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# PERISTALTIC TRANSPORT OF A MAGNETIC FLUID THROUGH POROUS MEDIA IN A UNIFORM AND NON-UNIFORM ANNULUS

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*Abstract:* The aim of the present investigation is to study the peristaltic transport through the gap between coaxial tubes, where the outer tube is non uniform and the inner tube is rigid. The necessary theoretical results such as viscosity, pressure gradient and friction force on the inner and outer tubes have been obtained in terms of couple stress parameter. Out of these theoretical results the numerical solution of pressure gradient, outer friction, inert friction and flow rate are shown graphically for the better understanding of the problem.

Keyword: Peristaltic transport, Magnetic fluid, porous media.

## 1. INTRODUCATION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the esophagus, in urine transport form the kidney to the bladder through the urethra, in movement of chyme in the gastro –intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessel such as arterioles, venules and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical system. Also, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of analytical [1-8], numerical and experimental [9-13] studies of peristaltic flows of different fluids have been reported. A summary of most of the investigation reported up to the year 1983, has been presented by Srivastava and Srivastava [14], and some imported contribution of recent year, are reference in Srivastava and Saxsen [15]. Physiological organs are generally observed have the form of a non-uniform duct [16, 17]. In particular, the vas deferens in rhesus monkey is in the form of a diverging tube with a ration of exit to inlet dimensions of approximately four [18]. Hence, peristaltic analysis of a Newtonian fluid in a uniform geometry cannot be applied when explaining the mechanism of transport of fluid in most bio-systems. Recently, Srivastava et al [19] and Srivastava and Srivastava [20] studied peristaltic transport of Newtonian and non-Newtonian fluids in non-uniform geometries. Asha and Rathod[23,24] studied the effect of magnetic on peristaltic motion in uniform annulus.

With the above discussion in mind, we propose to study the peristaltic transport of a viscous incompressible fluid (creeping flow) through the gap between coaxial tubes, where the outer tube is non-uniform and has a sinusoidal wave traveling down its wall and the inner one is a rigid, uniform tube and moving with a constant velocity. This investigation may have application in many clinical applications such as the endoscopes problem.

# 2. FORMULATION OF THE PROBLEM

Consider the flow of an incompressible Newtonian fluid through coaxial tubes such that the outer tubes is non-uniform and has a sinusoidal wave traveling down and inner one rigid, and moving with a constant velocity. The geometry of the wall surface is

$$r_2 = a_1, \tag{1}$$

$$r_{2} = a_{2} + bSin(\frac{2\pi}{\lambda}(x - ct))$$
<sup>(2)</sup>

With

$$a_2(z) = a_{20}kz$$

With  $a_1$  is the radius of the inner tube  $a_2(z')$  is the radius of the outer tube at axial distance z' form inlet,  $a_{20}$  is the radius of the outer tube at the inlet,  $k(\ll 1)$  is the constant whose magnitude depends on the length of the outer tube, b is the amplitude,  $\lambda$  is the wave length, c is the propagation velocity,  $\eta$  is constant couple stress parameter, and t is the time. We choose a cylindrical coordinate system (r', z') where the z-axis lies along the centerline of the inner and the outer tubes and r' is the distance measured racially.

The equation of motion of the flow in the gap between the inner and the outer tubes are

Where u' and w' are the velocity components in the r' and w' direction respectively,  $\rho$  is the density, p' is the pressure and  $\mu$  is the viscosity,  $\sigma$  is Electric conductivity,  $B_0$  is applied magnetic field and k is porous media. The boundary conditions are

$$u' = 0, \quad w' = V_0, \quad at \quad r' = r_1'$$
  
 $u' = \frac{\partial r_2'}{\partial t'}, \quad w' = 0 \quad at \quad r' = r_2'$ 
(6a)
  
(6b)

It is convenient to non dimensionalize the variable appearing in equation (1-6) and introducing Reynolds number Re, wave number ratio  $\delta$ , and velocity parameter  $V_{a}$  as follows:

$$z = \frac{z}{\lambda}, r = \frac{r}{c}, u = \frac{\lambda u}{a_{20}c}, p = \frac{a_{20}^2}{\lambda \mu c} p'(z'),$$

$$t = \frac{tc}{\lambda}, \text{Re} = \frac{\rho c a_{20}}{\mu}, \quad \delta = \frac{a_{20}}{\lambda}, V_0 = \frac{V_o}{c}$$

