

HEAT TRANSFER IN MHD BOUNDARY LAYER FLOW OVER A SHRINKING SHEET WITH RADIATION AND HEAT SINK

Chandaneswar Midya

Department of Mathematics, Ghatal Rabindra Satbarsiki Mahavidyalaya,

Paschim Medinipur, West Bengal - 721212, India

Email: c_midya@yahoo.com

Abstract : In this work, the effect of radiation on heat transfer of an electrically conducting fluid flow over a linearly shrinking surface subject to heat sink and magnetic field applied normal to the plane of the flow is investigated analytically. The governing boundary layer equations for fluid flow and energy are reduced into ordinary differential equations by means of similarity transformations. Closed form exact solutions of the reduced energy equation have been obtained for both prescribed power-law surface temperature (PST) and power-law wall heat flux (PHF) boundary conditions and these solutions are valid for all $M > 1$, where M is the magnetic interaction parameter. It is found that the temperature within the fluid is reduced significantly with the increasing values of radiation parameter, Prandtl number, heat sink and magnetic field parameters for both PST and PHF cases. Some solutions involving negative temperature values are also noticed. In some cases, temperature overshoot near the wall is also observed.

Keywords: Shrinking sheet, Second-grade fluid, MHD, Porous medium, chemically reactive species, Exact Solution

NOMENCLATURE

a proportionality constant of sheet velocity
 c_p specific heat at constant pressure
 f non-dimensional stream function
 p wall temperature power index
 n wall temperature heat flux power index
 Pr Prandtl number
 q_w wall heat flux
 T temperature of the fluid
 T_w temperature of the wall of the surface
 T_∞ free-stream temperature
 u velocity component along the sheet
 v velocity component normal to the sheet
 x distance along the sheet
 y distance normal to the sheet
 Q volumetric rate of heat generation / absorption

q_r radiative heat flux
 M magnetic field parameter

Greek symbols

α Parameter related to suction
 ξ transformation parameter
 η Similarity variable
 κ coefficient of thermal conductivity
 μ dynamic viscosity
 ν Kinematic viscosity
 ρ density of the fluid
 σ Conductivity of the fluid
 θ non-dimensional temperature (PST case)
 ω non-dimensional temperature (PHF case)
 λ heat source or sink parameter

INTRODUCTION

The boundary layer flow of an incompressible viscous fluid over a shrinking sheet has received considerable attention of modern day researchers because of its increasing application to many engineering systems. Wang (1990) first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. Later, Miklavcic and Wang (2006) obtained an analytical solution for steady viscous hydrodynamic flow over a permeable shrinking sheet. Then, Hayat et al. (2007) derived both exact and series solution (using HAM) describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of stagnation flow towards a shrinking sheet was studied by Wang (2008). Nadeem and Awais (2008) studied thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity. Muhaimin et al. (2008) investigated the effects of heat and mass transfer on MHD boundary layer flow over a shrinking sheet in

the presence of suction. They obtained numerical solution using Runge Kutta Gill method. Viscous flow over an unsteady shrinking sheet with mass transfer was studied by Fang and Zhang (2009a). Nadeem and Hussain (2009) used homotopy analysis method to study the viscous flow on a nonlinear porous shrinking sheet. Fang and Zhang (2009b) solved the Full N-S equation analytically for two dimensional MHD viscous flow due to a shrinking sheet. Fang and Zhang (2010) recently investigated the heat transfer characteristics of the shrinking sheet problem with a linear velocity. Later on, Noor et al. (2010) studied the MHD viscous flow due to shrinking sheet using Adomian decomposition Method (ADM) and they obtained a series solution. Sajid and Hayat (2009) applied homotopy analysis method for the MHD viscous flow due to a shrinking sheet. Midya (2012a) studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhaimin et al. (2010). Midya (2012b) obtained a closed form analytical solution for the distribution of reactant solute in a MHD boundary layer flow over a shrinking sheet. Very recently, flow and mass transfer over a viscoelastic fluid was investigated in the presence of chemical reaction by Midya (2012c).

