of $PD_1[n]$.

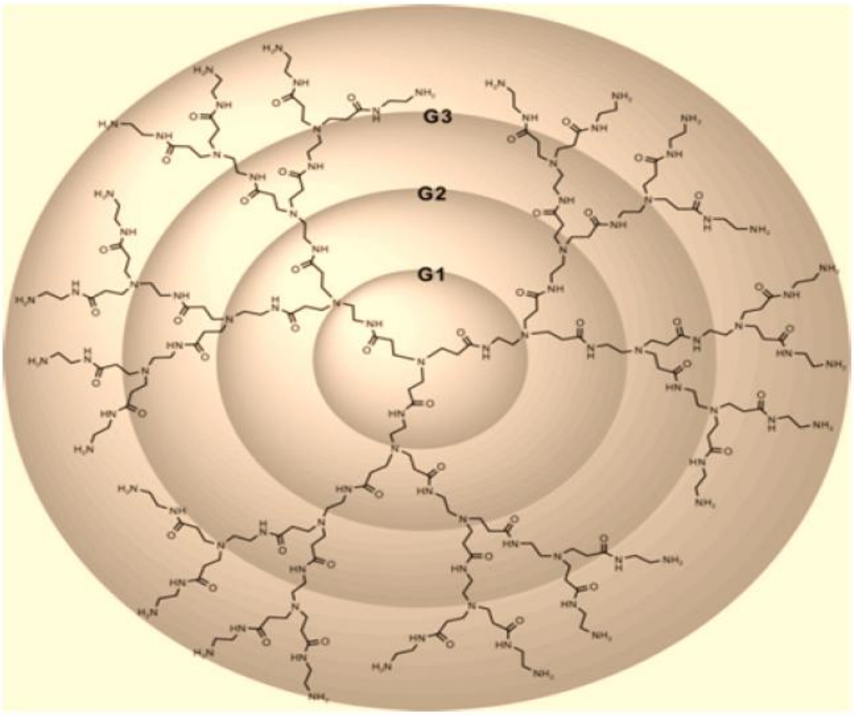

Figure 1. The molecular structure of PAMAM dendrimer $PD_1[n]$

Let G be the molecular graph of PAMAM dendrimer $PD_1[n]$. By calcuation, we have $|V(PD_1[n])|=12\times 2^{n+2}-23$ and $|E(PD_1[n])|=12\times 2^{n+2}-24$. Also the edge partition of the form (2, 3), (3, 4), (3, 5), (4, 5), (5,5), (5,6) for PAMAM dendrimer based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 1.

Table 1. Edge partition of $PD_1[n]$.

$S_G(u), S_G(v)\backslash uv\in E(G)$	(2, 3)	(3, 4)	(3, 5)	(4, 5)	(5, 5)	(5, 6)

Number of edges	3×2^n	3×2^n	$6\times2^n-3$	$9\times2^n-6$	$18\times2^n-9$	$9\times2^n-6$

Theorem 1. The general fifth multiplicative M_1-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$M_1^aG_5II(G)=5^{(3\times2^n)a}\times7^{(3\times2^n)a}\times8^{(6\times2^n-3)a}\times9^{(9\times2^n-6)a}\times10^{(18\times2^n-9)a}\times11^{(9\times2^n-6)a}. \qquad (4)$$

Proof: From equation (1) and Table 1, we deduce

$$M_1^aG_5II(G)=\prod_{uv\in E(G)}\left[S_G(u)+S_G(v)\right]^a$$

$$=5^{(3\times2^n)a}\times7^{(3\times2^n)a}\times8^{(6\times2^n-3)a}\times9^{(9\times2^n-6)a}\times10^{(18\times2^n-9)a}\times11^{(9\times2^n-6)a}.$$

Corollary 1.1. The fifth multiplicative M_1-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$M_1G_5II(G)=5^{3\times2^n}\times7^{3\times2^n}\times8^{6\times2^n-3}\times9^{9\times2^n-6}\times10^{18\times2^n-9}\times11^{9\times2^n-6}.$$

Proof: Put a = 1 in equation (4), we get the desired result.