$$r_1 = \frac{r_1}{a_{20}} = \varepsilon, \quad r_2 = \frac{r_2}{a_{20}} = 1 + \frac{\lambda k z}{a_{20}} + \phi(\frac{2\pi}{\lambda}(z-t))$$
where  $\phi$  amplitude  $-\frac{b}{a_{20}} \le 1$ 

$$(7)$$

The equation of motion and boundary conditions in the dimensionless form becomes

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0$$
(8)
$$\operatorname{Re} \delta^{3}\left\{\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right\} = -\frac{\partial p}{\partial r} + \delta^{2}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru)}{\partial r}\right) + \delta^{4}\frac{\partial^{2}u}{\partial z^{2}} - \delta^{2}\left(M^{2} + \frac{1}{Da}\right)u$$
(9)

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$$\operatorname{Re} \delta\{\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial(rw)}{\partial r}) + \delta^{2}\frac{\partial^{2}w}{\partial z^{2}} - (M^{2} + \frac{1}{Da})w$$

$$(10)$$

Where  $M^2 = \sqrt{\frac{\sigma}{\mu}a_{20}B_0}$  is the Hartman number and  $Da = \frac{k}{a_{20}^2}$  is the Darcy number.

The boundary condition are:

$$u=0 \qquad w=V_0 \quad \text{at } r=r_1=\mathcal{E}, \qquad (11a)$$
$$u=\frac{\partial r_2}{\partial y}, w=0 \text{ at } r=r_2=1+\frac{\lambda kz}{a_{20}}+\phi \sin[2\pi(z-t)] \qquad (11b)$$

Using the long wavelength approximation and dropping terms of order  $\delta$  it follows from equation (8-11) that the appropriate equation describing the flow in the laboratory frame of reference are

$$\frac{\partial p}{\partial r} = 0,$$
(12)
$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) - N^2 w$$
(13)
Where  $N^2 = M^2 + \frac{1}{Da}$ 

with dimensional boundary condition

w=V<sub>0</sub> 
$$\nabla^2(u, w)$$
 finite at r=  $r_1 = \mathcal{E}$ ,  
w = 0,  $\nabla^2(u, w) = 0$  at  $r = r_2 = 1 + \frac{\lambda kz}{a_{20}} + \phi \sin[2\pi(z-t)]$  (14)

Integrating equation and using the boundary condition one finds the expression for the velocity profile as

$$w(z,t) = -\frac{1}{4} \left(\frac{\partial p}{\partial z}\right) \left[ (r_2^2 - r_1^2) \left(\frac{In(r/r_1)}{In(r_2/r_1)}\right) - r^2 + r_1^2 \right] - \frac{V_0}{In(r_2/r_1)} In(r/r_2) \left[ 1 - \frac{r_1}{4} N^2 \right]$$
(15)

The instantaneous volume flow rate Q (z,t) is given by

$$Q(x,t) = \int_{r_1}^{r_2} 2\pi r w dr = -\frac{\pi}{8} \frac{\partial p}{\partial z} \{ (r_2^2 - r_1^2) [r_2^2 + r_1^2 - (\frac{r_2^2 - r_1^2}{\ln(r_2/r_1)})] \} - \pi V_o \{ \frac{r_1^2 - r_2^2}{\ln(r_2/r_1)} + r_1^2 \} [1 - \frac{r_1}{4} N^2]$$

$$\frac{\partial p}{\partial z} = \left( -8 \frac{Q/\pi \ln(r_2/r_1) + \frac{V_0}{2} (r_1^2 - r_2^2) + V_0 r_1^2 \ln(r_2/r_1) [1 - \frac{\pi}{4} N^2]}{(r_2^4 - r_1^4) \ln(r_2/r_1) - (r_2^2 - r_1^2)^2} \right)$$
(16)
(17)

The pressure rise  $\Delta p_L(t)$  and friction force (at the wall) on the outer and the inner tubes  $F_L^{(o)}(t)$  and  $F_L^{(i)}(t)$  respectively, in a tube of length L, in their non-dimensional forms, are given by

$$\Delta p_L(t) = \int_0^A \frac{\partial p}{\partial z} dz \tag{18}$$

$$\Delta F_L^{(o)}(t) = \int_0^A r_2^2 \left(-\frac{\partial p}{\partial z}\right) dz,\tag{19}$$

$$\Delta F_L^{(i)}(t) = \int_0^A r_1^2 \left(-\frac{\partial p}{\partial z}\right) dz,$$
(20)