Again, thermal radiation effect on boundary layer flow and heat transfer over a moving surface has important applications in modern industrial and technological equipments. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent of equipment. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, power generation systems, gas-turbines and various propulsion devices for air craft, missiles, satellites and space vehicles are examples of such engineering areas. Viskanta and Grosh (1962) dicovered the effects of thermal radiation on boundary layer flow and heat transfer over a wedge in an absorbing and emitting media. The radiation effect on fluid flow and heat transfer in different physical contexts was then investigated by Ali et al. (1984), Elbashby (1998), Ouaf (2005), Devi and Kayalvizhi (2010), Rajput and Kumar (2012) and others. Recently, Ali et al. (2010) obtained a numerical solution of the unsteady flow and heat transfer past an axisymmetric permeable shrinking sheet with radiation effect. The purpose of this paper is to investigate analytically the effects of radiation on the magnetohydrodynamic viscous incompressible fluid flow and heat transfer over a linearly shrinking sheet in the presence of heat sink. Both prescribed power-law wall temperature and power-law wall heat flux are considered as thermal boundary conditions. Closed form exact solutions of the boundary layer energy equation are obtained for both the cases under certain conditions. Heat transfer distributions are presented and discussed for various controlling parameters.

MATHEMATICAL FORMULATION

Consider the flow of an electrically conducting incompressible fluid over a flat plate coinciding with the plane $y = 0$. The flow is confined to $y > 0$. Two equal and opposite forces are applied opposite to the x -axis so that the wall is shrunk keeping the origin fixed. A magnetic induction B_0 is applied perpendicular to the shrinking surface. The shrinking sheet velocity is proportional to the distance i.e. $u_w = -ax$, ($a > 0$). Using boundary layer approximation and neglecting the induced magnetic field (by assuming the magnetic Reynolds number R_m for the flow to be very small i.e. $R_m \ll 1$ [see Midya et al. (2003)], the equations for steady two-dimensional flow and the reactive temperature equation can be written in usual notation as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho c_p} (T - T_\infty) \quad (3)$$

where u and v are the components of velocity respectively in the x and y directions, T is the temperature, T_∞ is the temperature far from the sheet, ρ is the fluid density (assumed constant), σ is the electrical conductivity of the fluid, $\nu (= \mu/\rho)$ is the coefficient of fluid viscosity, D is the mass diffusion coefficient, κ is the thermal conductivity, q_r is the radiative heat flux, Q is the volumetric rate of internal heat generation / absorption.

The boundary conditions for the velocity components and temperature are given by

$$u = -ax, \quad v = 0, \quad T = T_w \quad \text{at } y = 0 \quad (4)$$

and

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{at } y \rightarrow \infty \quad (5)$$

where T_w is the wall temperature.

Now, Rosseland's approximation for radiation gives $q_r = -(4\sigma^*/3k_1) \partial T^4 / \partial y$, where σ^* is the Stefan-Boltzmann constant, k_1 is the absorption coefficient (see Brewster (1972)). It is assumed that the temperature variation within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher order terms, we have $T^4 = 4T_\infty^3 T - 3T_\infty^4$.

Therefore, Eq. (3) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3k_1 \rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) \quad (6)$$

SOLUTION OF THE PROBLEM

Equations (1-2) admit self-similar solutions of the form

$$u = axf'(\eta), \quad v = -\sqrt{av}f(\eta), \quad \eta = y\sqrt{\frac{a}{v}} \quad (7)$$

where f is the dimensionless stream function and η is the similarity variable. Substituting these, Eqs. (2) become

$$\frac{d^3 f}{d\eta^3} + \frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - M^2 \frac{df}{d\eta} = 0 \quad (8)$$

where $M = \sqrt{\sigma B_0^2 / (a\rho)}$ is the magnetic interaction parameter.

The boundary conditions are

$$f'(0) = -1, \quad f(0) = 0, \quad \text{and} \quad f'(\infty) = 0 \quad (9)$$

There is an analytical solution (see Fang and Zhang (2009b)) for the equation with the boundary conditions given by

$$f(\eta) = \frac{1}{\alpha} (e^{-\alpha\eta} - 1), \quad \alpha = \sqrt{M^2 - 1} \quad (10)$$

It is, therefore, seen that there is an exponential solution for this equation for any $M > 1$.

The non-dimensional horizontal velocity component is given by

$$f'(\eta) = -e^{-\alpha\eta} \quad (11)$$

The shear stress at the wall is denoted by τ_w and is defined as

$$\tau_w = \mu(\partial u / \partial y)_{y=0} = \mu ax \sqrt{\frac{a}{v}} f''(0) = \mu ax \sqrt{\frac{a}{v}} \alpha \quad (12)$$

The skin friction coefficient C_f at the wall is obtained as

$$C_f = \frac{\tau_w}{\left(\mu ax \sqrt{\frac{a}{v}}\right)} = f''(0) = \alpha \quad (13)$$

HEAT TRANSFER ANALYSIS

First, we consider power-law surface temperature (PST) as surface boundary conditions and then power-law wall heat flux (PHF) case will be discussed.