Corollary 1.2. The fifth multiplicative M_1 hyper-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$HM_1G_5II(G)=5^{3\times2^{n+1}}\times7^{3\times2^{n+1}}\times8^{6\times2^{n+1}-6}\times9^{9\times2^{n+1}-12}\times10^{18\times2^{n+1}-18}\times11^{9\times2^{n+1}-12}.$$

Proof: Put a = 2 in equation (4), we get the desired result.

Corollary 1.3. The fifth multiplicative sum connectivity index of a PAMAM dendrimer $PD_1[n]$ is

$$S_5II(G)=\left(\frac{1}{\sqrt{5}}\right)^{3\times2^n}\times\left(\frac{1}{\sqrt{7}}\right)^{3\times2^n}\times\left(\frac{1}{\sqrt{8}}\right)^{6\times2^n-3}\times\left(\frac{1}{\sqrt{9}}\right)^{9\times2^n-6}\times\left(\frac{1}{\sqrt{10}}\right)^{18\times2^n-9}\times\left(\frac{1}{\sqrt{11}}\right)^{9\times2^n-6}$$

Proof: Put $a = -\frac{1}{2}$ in equation (4), we get the desired result.

Theorem 2. The general fifth multiplicative M_2-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$M_2^aG_5II(G)=6^{(3\times2^n)a}\times12^{(3\times2^n)a}\times15^{(6\times2^n-3)a}\times20^{(9\times2^n-6)a}\times25^{(18\times2^n-9)a}\times30^{(9\times2^n-6)a}. \qquad (5)$$

Proof: From equation (1) and Table 1, we deduce

$$M_2^aG_5II(G)=\prod_{uv\in E(G)}\left[S_G(u)S_G(v)\right]^a$$

$$=6^{(3\times2^n)a}\times12^{(3\times2^n)a}\times15^{(6\times2^n-3)a}\times20^{(9\times2^n-6)a}\times25^{(18\times2^n-9)a}\times30^{(9\times2^n-6)a}.$$

Corollary 2.1. The fifth multiplicative M_2-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$M_2G_5II(G)=6^{3\times2^n}\times12^{3\times2^n}\times15^{6\times2^n-3}\times20^{9\times2^n-6}\times25^{18\times2^n-9}\times30^{9\times2^n-6}.$$

Proof: Put a = 1 in equation (5), we get the desired result.

Corollary 2.2. The fifth multiplicative M_2 hyper-Zagreb index of a PAMAM dendrimer $PD_1[n]$ is

$$HM_2G_5II(G)=6^{3\times2^{n+1}}\times12^{3\times2^{n+1}}\times15^{6\times2^{n+1}-6}\times20^{9\times2^{n+1}-12}\times25^{18\times2^{n+1}-18}\times30^{9\times2^{n+1}-12}.$$

Proof: Put a = 2 in equation (5), we get the desired result.

Corollary 2.3. The fifth multiplicative product connectivity index of a PAMAM dendrimer $PD_1[n]$ is

$$P_5II(G)=\left(\frac{1}{\sqrt{6}}\right)^{3\times2^n}\times\left(\frac{1}{\sqrt{12}}\right)^{3\times2^n}\times\left(\frac{1}{\sqrt{15}}\right)^{6\times2^n-3}\times\left(\frac{1}{\sqrt{20}}\right)^{9\times2^n-6}\times\left(\frac{1}{\sqrt{25}}\right)^{18\times2^n-9}\times\left(\frac{1}{\sqrt{30}}\right)^{9\times2^n-6}.$$

Proof: Put $a = -\frac{1}{2}$ in equation (5), we get the desired result.