Where A=L/ $\lambda$ ,

Substituting from equation (17) in equation (18-20) and with  $r_1 = \varepsilon$  and  $2k_7$ 

$$r_{2}(z,t) = 1 + \frac{\lambda kz}{a_{20}} + \phi \sin[2\pi(z-t)], \text{ we get}$$

$$\Delta p_{L}(t) = \int_{0}^{4} -8\{\frac{Q(z,t)}{\pi} In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon}] + \frac{V_{0}}{2}[\varepsilon^{2} - (1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t))^{2}] + V_{0}\varepsilon^{2}In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon}]\}[1 - \frac{r_{1}}{4}N^{2}] - \{1/[(([1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)]^{4} - \varepsilon^{4}) + V_{0}\varepsilon^{2}In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon}]\}[1 - \frac{r_{1}}{4}N^{2}] - \{1/[(([1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)]^{4} - \varepsilon^{4}) + In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi(z-t)}{\varepsilon}]\}$$

$$\left(\left(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)^2 - \varepsilon^2\right)^2\right) dz$$
 (21)

$$\Delta F_{L}^{(o)}(t) = \int_{0}^{A} 8\{(1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t))^{2}\} \{\frac{Q(z, t)}{\pi} In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)}{\varepsilon}] + \frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)}{\varepsilon}] + V_{0}\varepsilon^{2}In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)}{\varepsilon}] \} [1 - \frac{r_{1}}{4}N^{2}]$$

$$\{1/[((1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t))^{4} - \varepsilon^{4})In[\frac{1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)}{\varepsilon}] - \frac{((1 + \frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z - t)^{2} - \varepsilon^{2})^{2})\} dz$$

$$(22)$$

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$$\Delta F_{L}^{(i)}(t) = \int_{0}^{A} 8\varepsilon^{2} \{ \frac{Q(z,t)}{\pi} In[\frac{1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t)}{\varepsilon}] + \frac{V_{0}}{2} [\varepsilon^{2} - (1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t))^{2}] + V_{0}\varepsilon^{2} In[\frac{1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t)}{\varepsilon}] [1-\frac{r_{1}}{4}N^{2}] \{ 1/[((1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t))^{4} - \varepsilon^{4}) - In[\frac{1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t)}{\varepsilon}] - ((1+\frac{\lambda kz}{a_{20}} + \phi \sin 2\pi (z-t))^{2} - \varepsilon^{2})^{2} ] \} dz$$
(23)

The limiting of equation (15-17) as  $r_1$  tends to zero gives the forms of the axial velocity and the pressure gradient for peristaltic flow in non uniform tube(without endoscope,  $\mathcal{E} = 0$ ), these are

$$w(r, z, t) = -\frac{1}{4} \left(\frac{\partial p}{\partial z}\right) (r_2^2 - r^2),$$
(24)

$$\frac{\partial p}{\partial z} = -\frac{8Q}{\pi r_2^4},\tag{25}$$

Hence the pressure rise and the outer friction force, in this case respectively, take the form

$$\Delta p_{L}(t) = -8 \int_{0}^{A} \frac{Q(z,t)/\pi}{\left(1 + \frac{\lambda k z}{a_{20}} + \phi \sin 2\pi (z-t)\right)^{4}} dz$$
(26)  
$$\Delta F_{L}^{(o)}(t) = 8 \int_{0}^{A} \frac{Q(z,t)/\pi}{\left(1 + \frac{\lambda k z}{a_{20}} + \phi \sin 2\pi (z-t)\right)^{2}} dz$$
(27)

Equation (26) and equation (27) are the same result as those obtained by Gupta and Seshadri [21], Srivastava and Srivastava for non Newtonian fluid [20] when the power law index and also those obtained by Srivastava et al.[19] for a constant viscosity if  $\delta = 0$ . Further, if k=0 in equations (26) and (27), we get expression for the pressure rise and friction force in a uniform tube. The analytical interpretation of our analysis with other theories are difficult to make at this stage, as the integrals in equation (21-23) and equation (26) and (27) are not integrable in closed form , neither for non-uniform nor uniform geometry (k=0). Thus further studies of our analysis are only possible after numerical evaluation of these integrals.

