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# INTUITIONISTIC FUZZY LATTICE ORDERED M-GROUPS

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**ABSTRACT:** This paper contains some definitions and results in intuitionistic fuzzy lattice ordered M-groups, which are required in the sequel. Some properties of homomorphism and anti-homomorphism of intuitionistic L-fuzzy M-subgroups are also established.

**Keywords**-fuzzy m subgroup, lattice ordered m-group, Anti L-fuzzy M-subgroup, intuitionistic fuzzy lattice ordered M-groups

## 1. INTRODUCTION:

The notion of fuzzy sets was introduced by L.A. Zadeh [10].Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, Rosenfield [1] introduced the concept of fuzzy subgroup. Motivated by this,many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], Biswas introduced the concept of anti-fuzzy subgroups of groups. Palaniappan. N and Muthuraj**,** [7] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups .G.S.V. Satya Saibaba [4] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l-groups. Pandiammal P, Natarajan R and Palaniappan N, [9] defined the homomorphism, antihomomorphism of an anti L-fuzzy M-subgroup, Institutionistic L- fuzzy m-groups. In this paper we define a new algebraic structure of intuitionistic fuzzy lattice ordered M-group and studied some related properties.

## 2. PRELIMINARIES:

**2.1 Definition:** Let G be a group, M be any set if i) m x ϵ G. ii) m (x y) = (m x) y = x m y for all x, yϵ G, m ϵ M. Then G is called a M group.

**2.2 Definition :** Let μ: X→ [0, 1] be a fuzzy set & G be a M group G. A fuzzy set on G,
G ϵ □(X)) is called a fuzzy m group if i) $\mu(m(x\,y)) \geq \min\{\mu(m\,x), \mu(my)\}$
ii) $\mu(mx^{-1}) \geq \mu(m\,x)$ for all x, yϵ G, m ϵ M

**2.3 Definition:** Let G be a M-group. A L-fuzzy subset A of G is said to be L-fuzzy M-subgroup (LFMSG) of G if it satisfies the following axioms:
(i) $\mu_A(mxy) \geq \min\{\mu_A(mx), \mu_A(y)\}$
(ii) $\mu_A(mx^{-1}) \geq \mu_A(mx)$, for all x and y in G.
**2.4 Definition:** Let G be a M-group. A L-fuzzy subset A of G is said to be anti L-fuzzy M-subgroup (LFMSG) of G if it satisfies the following axioms:
(i) $\mu_A(mxy) \leq \min\{\mu_A(mx), \mu_A(y)\}$
(ii)$\mu_A(mx^{-1}) \leq \mu_A(mx)$, for all x and y in G.

**2.5 Definition:** A lattice ordered group is a system ( G,.,≤) if i) (G, ·) is a group ii) (G,≤ ) is a lattice . iii ) x ≤ y implies a x b ≤ a y b ( compatibility) For a, b, x, yϵ G

**2.6 Definition :** Let μ: X → [0, 1] be a fuzzy set & G is a lattice ordered set, G ϵ □(X).
A function μ on G is said to be a fuzzy lattice ordered group if i) $\mu(x\,y) \geq \min\{\mu(x), \mu(y)\}$
ii) $\mu(x^{-1}) \geq \mu(x)$ for all x, yϵ G

**2.7 Definition:** μ: X to [0, 1], G ϵ □(X), M □ X.
A function μ on G is said to be a fuzzy lattice ordered m-group if

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i) (G, ·) is a M-group.
ii) (G,·, ≤) is a lattice ordered group.
iii) $\mu ( m(x y) ) \geq \min \{\mu (m x ), \mu(my)\}$
iv) $\mu ( (m x)^{-1} ) \geq \mu (m x )$
v) $\mu (m x \vee my ) \geq \min \{\mu (m x ), \mu(m y)\}$
vi) $\mu (m x \wedge my ) \geq \min \{\mu (m x ), \mu(my)\}$ For all x, y ϵ G

**2.8 Definition:** Let (G, ·) be a M-group. An intuitionistic L fuzzy subset A of G is said to be an intuitionistic L-fuzzy M subgroup (ILFMSG) of G if the following conditions are satisfied:
(i) $\mu_A ( mxy ) \geq \min \{ \mu_A (mx) , \mu_A (my) \}$
(ii) $\mu_A ( mx^{-1}) \geq \mu_A ( mx )$,
(iii) $\nu_A ( mxy ) \leq \max \{\nu_A (mx) \mu_A (my) \}$
(iv) $\nu_A ( mx^{-1} ) \leq \nu_A (mx )$, for all x and y in G.

