

INTUITIONISTIC FUZZY CONTRA π GENERALIZED SEMI OPEN MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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Abstract: In this paper we introduce intuitionistic fuzzy contra π generalized semi open mappings, intuitionistic fuzzy contra π generalized semi closed mappings and intuitionistic fuzzy contra $M\pi$ -generalized semi open mappings in intuitionistic fuzzy topological spaces and some of their basic properties are studied.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy contra π generalized semi open mappings, intuitionistic fuzzy contra π generalized semi closed mappings and intuitionistic fuzzy contra $M\pi$ -generalized semi open mappings.

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INTRODUCTION

Zadeh [15] introduced the notion of fuzzy sets. After which there have been a number of generalizations on this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [1] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [2] introduced the notion of intuitionistic fuzzy topological space. In this paper we introduce the notion of intuitionistic fuzzy contra π generalized semi open mappings and intuitionistic fuzzy contra π generalized semi closed mappings. We also introduce intuitionistic fuzzy contra $M\pi$ -generalized semi open mappings. We investigate some of their properties and arrive at some characterizations of intuitionistic fuzzy contra π - generalized semi open mappings and intuitionistic fuzzy contra π - generalized semi closed mappings.

PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the values $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by $IFS(X)$.

Definition 2.2: [1] Let A and B be IFS's of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and

$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$,

$$(d) A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \},$$

$$(e) A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}.$$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT in short) on a non empty set X is a family τ of IFS in X satisfying the following axioms:

$$(a) 0_\sim, 1_\sim \in \tau$$

$$(b) G_1 \cap G_2 \in \tau, \text{ for any } G_1, G_2 \in \tau$$

$$(c) \cup G_i \in \tau \text{ for any arbitrary family } \{G_i / i \in J\} \subseteq \tau.$$

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X .

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.4: [2] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition 2.5: An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) [12] intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,

(ii) [10] intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,

(iii) [11] intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$.

The family of all IFSCSs, IF α CSs and IFPCSs (respectively IFSOs, IF α OSs and IFPOs) of an IFTS (X, τ) is denoted by IFSC(X), IF α C(X) and IFPC(X) (respectively IFSO(X), IF α O(X) and IFPO(X)).

Definition 2.6: [14] Let A be an IFS in an IFTS (X, τ) . Then

$$\text{sint}(A) = \cup \{ G / G \text{ is an IFSO in } X \text{ and } G \subseteq A \},$$

$$\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{scl}(A^c) = (\text{sint}(A))^c$ and $\text{sint}(A^c) = (\text{scl}(A))^c$.

Definition 2.7: [13] The IFS $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$ where $\alpha \in (0, 1]$, $\beta \in [0, 1)$ and $\alpha + \beta \leq 1$ is called an intuitionistic fuzzy point (IFP) in X .

Definition 2.8: [13] Two IFSs are said to be q -coincident ($A \text{ }_q \text{ } B$) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.9: [13] An IFS A in an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.10: [9] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition 2.11: [9] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X .

Definition 2.12: [4] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π -generalized semi closed set (IF π GSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .

Result 2.13: [8] Every IFCS, IFGCS, IFRCs, IF α CS, IFGSCS is an IF π GSCS but the converses may not be true in general.

Definition 2.14: [6] Let A be an IFS in an IFTS (X, τ) . Then

$$\pi\text{gsint}(A) = \cup \{ G / G \text{ is an IF}\pi\text{GSOS in } X \text{ and } G \subseteq A \},$$

$$\pi\text{gscl}(A) = \cap \{ K / K \text{ is an IF}\pi\text{GSCS in } X \text{ and } A \subseteq K \}.$$

Definition 2.15:[5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi T_{1/2}$ (IF $\pi T_{1/2}$ in short) space if every IF π GSOS in X is an IFOS in X .

Definition 2.16:[5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\alpha T_{1/2}$ (IF $\pi\alpha T_{1/2}$ in short) space if every IF π GSCS in X is an IFCS in X .

Definition 2.17:[5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi b T_{1/2}$ (IF $\pi b T_{1/2}$ in short) space if every IF π GSCS in X is an IFGCS in X .

Definition 2.18:[5] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi c T_{1/2}$ (IF $\pi c T_{1/2}$ in short) space if every IF π GSCS in X is an IFGSCS in X .