#### 3. NUMERICAL RESULT AND DISCUSSION

To discuss the results obtained above quantitatively we shall assume the form of the instantaneous volume rate of the flow Q(z, t), periodic in (z-t) as [19,21]

$$\frac{Q(z,t)}{\pi} = \frac{Q}{\pi} - \frac{\phi^2}{2} + 2\phi \sin(2\pi(z-t)) + \frac{2\lambda kz}{a_{20}}\phi \sin(2\pi(z-t)) + \phi^2 \sin^2(2\pi(z-t))$$

where Q is the time average of the flow over one period of the wave .This form Q(z, t) has been assumed in view of the fact that the constant value of Q(z,t) gives  $\Delta P_L(t)$  always negative, and hence will be no pumping action. Using this form of Q © JGRMA 2014, All Rights Reserved 48 (z, t), we shall now compute the dimensionless pressure rise  $\Delta P_L(t)$ , the inner friction force  $F_L^{(i)}(t)$  (on the inner surface) and the outer friction force  $F_L^{(o)}(t)$  (on the outer tube) over the tube length for various value of the dimensionless time t, dimensionless flow average Q, amplitude ratio  $\phi$ , radius ratio  $\mathcal{E}$ , couple stress parameter  $\gamma$ , and the velocity of the inner tube V<sub>0</sub>. The average rise in pressure  $\Delta P_L$ , outer friction force  $F_{(L)}^{-(o)}$  and the inner friction force  $F_L^{-(i)}(t)$  are then evaluated by averaging  $\Delta P_L(t)$ ,  $F_L^{(o)}(t)$ , and  $F_L^{(i)}(t)$  over one period of the wave. As integrals in equation (21-23) are not integrable in closed form, they are evaluated numerically using digital computer. Following Srivastava [20], we use the value of the various parameters in equation (21-23) as:

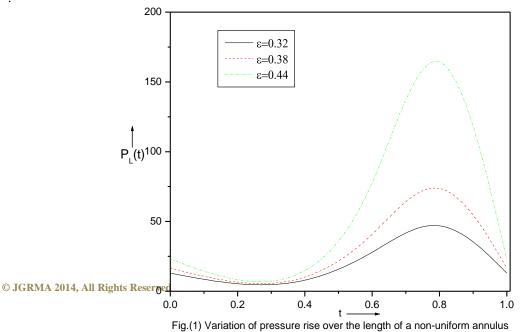
$$k = \frac{3a_{20}}{\lambda}$$
.  $a_{20} = 1.25$  cm,  $L = \lambda = 8.01$  cm

Furthermore, since most routine upper gastrointestinal endoscopes are between 8-11 mm in diameter as reported Cotton and Williams [22] and the radius of the small intestine is 1.25 cm as reported in Srivastava [20] then the radius ratio  $\mathcal{E}$ , take the values 0.32, 0.38, and 0.44.

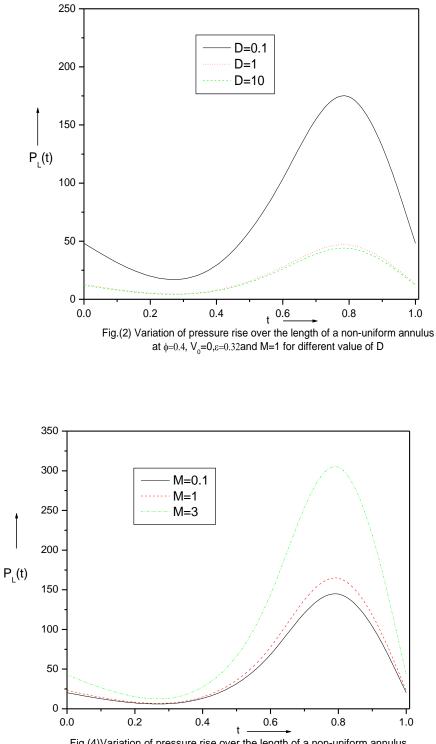
Figure (1) represent the variation of the dimensionless pressure with dimensionless time t for  $\phi = 0.4$ , V<sub>0</sub>=0, M=1, Da=0.1 and radius ratio  $\mathcal{E} = 0.32$ , 0.38, and 0.44. Shows that the pressure rise increases as the radius ratio increases, *i.e.*, as the inner radius of the tube increases. Figure (2) illustrate the variation of pressure rise with time for  $\phi = 0.4$ , V<sub>0</sub>=0, M=1, and radius ratio  $\mathcal{E} = 0.32$ , different values of Darcy number here it is observed that as Darcy number increases the pressure decreases.