**2.9 Definition:** μ: X to [0, 1], G ϵ □(X), M □ X. A function μ on G is said to be an Intuitionistic fuzzy lattice ordered m-group ( IFLOMG) if
i) (G, ·) is a M-group.
ii) (G,·, ≤) is a lattice ordered group.
iii) $\mu ( m(x y) ) \geq \min \{\mu (m x ), \mu(my)\}$
iv) $\mu ( (m x)^{-1} ) \geq \mu (m x )$
v) $\mu (m x \vee my ) \geq \min \{\mu (m x ), \mu(m y)\}$
vi) $\mu (m x \wedge my ) \geq \min \{\mu (m x ), \mu(my)\}$
vii) $\nu_A ( mxy ) \leq \max \{\nu_A (mx), \mu_A (my) \}$
viii) $\nu_A ( (m x)^{-1} ) \leq \nu_A (m x )$
ix) $\nu_A ( mx \vee my ) \leq \max \{\nu_A (mx), \mu_A (my) \}$
x) $\nu_A ( mx \wedge my ) \leq \max \{\nu_A (mx), \mu_A (my) \}$

**2.10 Definition:** Let ( G, · ) and ( G', · ) be any two M-groups. Let f : G → G' be any function and A be an IFLOMG in G, V be an IFLOMG in f(G) = G', defined by $\mu_v(y) = \sup\{ \mu_A(x)) /$ $x\epsilon\ f^{-1}(y) \}$ and $\nu_A(y) = \inf\{ \nu_A(x)) / x\epsilon\ f^{-1}(y) \}$ , for all x in G and y in G'. Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.
**2.11 Definition:** Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B denoted by A x B , is defined as
A x B = {< (x, y ), $\mu_{A x B}( x, y )$ , $\nu_{AxB}( x, y )$ >/ for all xϵ G and y ϵ H },where
$\mu_{A x B}( x, y ) = \mu_A(x) \wedge \mu_B(y)$ } and $\nu_{A x B}( x, y ) = \nu_A(x) \vee \nu_B(y)$
**2.12 Definition:** Let A and B be any two IFLOMG. Then A and B are said to be conjugate IFLOMG if for some g in G, $\mu_A (x) = \mu_B (m g^{-1} x mg)$ and $\nu_A (x) = \nu_B (mg^{-1} x mg)$, for every x in G.
**2.13 Definition:** Let A be an intuitionistic L-fuzzy subset in a set S, the strongest intuitionistic L-fuzzy relation on S, that is an intuitionistic L-fuzzy relation on A is V given by
$\mu_v( x, y ) = \min \{\mu_A(x) , \mu_A(y) \}$ and $\nu_v (x, y ) = \max \{ \nu_A(x) ,\nu_A(y) \}$, for all x and y in S.

## 3 – PROPERTIES OF INTUITIONISTIC L-FUZZY M-SUBGROUPS:

**3.1 Theorem:** If A is an IFLOMG of a M-group (G, ·), then $\mu_A ( mx^{-1}) = \mu_A(mx)$ and $\nu_A( mx^{-1})=\nu_A(mx)$, $\mu_A(mx) \leq \mu_A(me)$ and $\nu_A(mx) \geq \nu_A(me)$, for x in G, where e is the identity element in G.
**Proof:** For x in G and e is the identity element in G.
Now, $\mu_A(mx) = \mu_A( (mx^{-1} )^{-1} ) \geq \mu_A( m x^{-1} ) \geq \mu_A(m x )$.
Therefore, $\mu_A(mx^{-1}) = \mu_A (m x)$. And ,
$\nu_A (m x ) = \nu_A( (mx^{-1})^{-1} ) \leq \nu_A ( mx^{-1}) \leq \nu_A(m x)$.
Therefore, $\nu_A(mx^{-1}) = \nu_A(mx)$, for all x in G.
Now, $\mu_A(me) = \mu_A(m xx^{-1}) \geq \min \{\mu_A(mx) , \mu_A (mx^{-1})\} = \min \{\mu_A(mx) , \mu_A (mx)\}= \mu_A(mx)$.
Therefore, $\mu_A(me) \geq \mu_A(m x)$. And,
$\nu_A(me) = \nu_A(m xx^{-1}) \leq \max \{\nu_A(mx), \nu_A(mx^{-1}) \}= \max \{\nu_A(mx), \nu_A(mx) \} = \nu_A(mx)$.
Therefore, $\nu_A(me) = \nu_A(mx)$ , for all x in G.

**3.2 Theorem:** If A is an IFLOMG of a M-group ( G, · ), then
(i) $\mu_A( mxy^{-1}) = \mu_A(me)$ gives $\mu_A(mx) = \mu_A(my)$,
(ii) $\nu_A(m xy^{-1}) = \nu_A (me)$ gives $\nu_A (mx) = \nu_A (my)$, for x & y in G, where e is the identity element in G.
Proof: Let x & y in G and e is the identity element in G.
Now, $\mu_A (mx) = \mu_A (m xy^{-1}y ) \geq \min \{\mu_A (mxy^{-1}) ,\mu_A (my) \}$
$= \min \{\mu_A (me), \mu_A (my) \}$
$= \mu_A (my) = \mu_A ( myx^{-1}x )$
$\geq \min \{\mu_A (myx^{-1}) ,\mu_A (mx) \}$

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= min $\{\mu_A(me), \mu_A(mx)\}$
= $\mu_A(mx)$.
Therefore, $\mu_A(mx) = \mu_A(my)$, for all x and y in G.
And, $\nu_A(mx) = \nu_A(mxy^{-1}y) \leq \max\{\nu_A(mxy^{-1}), \nu_A(my)\}$
= $\max\{\nu_A(me), \nu_A(my)\}$
= $\nu_A(my)$
= $\nu_A(myx^{-1}x)$
$\leq \max\{\nu_A(myx^{-1}), \nu_A(mx)\}$
= $\max\{\nu_A(me), \nu_A(mx)\}$
= $\nu_A(mx)$.
Therefore, $\nu_A A(mx) = \nu_A A(my)$, for all x and y in G.