POWER-LAW SURFACE TEMPERATURE (PST) CASE

In this case the boundary conditions are

$$T = T_w = T_\infty + Ax^p \quad \text{at} \quad y = 0 \quad (14)$$

$$T \rightarrow T_\infty \quad \text{at} \quad y \rightarrow \infty \quad (15)$$

Defining the non-dimensional temperature $\theta(\eta)$, Prandtl number Pr and heat source / sink parameter λ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad Pr = \frac{\mu c_p}{\kappa}, \quad \lambda = \frac{Q}{\rho c_p a} \quad (16)$$

Using Eq. (7), we have from Eq. (6)

$$\frac{d^2 \theta}{d\eta^2} + DPr f \frac{d\theta}{d\eta} + DPr \left(\lambda - p \frac{df}{d\eta} \right) \theta = 0 \quad (17)$$

Here $D = 3R / (3R + 4)$ and R is the thermal radiation parameter given by $R = \kappa k_1 / 4\sigma^* T_\infty^3$.

The boundary conditions become

$$\theta(0) = 1, \quad \text{and} \quad \theta(\infty) = 0. \quad (18)$$

Substituting the solution for the momentum transport the above Eq.(17) reduces to

$$\frac{d^2 \theta}{d\eta^2} + \frac{DPr}{\alpha} (e^{-\alpha\eta} - 1) \frac{d\theta}{d\eta} + DPr (\lambda + p e^{-\alpha\eta}) \theta = 0 \quad (19)$$

Now, let us introduce a new variable $\xi = \frac{Sc}{\alpha^2} e^{-\alpha\eta}$ so that the above equation transforms to

$$\xi \frac{d^2 \theta}{d\xi^2} + \left(1 + \frac{DPr}{\alpha^2} - \xi \right) \frac{d\theta}{d\xi} + \left(\frac{DPr\lambda}{\alpha^2 \xi} + p \right) \theta = 0 \quad (20)$$

The boundary conditions (18) then become

$$\theta \left(\frac{DPr}{\alpha^2} \right) = 1, \quad \text{and} \quad \theta(0) = 0 \quad (21)$$

Now, transforming the above equation (20) into confluent hypergeometric equation, we can obtain the solution (see Abramowitz and Stegun (1972)) given by

$$\theta(\xi) = (\alpha^2 \xi / DPr)^\beta \Phi(\beta - p, 1 + b_0, \xi) / \Phi(\beta - p, 1 + b_0, DPr / \alpha^2), \quad (22)$$

where $\beta = (b_0 - a_0)/2$, $a_0 = DPr / \alpha^2$, $b_0 = \sqrt{a_0^2 - 4a_0\lambda}$ and $\Phi(a', b', x)$ is the confluent hypergeometric function of the first kind or Kummer function.

Now, because of the boundary condition $\theta(0)=0$, β should be positive and hence λ should be negative i.e. heat sink is needed. Thus, in order to get a boundary layer solution for energy equation, we have to consider the heat sink case only. For heat source case i.e., for $\lambda > 0$, the boundary layer solution of the energy equation does not exist although the solution for momentum equation exists.

Therefore,

$$\theta(\eta) = e^{-\alpha\beta\eta} \Phi(\beta - p, 1 + b_0, DPre^{-\alpha\eta} / \alpha^2) / \Phi(\beta - p, 1 + b_0, DPr / \alpha^2). \quad (23)$$

The dimensionless wall temperature gradient $\theta'(0)$ is obtained as

$$\theta'(0) = -\alpha\beta - \frac{DPr}{\alpha} \left(\frac{\beta-p}{1+b_0} \right) \frac{\Phi(1+\beta-p, 2+b_0, DPr/\alpha^2)}{\Phi(\beta-p, 1+b_0, DPr/\alpha^2)}. \quad (24)$$

POWER-LAW HEAT FLUX (PHF) CASE

Here the boundary conditions become

$$-\kappa \frac{\partial T}{\partial y} = q_w = Ex^n \quad \text{at } y=0 \quad (25)$$

$T \rightarrow T_\infty$ at $y \rightarrow \infty$

where E is a constant, n the heat flux power index, q_w is the wall heat flux.