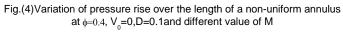
Figure (4) shows the variation of dimensionless pressure rise with the dimensionless time  $\phi = 0.4$ , V<sub>0</sub>=0, Da=0.1 and  $\varepsilon = 0.32$ , for different values of Hartman number. This shows that as the Hartman number increases the pressure increases. Figure (5) shows the pressure with flow rate for different values Darcy number and velocity here it is observed that as the Darcy number increases the flow rate Q decreases. In figure (6) it is observed that the as the velocity decreases with increase of Hartman number M.

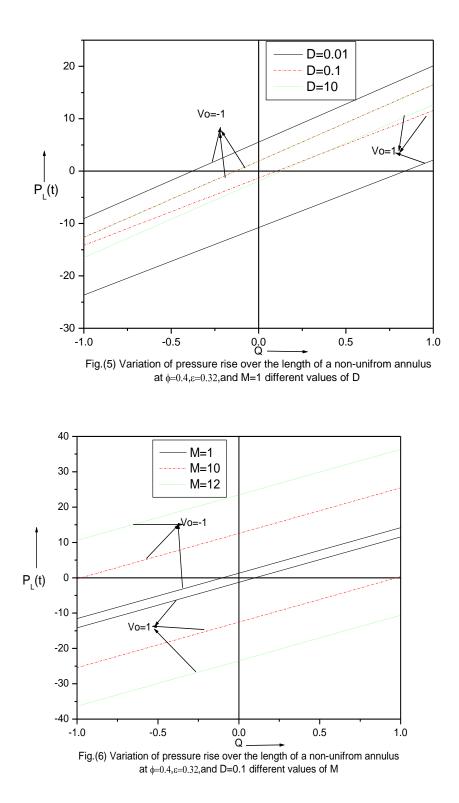
Form figure (7) to figure (10) we plot the effect of Darcy number, Hartmann number and velocity on the inner and outer friction it is observed that the inner friction and outer friction have same effect, it decrease with increase of Darcy number Da and increases with increase of Hartmann number M. It is clear that as the velocity increase the inner and outer friction force decreases for different values of Hartmann number and Darcy number. When radius ratio is constant and magnitude of velocity is varying and is not affected when velocity is zero.

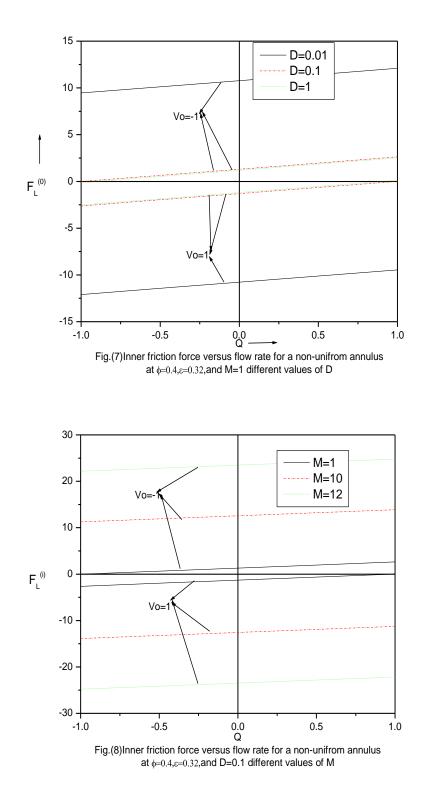


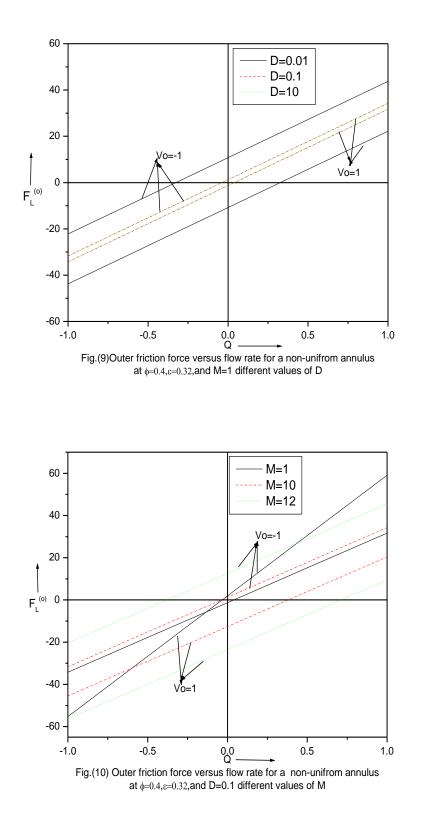
at  $\phi=0.4$ , V\_=0,D=0.1 and M=1 for different values of  $\varepsilon$ 











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