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ON THE HYPER-WIENER INDEX OF THORNY-COMPLETE GRAPH

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Abstract: Let G be the graph. The Wiener Index W(G) is the sum of all distances between vertices of G, where as the Hyper-Wiener index WW(G) is defined as WW(G) = W(G) + $\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$. In this paper we prove some general results on Hyper-Wiener Index of Thorny-Complete graphs.

Mathematics Subject Classification: 05C12.

Keywords: Thorny-complete graph, Wiener index and hyper-Wiener index.

1. INTRODUCTION:

In this paper we consider graphs means simple connected graphs, connected graphs without loops and multiple edges. In mathematical terms a graph is represented as G = (V,E) where V is the set of vertices and E is the set of edges. The distance between the vertices u and v of V(G) is denoted by d(u,v) and it is defined as the number of edges in a minimal path connecting the vertices u and v.

In chemical graph theory, the Wiener index (also called Wiener number) is a topological index of a molecule, defined as the sum of the lengths of the shortest paths between all pairs of vertices in the chemical graph represented the non-hydrogen atom in the molecule. The Wiener index is named after Harry Wiener, who introduced it in 1947; at the time, Wiener called it the **"Path Number"**. It is the oldest topological index related to molecular branching.

Wiener Index given by

$$W(G) = \sum_{u<v} d(u,v)$$

The Hyper-Wiener index "WW" is distance based graph invariants, used as a structure descriptor for predicting physico-chemical properties of organic compounds. The hyper-Wiener index of acyclic graphs was introduced by Milan Randic in 1993. Then Klein, generalized Randic's definition for all connected graphs, as a generalization of the Wiener index. It is defined as

$$WW(G) = W(G) + \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$$

The Hyper-Wiener index of Complete graph K_n, Path graph P_n, Star graph $K_{1,n-1}$ and Cycle graph C_n is given by the expressions

WW(K_n) = $\frac{n(n-1)}{2}$, WW(P_n) = $\frac{n^4+2n^3-n^2-2n}{24}$, WW($K_{1,n-1}$) = $\frac{1}{2}(n-1)(3n-4)$

$$WW(C_n) = \begin{cases} \frac{n^2(n+1)(n+2)}{48}, n \text{ is even} \\ \end{cases}$$

$$\boldsymbol{\frac{n(n^2-1)(n+3)}{48}}\ ,\boldsymbol{n\ is\ odd}$$

We have three methods for calculation of the Hyper-Wiener Index of molecular graphs.

(i) Distance Formula:

$$WW(G) = W(G) + \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$$

(ii) Cut Method:

(iii) The Method of Hosaya Polynomials:

Let G a connected n- vertex graph with vertex set V(G) = $\{v_1, v_2, \ldots\ldots, v_n\}$ and p = $(p_1, p_2, \ldots\ldots, p_n)$ be an n-tuple of non-negative integers. The ***thorn graph G_p*** is the graph obtained by attaching p_i pendent vertices to the vertex v_i of G for i = 1,2,….,n. The p_i pendent vertices attached to the vertex v_i will be called the thorns of v_i. The concept of thorny graphs was introduced by Ivan Gutman.

Following results are proved using distance formula

2. MAIN RESULTS:

Theorem 2.1: Let K_n be the compete graph on n vertices. The graph G obtained by attaching S- number of Pendent vertices to each vertex of K_n with common vertex then its Hyper-Wiener index given by

WW(G) = $\frac{1}{2}\{n^2 - n - 2Sn + 6Pn - 5P - 3PS + 6P^2\}$

Where n- cardinality of complete graph

S - Number of Pendent vertices attached to each vertex of n.

P – Total number of Pendent vertices present in G.