Defining the non-dimensional temperature by

$$\omega(\eta) = \frac{T-T_\infty}{\frac{Ex^n}{\kappa} \sqrt{\frac{\nu}{a}}} \quad (26)$$

Using Eq. (7), we have from Eq. (6)

$$\frac{d^2\omega}{d\eta^2} + DPrf \frac{d\omega}{d\eta} + DPr \left(\lambda - n \frac{df}{d\eta} \right) \omega = 0 \quad (27)$$

The boundary conditions become

$$\omega'(0) = -1, \quad \text{and} \quad \omega(\infty) = 0. \quad (28)$$

Substituting the solution for the momentum transport the above Eq.(27) reduces to

$$\frac{d^2\omega}{d\eta^2} + \frac{DPr}{\alpha} (e^{-\alpha\eta} - 1) \frac{d\omega}{d\eta} + DPr(\lambda + ne^{-\alpha\eta})\omega = 0 \quad (29)$$

Now, let us introduce a new variable $\xi = \frac{Sc}{\alpha^2} e^{-\alpha\eta}$ so that the above equation transforms to

$$\xi \frac{d^2\omega}{d\xi^2} + \left(1 + \frac{DPr}{\alpha^2} - \xi \right) \frac{d\omega}{d\xi} + \left(\frac{DPr\lambda}{\alpha^2\xi} + n \right) \omega = 0 \quad (30)$$

The boundary conditions (28) then become

$$\omega' \left(\frac{DPr}{\alpha^2} \right) = \frac{\alpha}{DPr}, \quad \text{and} \quad \omega(0) = 0 \quad (31)$$

Now, the solution of above equation (30) with the boundary conditions is

$$\omega(\xi) = \frac{\alpha(1+b_0)(\alpha^2\xi/Sc)^\beta \Phi(\beta-n, 1+b_0, \xi)}{\alpha^2\beta(1+b_0)\Phi(\beta-n, 1+b_0, DPr/\alpha^2) + DPr(\beta-n)\Phi(1+\beta-n, 2+b_0, DPr/\alpha^2)} \quad (32)$$

As in the PST case, the boundary layer solution for energy equation can be obtained in heat sink case only.

The solution then becomes

$$\omega(\eta) = \frac{\alpha(1+b_0)e^{-\alpha\beta\eta} \Phi(\beta-n, 1+b_0, \frac{DPr}{\alpha^2}e^{-\alpha\eta})}{\alpha^2\beta(1+b_0)\Phi(\beta-n, 1+b_0, DPr/\alpha^2) + DPr(\beta-n)\Phi(1+\beta-n, 2+b_0, DPr/\alpha^2)} \quad (33)$$

in terms of η .

Therefore,

$$\omega(0) = \frac{\alpha(1+b_0)\Phi(\beta-n, 1+b_0, \frac{DPr}{\alpha^2})}{\alpha^2\beta(1+b_0)\Phi(\beta-n, 1+b_0, DPr/\alpha^2) + DPr(\beta-n)\Phi(1+\beta-n, 2+b_0, DPr/\alpha^2)} \quad (34)$$

RESULTS AND DISCUSSION

Some examples for temperature distributions in the fluid are presented here for certain values of the controlling parameters.

POWER-LAW SURFACE TEMPERATURE (PST) CASE

Figure 1(a) presents the influence of radiation parameter R on the temperature profile for $Pr = 0.5$, $\alpha = 2.83$ ($M = 3$), $\lambda = -0.2$ and $p = 0$. It is observed that increasing radiation parameter R is to decrease temperature throughout the boundary layer. This can be explained by the fact that the increase of radiation parameter R implies the release of heat energy from the flow region by means of radiation.

The temperature profiles for power-law surface temperature case are depicted in Figure 1(b) for different values of the Prandtl number Pr ($Pr = 0.5, 1.0, 1.5$). Here the radiation parameter is taken as $R = 0.6$, $M = 2$ ($\alpha = 1.73$), $\lambda = -0.3$ and power index $p = 2$. It is seen that the increase of Prandtl number results in the decrease of temperature distribution. The increase of Prandtl number means slow rate of thermal diffusion. Because of reduced thermal conductivity, there would be a thinning of the thermal boundary layer and this leads to the decrease in the temperature.