Theorem 3. The fourth multiplicative atom bond connectivity index of a PAMAM dendrimer $PD_1[n]$ is

$$ABC_4II(G)=\left(\sqrt{\frac{1}{2}}\right)^{3\times2^n}\times\left(\sqrt{\frac{5}{12}}\right)^{3\times2^n}\times\left(\sqrt{\frac{2}{5}}\right)^{6\times2^n-3}\times\left(\sqrt{\frac{7}{20}}\right)^{9\times2^n-6}\times\left(\sqrt{\frac{8}{25}}\right)^{18\times2^n-9}\times\left(\sqrt{\frac{3}{10}}\right)^{9\times2^n-6}.$$

Proof: From equation (2) and Table 1, we deduce

$$ABC_4II(G)=\prod_{uv\in E(G)}\sqrt{\frac{S_G(u)+S_G(v)-2}{S_G(u)S_G(v)}}$$

$$=\left(\sqrt{\frac{2+3-2}{2\times 3}}\right)^{3\times 2^n}\times\left(\sqrt{\frac{3+4-2}{3\times 4}}\right)^{3\times 2^n}\times\left(\sqrt{\frac{3+5-2}{3\times 5}}\right)^{6\times 2^n-3}$$

$$\times\left(\sqrt{\frac{4+5-2}{4\times 5}}\right)^{9\times 2^n-6}\times\left(\sqrt{\frac{5+5-2}{5\times 5}}\right)^{18\times 2^n-9}\times\left(\sqrt{\frac{5+6-2}{5\times 6}}\right)^{9\times 2^n-6}.$$

$$=\left(\sqrt{\frac{1}{2}}\right)^{3\times 2^n}\times\left(\sqrt{\frac{5}{12}}\right)^{3\times 2^n}\times\left(\sqrt{\frac{2}{5}}\right)^{6\times 2^n-3}\times\left(\sqrt{\frac{7}{20}}\right)^{9\times 2^n-6}\times\left(\sqrt{\frac{8}{25}}\right)^{18\times 2^n-9}\times\left(\sqrt{\frac{3}{10}}\right)^{9\times 2^n-6}.$$

Theorem 4. The fifth multiplicative geometric-arithmetic index of a PAMAM dendrimer $PD_1[n]$ is

$$GA_5II(G)=\left(\frac{2\sqrt{6}}{5}\right)^{3\times 2^n}\times\left(\frac{4\sqrt{3}}{7}\right)^{3\times 2^n}\times\left(\frac{\sqrt{15}}{4}\right)^{6\times 2^n-3}\times\left(\frac{2\sqrt{20}}{9}\right)^{9\times 2^n-6}\times\left(\frac{2\sqrt{30}}{11}\right)^{9\times 2^n-6}.$$

Proof: From equation (3) and Table 1, we deduce

$$GA_5II(G)=\prod_{uv\in E(G)}\frac{2\sqrt{S_G(u)S_G(v)}}{S_G(u)+S_G(v)}$$

$$=\left(\frac{2\sqrt{2\times 3}}{2+3}\right)^{3\times 2^n}\times\left(\frac{2\sqrt{3\times 4}}{3+4}\right)^{3\times 2^n}\times\left(\frac{2\sqrt{3\times 5}}{3+5}\right)^{6\times 2^n-3}$$

$$\times\left(\frac{2\sqrt{4\times 5}}{4+5}\right)^{9\times 2^n-6}\times\left(\frac{2\sqrt{5\times 5}}{5+5}\right)^{18\times 2^n-9}\times\left(\frac{2\sqrt{5\times 6}}{5+6}\right)^{9\times 2^n-6}.$$

$$=\left(\frac{2\sqrt{6}}{5}\right)^{3\times 2^n}\times\left(\frac{4\sqrt{3}}{7}\right)^{3\times 2^n}\times\left(\frac{\sqrt{15}}{4}\right)^{6\times 2^n-3}\times\left(\frac{2\sqrt{20}}{9}\right)^{9\times 2^n-6}\times\left(\frac{2\sqrt{30}}{11}\right)^{9\times 2^n-6}.$$

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