Proof: To find Hyper-Wiener index of the graph we need to find following two parts,

To find W(G):

W(G) = $\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$

$$=\frac{1}{2}\{[(\underbrace{1+1+\cdots+1}_{n+S-1\text{ times}}+\underbrace{2+2+\cdots+2}_{\text{P-S times}})+\cdots+(\underbrace{1+1+\cdots+1}_{n+S-1\text{ times}}+\underbrace{2+2+..+2}_{\text{P-S times}})]$$

$$+[(\underbrace{1+2+2+\cdots+2}_{n+S-2\text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})++(\underbrace{1+2+2+\cdots+2}_{n+S-1\text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})]\}$$

$$=\frac{1}{2}\{[\underbrace{n+S-1+2(\mathrm{P}-\mathrm{S})+\cdots+n+S-1+2(\mathrm{P}-\mathrm{S})}_{n\text{ times}}]$$

$$+[\underbrace{1+2(n+S-2)+3(\mathrm{P}-\mathrm{S})+\cdots+1+2(n+S-2)+3(\mathrm{P}-\mathrm{S})}_{\text{P times}}]$$

$$=\frac{1}{2}\{n[n+S-1+2(\mathrm{P}-\mathrm{S})]+P[1+2(n+S-2)+3(\mathrm{P}-\mathrm{S})]\}$$

$$\mathrm{W(G)}=\frac{1}{2}\{n^2-Sn-n+4Pn-3P-PS+3P^2\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(a)}$$

To find WW*(G):

$$\mathrm{WW^*(G)}=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$$

$$\mathrm{WW^*(G)}=\frac{1}{2}\{[\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}+\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}+\cdots+\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}]$$

$$+(\underbrace{1+1+\cdots+1}_{n+S-2\text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})+\cdots+(\underbrace{1+1+\cdots+1}_{n+S-2\text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})\}$$

$$=\frac{1}{2}\{[\underbrace{(\mathrm{P}-\mathrm{S})+(\mathrm{P}-\mathrm{S})+\cdots+(\mathrm{P}-\mathrm{S})}_{n\text{ times}}]$$

$$+[\underbrace{(n+S-2)+3(\mathrm{P}-\mathrm{S})+\cdots+(n+S-2)+3(\mathrm{P}-\mathrm{S})}_{\text{P times}}]\}$$

$$\mathrm{WW^*(G)}=\frac{1}{2}\{n(P-S)+P[(n+S-2)+3(\mathrm{P}-\mathrm{S})]\}$$

$$\mathrm{WW^*(G)}=\frac{1}{2}\{2Pn-Sn-2PS-2P+3P^2\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(b)}$$

Since WW(G) = $W(G)+WW^*$(G)

Therefore WW(G) = $\frac{1}{2}\{n^2-Sn-n+4Pn-3P-PS+3P^2\}$

$$+\frac{1}{2}\{2Pn-Sn-2PS-2P+3P^2\}$$

Combining (a) and (b) gives

$$\mathrm{WW(G)}=\frac{1}{2}\{n^2-n-2Sn+6Pn-5P-3PS+6P^2\}$$ ■

Corollary 2.1.1: Let K_n be the compete graph on n vertices. The graph G obtained by attaching three Pendent vertices to each vertex of K_n with common vertex then its Hyper-Wiener index given by

$$\mathrm{WW(G)}=\frac{1}{2}\{n^2-7n+6Pn-14P+6P^2\}$$

Proof: Substituting S=3 in above theorem, gives the result. ■

Corollary 2.1.2: Let K_n be the compete graph on n vertices. The graph G obtained by attaching two Pendent vertices to each vertex of K_n with common vertex then its Hyper-Wiener index given by

WW(G) = $\frac{1}{2}\{n^2 - 5n + 6Pn - 11P + 6P^2\}$

Proof: Substituting S=2 in above theorem, gives the result. ■

Corollary 2.1.3: Let K_n be the compete graph on n vertices. The graph G obtained by attaching one Pendent vertex to each vertex of K_n with common vertex then its Hyper-Wiener index given by

WW(G) = $\frac{1}{2}\{n^2 - 3n + 6Pn - 8P + 6P^2\}$

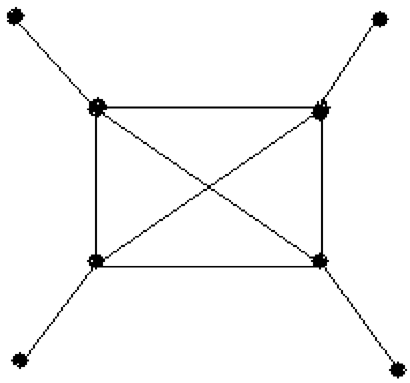
Proof: Substituting S=1 in above theorem, gives the result. ■