The temperature profiles are depicted in Figure 1(c) for different values of the heat sink parameters λ ($\lambda = -0.1, -0.2, -0.3$). Here the radiation parameter is taken as $R = 0.8$, $M = 2$, power index $p = 1$ and $Pr = 0.6$. It is seen that the temperature within the fluid sharply decreased if λ is increased. This is logical because internal heat energy absorption results in a decrease of heat transfer close to the shrinking sheet and this will reduce more around the flow along the sheet.

Figure 1(d) represents the temperature distribution for various values of the power index p . Here $Pr = 0.9$, $\alpha = 1.118$ ($M = 1.5$), $\lambda = -0.2$ and $R = 0.7$. When $p = 0$, the temperature distribution is strictly decreased and ultimately tends to zero for higher values of η . But for $p = 2$, the temperature near the wall is increased first and then decreased for higher values of η . Thus a temperature overshoot at the wall is observed in this case. Similar behavior is also seen for the case $p = 4$.

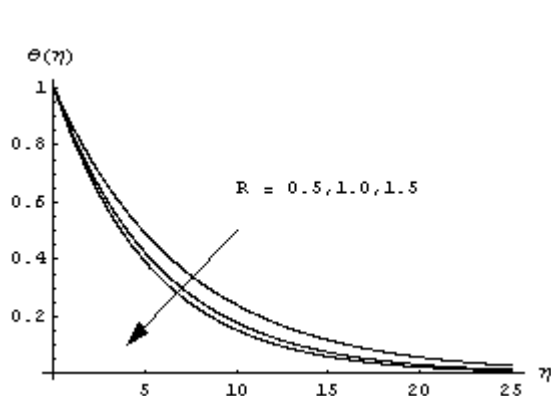


Figure 1(a) Variation of temperature for several values of R with $M = 3$ ($\alpha = 2.83$), $Pr = 0.5$, $\lambda = -0.2$ and $p = 0$.

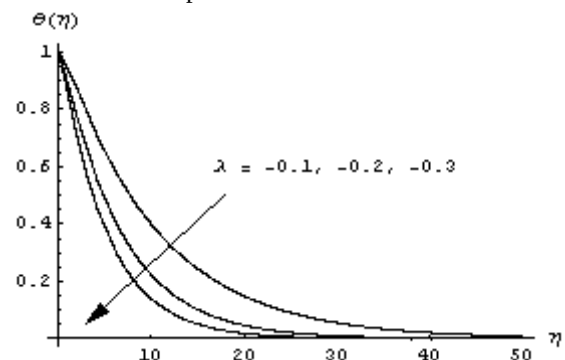


Figure 1(c) The temperature profiles under different values of λ for $Pr = 0.6$, $R = 0.8$, $M = 2$ ($\alpha = 1.73$) and $p = 1$.

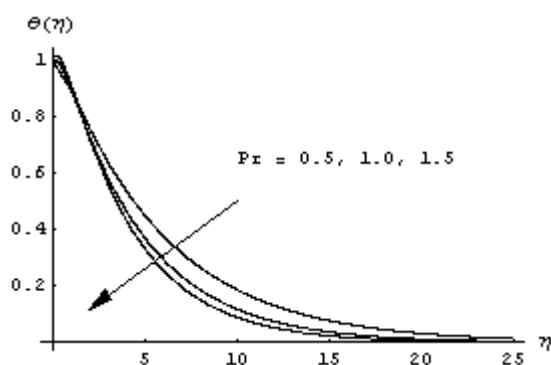


Figure 1(b) The temperature distribution for several values of Pr with $M = 2$ ($\alpha = 1.73$), $R = 0.6$, $\lambda = -0.3$ and $p = 2$.

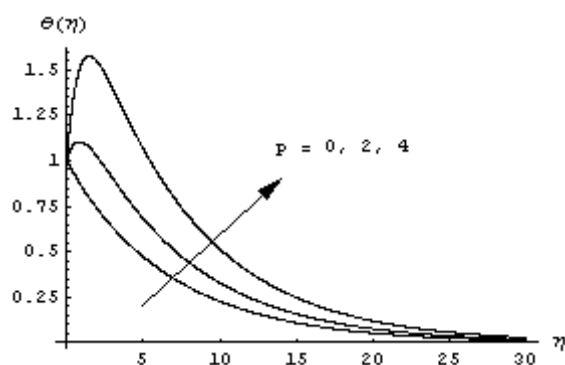


Figure 1(d) The temperature profiles for $M = 1.5$ ($\alpha = 1.118$), $Pr = 0.9$, $\lambda = -0.2$ and $R = 0.7$ under different values of p .