3. INTUITIONISTIC FUZZY CONTRA π GENERALIZED SEMI OPEN MAPPINGS

In this section we have introduced intuitionistic fuzzy contra π generalized semi open mappings. We investigated some of its properties.

Definition 3.1: A mapping $f: X \rightarrow Y$ is said to be an intuitionistic fuzzy contra π generalized semi open mapping (IFC π GSOM in short) if $f(A)$ is an IF π GSCS in (Y, σ) for every IFOS A in (X, τ) .

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFC π GSOM.

Definition 3.3: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy contra π generalized semi closed mapping (IFC π GS closed in short) if for every IFCS A of (X, τ) , $f(A)$ is an IF π GSOS in (Y, σ) .

Definition 3.4: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi d T_{1/2}$ (IF $\pi d T_{1/2}$ in short) space if every IF π GSCS in X is an IFSCS in X .

Theorem 3.5: For a bijective mapping $f: X \rightarrow Y$, where Y is an IF $\pi d T_{1/2}$ space, the following are equivalent.

- (i) f is an IFC π GSOM
- (ii) for every IFCS A in X , $f(A)$ is an IF π GSOS in Y
- (iii) for every IFOS B in X , $f(B)$ is an IF π GSCS in Y
- (iv) for any IFCS A in X and for any IFP $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta))_q A$, then $c(\alpha, \beta)_q \text{sint}(f(A))$
- (v) For any IFCS A in X and for any $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta))_q A$, then there exists an IF π GSOS B such that $c(\alpha, \beta)_q B$ and $f^{-1}(B) \subseteq A$.

Proof: (i) $\Rightarrow$ (ii) Let A be an IFCS in X . Then A^c is an IFOS in X . By hypothesis, $f(A^c)$ is an IF π GSCS in Y . That is $f(A)^c$ is an IF π GSCS in Y . Hence $f(A)$ is an IF π GSOS in Y . (ii) $\Rightarrow$ (i) Let A be an IFOS in X . Then A^c is an IFCS in X . By hypothesis, $f(A^c)$ = $[f(A)]^c$ is an IF π GSOS in Y . Hence $f(A)$ is an IF π GSCS in Y . Thus f is an IFC π GSOM. (ii) $\Leftrightarrow$ (iii) is obvious. (ii) $\Rightarrow$ (iv) Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f(A)$. By hypothesis, $f(A)$ is an IF π GSOS in Y . Since Y is an IF $\pi d T_{1/2}$ space, $f(A)$ is an IFSOS in Y . This implies $\text{sint}(f(A)) = f(A)$. Hence $c(\alpha, \beta)_q \text{sint}(f(A))$. (iv) $\Rightarrow$ (ii) Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f(A)$. By hypothesis this implies $c(\alpha, \beta)_q \text{sint}(f(A))$. That is $f(A) \subseteq \text{sint}(f(A))$. Therefore $f(A) = \text{sint}(f(A))$ is an IFSOS in Y and hence an IF π GSOS in Y . (iv) $\Rightarrow$ (v) Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f(A)$. This implies $c(\alpha, \beta)_q \text{sint}(f(A))$. Thus $f(A)$ is an IFSOS in Y and

hence an $IF\pi GSOS$ in Y . Let $f(A) = B$. Therefore $c(\alpha, \beta)_q B$ and $f^{-1}(B) = f^{-1}(f(A)) = A$, since f is a bijective mapping. (v) $\Rightarrow$ (iv)

Let $A \subseteq X$ be an IFCS and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta))_q A$. Then $c(\alpha, \beta)_q f(A)$. By hypothesis there exists an $IF\pi GSOS$ B in Y such that $c(\alpha, \beta)_q B$ and $f^{-1}(B) \subseteq A$. Let $B = f(A)$. Then $c(\alpha, \beta)_q f(A)$. Since Y is an $IF\pi dT_{1/2}$ space, $f(A)$ is an IFSOS in Y . Therefore $c(\alpha, \beta)_q \text{sint}(f(A))$.

Theorem 3.6: Let $f: X \rightarrow Y$ be a bijective mapping. Suppose that one of the following properties hold.

- (i) $f^{-1}(\text{scl}(A)) \subseteq \text{int}(f^{-1}(A))$ for each IFS A in Y
- (ii) $\text{scl}(f(B)) \subseteq f(\text{int}(B))$ for each IFS B in X
- (iii) $f(\text{cl}(B)) \subseteq \text{sint}(f(B))$ for each IFS B in X

Then f is an $IFC\pi GSOM$.