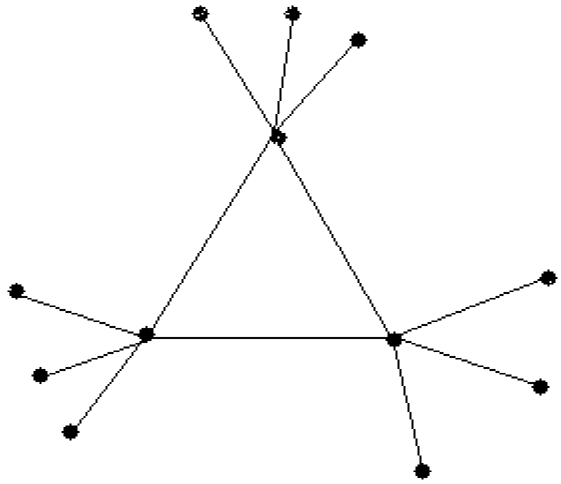
Corollary 2.1.4: Hyper-Wiener index of complete graph given by

WW(G) = $\frac{1}{2}\{n^2 - n\}$

Proof: Substituting S=0 and P=0 in above theorem, gives the result. ■

Illustrations:





$n = 4, S = 1, P = 4$ $W(G) = 52$ $WW(G) = 82$ $n = 3, S = 3, P = 9$ $W(G) = 147$ $WW(G) = 255$

Theorem 2.2: Let K_n be the compete graph on n vertices (n is even). The graph G obtained by attaching S- number of Pendent vertices to alternative vertices of K_n with common vertex then its Hyper-Wiener index given by

WW(G) = $\frac{1}{4}\{2n^2 + 12Pn - 2Sn - 2n - 10P - 6PS + 12P^2\}$

Where n- cardinality of complete graph and n is even

S - Number of Pendent vertices attached to alternative vertices of n.

P – Total number of Pendent vertices present in G.

Proof: To find Hyper-Wiener index of the graph we need to find following two parts,

To find W(G):

W(G) = $\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$

$$= \frac{1}{2}\{[\underbrace{(1 + 1 + \cdots + 1)}_{n-1 \text{ times}} + \underbrace{(2 + 2 + \cdots + 2)}_{P \text{ times}} + \cdots + \underbrace{(1 + 1 + \cdots + 1)}_{n-1 \text{ times}} + \underbrace{(2 + 2 + \cdots + 2)}_{P \text{ times}}]$$

$$+[(\underbrace{1 + 1 + \cdots + 1}_{n+S-1 \text{ times}} + \underbrace{2 + 2 + \cdots + 2}_{P\text{-}S \text{ times}}) + \cdots + (\underbrace{1 + 1 + \cdots + 1}_{n+S-1 \text{ times}} + \underbrace{2 + 2 + \cdots + 2}_{P\text{-}S \text{ times}})]$$

$$+[(\underbrace{1+2+2+\cdots+2}_{n+S-2 \text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})+\cdots+(\underbrace{1+2+2+\cdots+2}_{n+S-2 \text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})]\}$$

$$=\frac{1}{2}\{[\underbrace{(n-1+2P)+(n-1+2P)+\cdots+(n-1+2P)}_{\frac{n}{2} \text{ times}}]$$

$$+[\underbrace{(n-1+S+2P-2S)+\cdots+(n-1+S+2P-2S)}_{\frac{n}{2} \text{ times}}]$$

$$+[\underbrace{1+2(n+S-2)+3(P-s)+\cdots+1+2(n+S-2)+3(P-s)}_{\text{P times}}]\}$$

W(G)= $\frac{1}{2}\left\{\frac{n}{2}[n-1+2p]+\frac{n}{2}[n-1+S+2P-2S]+P[1+2(n+S-2)+\ 3(P-s)]\right\}$

W(G) = $\frac{1}{4}[2n^2+8Pn-Sn-2n-6P-2PS+6P^2]$………………….(a)

To find WW*(G):