The temperature profiles are depicted in Figure 1(e) for different values of magnetic parameter M ($M = 1.1, 1.5, 2.0$). Here the radiation parameter is taken as $R = 0.7$, $Pr = 0.9$, $\lambda = -0.2$ and power index $p = 1$. We notice that the effect of magnetic parameter is to decrease the temperature in the boundary layer.

POWER-LAW HEAT FLUX (PHF) CASE

The impact of radiation parameter R on the temperature field is presented in Figure 2(a) for $Pr = 0.8$, $\lambda = -0.2$, $M = 2$ and $n = 0$. From the figure, it is noticed that for increasing R , the temperature within the fluid decreases. This result is quite normal because increase in radiation enhances the rate of release of heat from the flow field.

Temperature distribution for various values of Prandtl number Pr is shown in Figure 2(b) for $\lambda = -0.2$, $M = 2$, $R = 0.9$ and $n = 0$. Here three values of Pr considered are 0.5, 1.0 and 1.5. It is observed that with increasing Pr , the dimensionless temperature profile as well as thermal boundary layer thickness decrease. An increase in Prandtl number means a decrease of fluid thermal conductivity which causes a decrease in temperature.

The temperature field for different values of the heat sink parameters λ ($\lambda = -0.2, -0.3, -0.4$) is depicted in Figure 2(c). Here $R = 0.8$, $Pr = 0.6$, $M = 2$ and power index $n = 1$. It is noted from the figure that the dimensionless temperature $\omega(\eta)$ decreases for increasing values of heat sink. The thickness of thermal boundary layer also reduces for the increase of heat sink. Physically, $\lambda < 0$ implies $T_w > T_\infty$ and there will be heat transfer from the flow region to the wall. This result is very much significant for the flow where heat transfer is given prime importance.

Figure 2(d) represents the temperature distribution for various values of the power index n . Here $Pr = 0.9$, $\lambda = -0.1$, $\alpha = 1.73$ ($M = 2$) and $R = 0.6$. When $n = 0$, the temperature distribution is seen to be positive. But for $n = 1$ and 2, the temperature becomes negative. In reality, these negative temperature values may not be applicable.

The temperature profiles are depicted in Figure 2(e) for different values of magnetic parameter M ($M = 1.5, 2, 3$) for $R = 0.7$, $Pr = 0.9$, $\lambda = -0.1$ and power index $n = 0$. Here, the increase of magnetic force causes significant decrease of thermal boundary layer thickness.

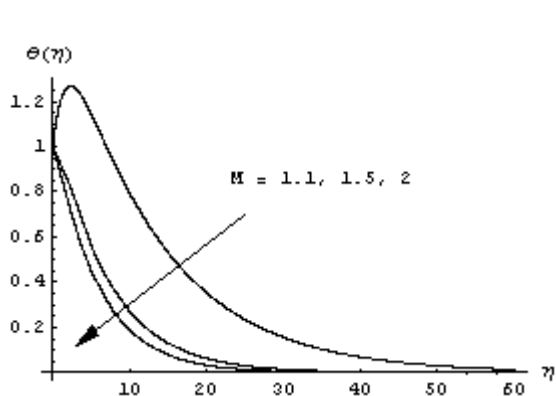


Figure 1(e) The temperature profiles for several values of M with $R = 0.7$, $Pr = 0.9$, $\lambda = -0.2$ and $p = 1$.

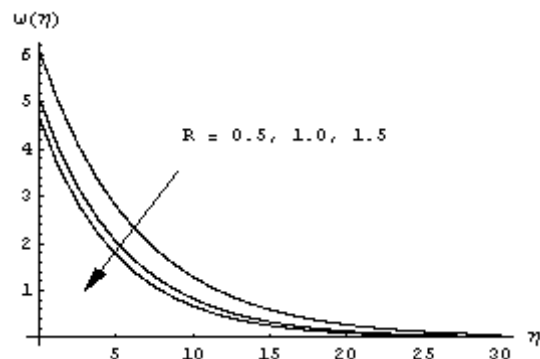


Figure 2(a) Variation of temperature for several values of R with $M = 2$ ($\alpha = 1.73$), $Pr = 0.8$, $\lambda = -0.2$ and $n = 0$.