Proof: (i) $\Rightarrow$ (ii) Let $B \subseteq X$. Then $f(B)$ is an IFS in Y . By hypothesis, $f^{-1}(\text{scl}(f(B))) \subseteq \text{int}(f^{-1}(f(B))) = \text{int}(B)$. Now $\text{scl}(f(B)) = f(f^{-1}(\text{scl}(f(B)))) \subseteq f(\text{int}(B))$. (ii) $\Rightarrow$ (iii) is obvious by taking complement in (ii). Suppose (iii) holds. Let A be an IFCS in X . Then $\text{cl}(A) = A$ and $f(A)$ is an IFS in Y . Now $f(A) = f(\text{cl}(A)) \subseteq \text{sint}(f(A)) \subseteq f(A)$, by hypothesis. This implies $f(A)$ is an IFSOS in Y and hence an $IF\pi GSOS$ in Y . Thus f is an $IFC\pi GSOM$ by Theorem 3.5.

Theorem 3.7: Let $f: X \rightarrow Y$ be a bijective mapping. Then f is an $IFC\pi GSOM$ if $\text{cl}(f^{-1}(A)) \subseteq f^{-1}(\text{sint}(A))$ for every IFS A in Y .

Proof: Let A be an IFCS in X . Then $\text{cl}(A) = A$ and $f(A)$ is an IFS in Y . By hypothesis $\text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{sint}(f(A)))$. Since f is one to one $f^{-1}(f(A)) = A$. Therefore $A = \text{cl}(A) = \text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(\text{sint}(f(A)))$. Now $f(A) \subseteq f(f^{-1}(\text{sint}(f(A)))) = \text{sint}(f(A)) \subseteq f(A)$. Hence $f(A)$ is an IFSOS in Y and hence an $IF\pi GSOS$ in Y . Thus f is an $IFC\pi GSOM$ by Theorem 3.5.

Theorem 3.8: If $f: X \rightarrow Y$ is an $IFC\pi GSOM$, where Y is an $IF\pi dT_{1/2}$ space, then the following conditions hold.

- (i) $\text{scl}(f(B)) \subseteq f(\text{int}(\text{scl}(B)))$ for every IFOS B in X
- (ii) $f(\text{cl}(\text{sint}(B))) \subseteq \text{sint}(f(B))$ for every IFCS B in X

Proof: (i) Let $B \subseteq X$ be an IFOS. Then $\text{int}(B) = B$. By hypothesis $f(B)$ is an $IF\pi GSCS$ in Y . Since Y is an $IF\pi dT_{1/2}$ space, $f(B)$ is an IFSCS in Y . This implies $\text{scl}(f(B)) = f(B) = f(\text{int}(B)) \subseteq f(\text{int}(\text{scl}(B)))$.

(ii) can be proved easily by taking complement in (i).

Theorem 3.9: A mapping $f: X \rightarrow Y$ is an $IFC\pi GSOM$ if $f(\text{scl}(B)) \subseteq \text{int}(f(B))$ for every IFS B in X .

Proof: Let $B \subseteq X$ be an IFCS. Then $\text{cl}(B) = B$. Since every IFCS is an IFSCS, $\text{scl}(B) = B$. Now by hypothesis, $f(B) = f(\text{scl}(B)) \subseteq \text{int}(f(B)) \subseteq f(B)$. This implies $f(B)$ is an IFOS in Y . Therefore $f(B)$ is an $IF\pi GSOS$ in Y . Hence f is an $IFC\pi GSOM$.

Theorem 3.10: A mapping $f: X \rightarrow Y$ is an $IFC\pi GSOM$, where Y is an $IF\pi dT_{1/2}$ space if and only if $f(\text{scl}(B)) \subseteq \text{sint}(f(\text{cl}(B)))$ for every IFS B in X .

Proof: Necessity: Let $B \subseteq X$ be an IFS. Then $\text{cl}(B)$ is an IFCS in X . By hypothesis, $f(\text{cl}(B))$ is an $IF\pi GSOS$ in Y . Since Y is an $IF\pi dT_{1/2}$ space, $f(\text{cl}(B))$ is an IFSOS in Y . Therefore $f(\text{scl}(B)) \subseteq f(\text{cl}(B)) = \text{sint}(f(\text{cl}(B)))$.