WW*(G) = $\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$

$$WW^*(G)=\frac{1}{2}\{[\underbrace{(1+1+\cdots+1)}_{\text{P times}}+\underbrace{(1+1+\cdots+1)}_{\text{P times}}+\cdots+\underbrace{(1+1+\cdots+1)}_{\text{P times}}]$$

$$+[\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}+\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}+\cdots+\underbrace{(1+1+\cdots+1)}_{\text{P-S times}}]$$

$$+(\underbrace{1+1+\cdots+1}_{n+S-2 \text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})+\cdots+(\underbrace{1+1+\cdots+1}_{n+S-2 \text{ times}}+\underbrace{3+3+\cdots+3}_{\text{P-S times}})\}$$

$$WW^*(G)=\{[\underbrace{P+P+\cdots+P}_{\frac{n}{2}\ times}]\ +\underbrace{(P-S)+(P-S)+\cdots+(P-S)}_{\frac{n}{2}\ times}]$$

$$+[\underbrace{(n+s-2+3P-3S)+\cdots+(n+s-2+3P-3S)}_{\text{P times}}]$$

WW*(G) = $\frac{1}{2}\{\frac{n}{2}P+\frac{n}{2}(P-S)+P(n+S-2+3P-3S)\}$

WW*(G) = $\frac{1}{4}\{4Pn-Sn-4SP+6P^2-4P\}$……………………….(b)

Since WW(G) = $W(G)+WW^*$(G)

Therefore

WW(G)=$\left\{\frac{1}{4}[2n^2+8Pn-Sn-2n-6P-2PS+6P^2]\right\}+\frac{1}{4}\{4Pn-Sn-4SP+6P^2-4P\}$

WW(G) $=\frac{1}{4}\{2n^2+12Pn-2Sn-2n-10P-6PS+12P^2\}$ ■

Corollary 2.2.1: Let K_n be the compete graph on n vertices (n is even). The graph G obtained by attaching three Pendent vertices to alternative vertices of $K_n(n\geq 4)$ with common vertex then its Hyper-Wiener index given by

WW(G) = $=\frac{1}{4}\{2n^2+12Pn-8n-28P+12P^2\}$

Proof: Substituting S=3 in above theorem, gives the result. ■

Corollary 2.2.2: Let K_n be the compete graph on n vertices (n is even). The graph G obtained by attaching two Pendent vertices to alternative vertices of $K_n(n\geq 4)$ with common vertex then its Hyper-Wiener index given by

WW(G) = $=\frac{1}{4}\{2n^2+12Pn-6n-22P+12P^2\}$

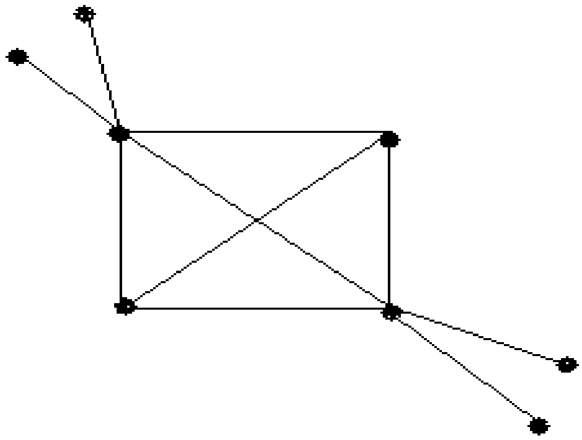
Proof: Substituting S=2 in above theorem, gives the result. ■

Corollary 2.2.3: Let K_n be the compete graph on n vertices (n is even). The graph G obtained by attaching one Pendent vertex to alternative vertices of $K_n(n\geq 4)$ with common vertex then its Hyper-Wiener index given by

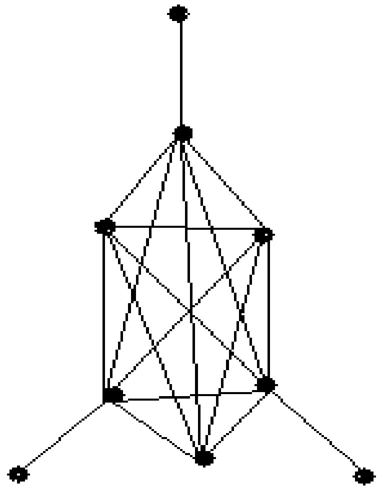
WW(G) = $=\frac{1}{4}\{2n^2+12Pn-4n-16P+12P^2\}$

Proof: Substituting S=1 in above theorem, gives the result. ■

Illustrations:



$|K_n| = 4, S = 2, P = 4, W(G) = 50\ \ WW(G) = 76$



$|K_n| = 6, S = 1, P = 3, W(G) = 57\ \ WW(G) = 81$

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