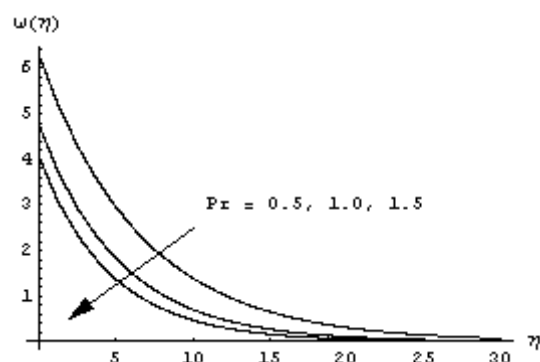


Figure 2(b) The temperature distribution for several values of Pr with $M = 2$ ($\alpha = 1.73$), $R = 0.9$, $\lambda = -0.2$ and $n = 0$.

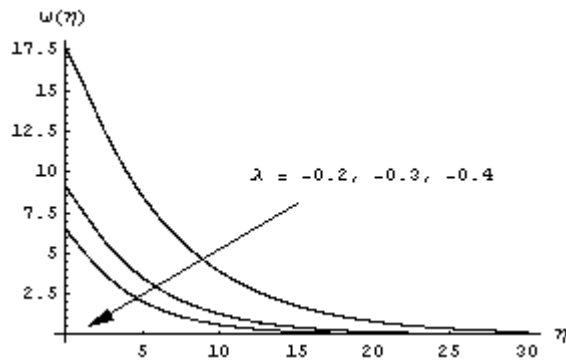


Figure 2(c) the temperature profiles under different values of λ for $Pr = 0.6$, $R = 0.8$, $M = 2$ ($\alpha = 1.73$) and $n = 1$.

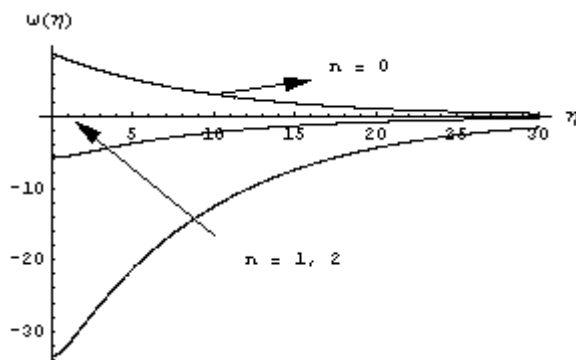


Figure 2(d) The temperature profiles for $M = 2$ ($\alpha = 1.73$), $Pr = 0.9$, $\lambda = -0.1$ and $R = 0.6$ under different values of n .

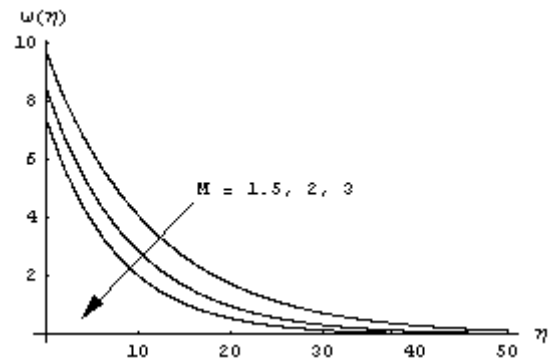


Figure 2(e) The temperature profiles for several values of M with $R = 0.7$, $Pr = 0.9$, $\lambda = -0.1$ and $n = 0$.

CONCLUSIONS

MHD viscous fluid flow and heat transfer over a linearly shrinking surface in the presence of radiation and heat sink is investigated analytically. The exact analytical solution of the boundary layer equation for fluid flow is used to solve the boundary layer equation for energy. Closed form exact solutions of the energy equation have been obtained for both power-law surface temperature and power-law heat flux cases. It is seen that these solutions are valid for all $M > 1$, where M is the magnetic interaction parameter. The effects of radiation parameter R , Prandtl number Pr , heat sink parameter λ on the temperature distribution are studied. The temperature overshoot at the wall is observed for some cases. For some positive power index, the solution has negative non-dimensional temperatures.

REFERENCES

- [1.] Abramowitz M, Stegun IA (1972). *Handbook of Mathematical Functions*, Dover Publications, New York.
- [2.] Ali FM, Nazar R, Arifin NM, Pop I (2010). Unsteady flow and heat transfer past an axisymmetric permeable shrinking sheet with radiation effect. *Int. J. Numer. Method in Fluids*,
- [3.] Ali MM, Chen TS, Armaly BF (1984). Natural convection-radiation interaction on boundary layer flow over horizontal surfaces. *AIAA J*, 22:1797-1803.