Sufficiency: Let $B \subseteq X$ be an IFCS. Then $\text{cl}(B) = B$. By hypothesis, $f(\text{scl}(B)) \subseteq \text{sint}(f(\text{cl}(B))) = \text{sint}(f(B))$. But $\text{scl}(B) = B$. Therefore $f(B) = f(\text{scl}(B)) \subseteq \text{sint}(f(B)) \subseteq f(B)$. This implies $f(B)$ is an IFSOS in Y and hence an $IF\pi GSOS$ in Y . Hence f is an $IFC\pi GSOM$.

Theorem 3.11: An IFOM $f: X \rightarrow Y$ is an $IFC\pi GSOM$ if $IF\pi GSO(Y) = IF\pi GSC(Y)$.

Proof: Let $A \subseteq X$ be an IFOS. By hypothesis, $f(A)$ is an IFOS in Y and hence is an $IF\pi GSOS$ in Y . By assumption $f(A)$ is an $IF\pi GSCS$ in Y . Therefore f is an $IFC\pi GSOM$.

Definition 3.12: A mapping $f: X \rightarrow Y$ is said to be an intuitionistic fuzzy contra $M\pi$ -generalized semi open mapping ($IFCM\pi GSOM$) if $f(A)$ is an $IF\pi GSCS$ in Y for every $IF\pi GSOS$ A in X .

Example 3.13: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v) \rangle$. Then $\tau = \{0., G_1, 1.\}$ and $\sigma = \{0., G_2, 1.\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an $IFCM\pi GSOM$.

Theorem 3.14: Let $f: X \rightarrow Y$ be a bijective mapping. Then the following are equivalent.

- (i) f is an $IFCM\pi GSOM$
- (ii) $f(A)$ is an $IF\pi GSOS$ in Y for every $IF\pi GSCS$ A in X

Proof: (i) $\Rightarrow$ (ii) Let A be an $IF\pi GSCS$ in X . Then A^c is an $IF\pi GSOS$ in X . By hypothesis, $f(A^c)$ is an $IF\pi GSCS$ in Y . That is $f(A)^c$ is an $IF\pi GSCS$ in Y . Hence $f(A)$ is an $IF\pi GSOS$ in Y . (ii) $\Rightarrow$ (i) Let A be an $IF\pi GSOS$ in X . Then A^c is an $IF\pi GSCS$ in X . By hypothesis, $f(A^c) = [f(A)]^c$ is an $IF\pi GSOS$ in Y . Hence $f(A)$ is an $IF\pi GSCS$ in Y . Thus f is an $IFCM\pi GSOM$.

Theorem 3.15: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if Y is an $IF\pi_a T_{1/2}$ space:

- (i) f is an $IFC\pi GSOM$
- (ii) f is an $IFC\pi GSOM$
- (iii) $\text{int}(\text{cl}(f(A))) \subseteq f(A)$ for every IFOS A in X .

Proof: (i) $\Rightarrow$ (ii): It is obviously true. (ii) $\Rightarrow$ (iii): Let A be an IFOS in X . Then $f(A)$ is an $IF\pi GSCS$ in Y . Since Y is an $IF\pi_a T_{1/2}$ space, $f(A)$ is an IFCS in Y . Therefore $\text{cl}(f(A)) = f(A)$. This implies $\text{int}(\text{cl}(f(A))) \subseteq f(A)$. (iii) $\Rightarrow$ (i): Let A be an IFCS in X . Then its complement A^c is an IFOS in X . By hypothesis, $\text{int}(\text{cl}(f(A^c))) \subseteq f(A^c)$. Hence $f(A^c)$ is an IFSCS in Y . Since every IFSCS is an $IF\pi GSCS$, $f(A^c)$ is an $IF\pi GSCS$ in X . Therefore $f(A)$ is an $IF\pi GSOS$ in X . Hence f is an $IFC\pi GSOM$.

Theorem 3.16: Every $IFCM\pi GSOM$ is an $IFC\pi GSOM$ but not conversely.

Proof: Let $f: X \rightarrow Y$ be an $IFCM\pi GSOM$. Let $A \subseteq X$ be an IFOS. Then A is an $IF\pi GSOS$ in X . By hypothesis, $f(A)$ is an $IF\pi GSCS$ in Y . Hence f is an $IFC\pi GSOM$.