- [4.] Brewster MQ (1972). *Thermal Radiative Transfer Properties*. John Wiley and Sons.
- [5.] Devi SPA, Kayalvizhi M (2010). Analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium. *International Journal of Applied Mathematics and Mechanics*, 6(7):82-106.
- [6.] Elbashbeshy EMA (1998). Heat transfer over a stretching surface with variable surface heat flux. *J. Phys. D Appl. Phys.*, 31:1951-1954.
- [7.] Fang T, Zhang J (2009a). Viscous flow over an unsteady shrinking sheet with mass transfer. *Chin. Phys. Lett.*, 26(1):014703-1-4.
- [8.] Fang T, Zhang J (2009b). Closed form exact solutions of MHD viscous flow over a shrinking sheet. *Commun. in Nonlinear Sc. and Numer. Simul.*, 14(7):2853-2857.
- [9.] Fang T, Zhang J (2010). Thermal boundary layer over a shrinking sheet : an analytical solution. *Acta Mech*, 209:325-343.
- [10.] Hayat T, Abbas Z, Sajid M (2007). On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet. *J. Appl. Mech. Trans ASME*, 74(6):1165-1171.
- [11.] Midya C (2012a). Hydromagnetic boundary layer flow and heat transfer over a linearly shrinking permeable surface. *Int. J. of Appl. Math. and Mech.*, 8(3):57-68.
- [12.] Midya C (2012b). Exact Solutions of Chemically Reactive Solute Distribution in MHD Boundary Layer Flow over a Shrinking Surface. *Chin. Phys. Lett.*, 29(1):014701-1-4.
- [13.] Midya C (2012c). Diffusion of chemically reactive species in a viscoelastic flow over a shrinking sheet in the presence of a magnetic field. *Int. J. of Appl. Math. and Mech.*, 8(18):64-78.
- [14.] Midya C, Layek GC, Gupta AS, Mahapatra TR (2003). Magnetohydrodynamics viscous flow separation in a channel with constrictions. *Trans. ASME J. Fluids Engg.*, 125:952-962.
- [15.] Miklavcic M, Wang CY (2006). Viscous flow due to a shrinking sheet. *Quart. of Appl. Math.*, 64(2):283-290.
- [16.] Muhaimin, Kandasamy R, Hashim I (2010). Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction. *Nuclear Engineering and Design*, 240(5):933-939.
- [17.] Muhaimin, Kandasamy R, Khamis AB (2008). Effects of heat and mass transfer on nonlinear MHD boundary layer flow over a shrinking sheet in the presence of suction. *Appl. Math. and Mech. (English Edition)*, 29(10):1309-1317.
- [18.] Nadeem S, Awais M (2008). Thin film flow on an unsteady shrinking sheet through porous medium with variable viscosity. *Physics Letters A*, 372:4965-4972.
- [19.] Nadeem S, Hussain A (2009). MHD flow of a viscous fluid on a nonlinear porous shrinking sheet with homotopy analysis method. *Appl. Math. Mech. (Engl. Ed.)*, 30(12):1569-1578.
- [20.] Noor NFM, Kechilb SA, Hashimc I (2010). Simple non-perturbative solution for MHD viscous flow due to a shrinking sheet. *Commun. in Nonlinear Sc. and Numer. Simul.*, 15(2):144-148.
- [21.] Ouaf MEM (2005). Exact solution of thermal radiation on MHD flow over a stretching porous sheet. *Appl Math Comput*, 170:1117-1125.
- [22.] Rajput US, Kumar S (2012). Radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. *Int. J. of Appl. Math. and Mech.*, 8(1):66-85.
- [23.] Sajid M, Hayat T (2009). The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet. *Chaos, Soliton & Fractals*, 39(3):1317-1323.
- [24.] Viskanta R, Grosh RJ (1962). Boundary layer in thermal radiation absorbing and emitting media. *Int. J. Heat Mass Transfer*, 5:795-806.
- [25.] Wang CY (1990). Liquid film on an unsteady stretching sheet. *Quart. Appl. Math.*, 48:601- 610.
- [26.] Wang CY (2008). Stagnation flow towards a shrinking sheet, *Int. J. Nonlinear Mech.* 43:377-382.