Example 3.17: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0_a, 0.3_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle y, (0.2_u, 0.4_v), (0.5_u, 0.4_v) \rangle$, $G_3 = \langle y, (0.1_u, 0.3_v), (0.3_u, 0.4_v) \rangle$, $G_4 = \langle y, (0.1_u, 0.3_v), (0.5_u, 0.4_v) \rangle$, $G_5 = \langle y, (0.2_u, 0.4_v), (0.3_u, 0.4_v) \rangle$ and $G_6 = \langle y, (0.4_u, 0.4_v), (0.3_u, 0.4_v) \rangle$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, G_3, G_4, G_5, G_6, 1\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an $IFC\pi GSOM$ but not an $IFCM\pi GSOM$, since $A = \langle x, (0_a, 0.3_b), (0.5_a, 0.4_b) \rangle$ is an $IF\pi GSOS$ in X but $f(A) = \langle y, (0_u, 0.3_v), (0.5_u, 0.4_v) \rangle$ is not an $IF\pi GSCS$ in Y .

Theorem 3.18: (i) If $f: X \rightarrow Y$ is an IFOM and $g: Y \rightarrow Z$ be an $IFC\pi GSOM$, then $g \circ f$ is an $IFC\pi GSOM$.

(ii) If $f: X \rightarrow Y$ is an $IFC\pi GSOM$ and $g: Y \rightarrow Z$ is an $IFM\pi GSCM$, then $g \circ f$ is an $IFC\pi GSOM$.

(iii) If $f: X \rightarrow Y$ is an $IF\pi GSOM$ and $g: Y \rightarrow Z$ is an $IFCM\pi GSOM$, then $g \circ f$ is an $IFC\pi GSOM$.

(iv) If $f: X \rightarrow Y$ is an $IFC\pi GSOM$ and $g: Y \rightarrow Z$ is an $IFCM\pi GSOM$, then $g \circ f: X \rightarrow Z$ is an $IF\pi GSOM$.

Proof: (i) Let A be an IFOS in X . Then $f(A)$ is an IFOS in Y . Therefore $g(f(A))$ is an $IF\pi GSCS$ in Z . Hence $g \circ f$ is an $IFC\pi GSOM$.

(ii) Let A be an IFOS in X . Then $f(A)$ is an $IF\pi GSCS$ in Y . Therefore $g(f(A))$ is an $IF\pi GSCS$ in Z . Hence $g \circ f$ is an $IFC\pi GSOM$.

(iii) Let A be an IFOS in X . Then $f(A)$ is an $IF\pi GSOS$ in Y . Therefore $g(f(A))$ is an $IF\pi GSCS$ in Z . Hence $g \circ f$ is an $IFC\pi GSOM$.

(iv) Let A be an IFOS in X . Then $f(A)$ is an $IF\pi GSCS$ in Y , since f is an $IFC\pi GSOM$. Since g is an $IFCM\pi GSOM$, $g(f(A))$ is an $IF\pi GSOS$ in Z . Therefore $g \circ f$ is an $IF\pi GSOM$.

Theorem 3.19: If $f: X \rightarrow Y$ is an $IFCM\pi GSOM$, then for any $IF\pi GSCS$ A in X and for any IFP $c(\alpha, \beta) \in Y$, if $f^{-1}(c(\alpha, \beta)) \subseteq A$, then $c(\alpha, \beta) \subseteq \pi \text{gsint}(f(A))$.

Proof: Let $A \subseteq X$ be an $IF\pi GSPCS$ and let $c(\alpha, \beta) \in Y$. Let $f^{-1}(c(\alpha, \beta)) \subseteq A$. Then $c(\alpha, \beta) \subseteq f(A)$. By hypothesis, $f(A)$ is an $IF\pi GSOS$ in Y . This implies $\pi \text{gsint}(f(A)) = f(A)$. Hence $c(\alpha, \beta) \subseteq \pi \text{gsint}(f(A))$.

Theorem 3.20: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an $IFC\pi GS$ closed mapping and Y is an $IF\pi_b T_{1/2}$ space, then $f(A)$ is an IFGOS in Y for every IFCS A in X .

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an $IFC\pi GS$ closed mapping and let A be an IFCS in X . Then by hypothesis $f(A)$ is an $IF\pi GSOS$ in Y . Since Y is an $IF\pi_b T_{1/2}$ space, $f(A)$ is an IFGOS in Y .

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