**3.3 Theorem:** A is an IFLOMG of a M-group (G, ·) if and only if
$\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$, $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$,
$\mu_A(mx \vee my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$, $\mu_A(mx \wedge my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$
$\nu_A(mx \vee my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$, $\nu_A(mx \wedge my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$
for all x , y in G.

**Proof:** Let A be an IFLOMG of a M-group (G, ·).
Then, $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} = \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), A(my)\}$ , for all x & y in G. And,
$\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} = \max\{\nu_A(mx), \nu_A(my)\}$.
Therefore, $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$ , for all x & y in G.
$\mu_A(mx \vee my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} = \min\{\mu_A(mx), \mu_A(my)\}$
Therefore $\mu_A(mx \vee my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$
$\mu_A(mx \wedge my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} = \min\{\mu_A(mx), \mu_A(my)\}$
Therefore $\mu_A(mx \wedge my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$
$\nu_A(mx \vee my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} = \max\{\nu_A(mx), \nu_A(my)\}$
Therefore $\nu_A(mx \vee my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$
$\nu_A(mx \wedge my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} = \max\{\nu_A(mx), \nu_A(my)\}$
Therefore $\nu_A(mx \wedge my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$

Conversely, if $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$ and $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), A(my)\}$ ,
replace y by x, then
$\mu_A(me) \geq \min\{\mu_A(mx), \mu_A(mx)\} = \mu_A(mx)$ and
$\nu_A(me) \leq \max\{\nu_A(mx), A(mx)\} = \nu_A(mx)$
i) $\mu_A(mx^{-1}) = \mu_A(mex^{-1}) \geq \min\{\mu_A(me), \mu_A(mx)\} = \mu_A(mx)$.
Therefore, $\mu_A(mx^{-1}) \geq \mu_A(mx)$.
ii) $\mu_A(mxy) = \mu_A(mx(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mxy) \geq \min\{\mu_A(mx), \mu_A(my)\}$ , for all x and y in G.
iii) $\nu_A(mx^{-1}) = \nu_A(mex^{-1}) \leq \max\{\nu_A(me), \nu_A(mx)\} = \nu_A(x)$.
Therefore, $\nu_A(mx^{-1}) \leq \nu_A(mx)$.
iv) $\nu_A(mxy) = \nu_A(mx(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\nu_A(mxy) \leq \max\{\nu_A(mx), \nu_A(my)\}$ for all x and y in G.
v) $\mu_A(mx \vee my) = \mu_A(mx \vee m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \geq \min\{\mu_A(mx), \mu_A(my)\}$ , for all x and y in G.
vi) $\mu_A(mx \wedge my) = \mu_A(mx \wedge m(y^{-1})^{-1}) \geq \min\{\mu_A(mx), \mu_A(my^{-1})\} \geq \min\{\mu_A(mx), \mu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \geq \min\{\mu_A(mx), \mu_A(my)\}$ , for all x and y in G.
vii) $\nu_A(mx \vee my) = \nu_A(mx \vee m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \vee my) \leq \max\{\mu_A(mx), \mu_A(my)\}$ , for all x and y in G.
viii) $\nu_A(mx \wedge my) = \nu_A(mx \wedge m(y^{-1})^{-1}) \leq \max\{\nu_A(mx), \nu_A(my^{-1})\} \leq \max\{\nu_A(mx), \nu_A(my)\}$
Therefore, $\mu_A(mx \wedge my) \leq \max\{\mu_A(mx), \mu_A(my)\}$ , for all x and y in G.
Hence A is an IFLOMG of G.

**3.4 Theorem:** Let A be an intuitionistic L-fuzzy subset of a group (G, ·). If $\mu_A(me) = 1$ and $\nu_A(me) = 0$ and $\mu_A(mxy^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$ and $\nu_A(mxy^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$,
$\mu_A(mx \vee my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$, $\mu_A(mx \wedge my^{-1}) \geq \min\{\mu_A(mx), \mu_A(my)\}$
$\nu_A(mx \vee my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$, $\nu_A(mx \wedge my^{-1}) \leq \max\{\nu_A(mx), \nu_A(my)\}$
for all x , y in G then A is an IFLOMG of a M-group G.
**Proof:** Let x & y in G and e is the identity element in G.
i) $\mu_A(mx^{-1}) = \mu_A(mex^{-1}) \geq \min\{\mu_A(me), \mu_A(mx)\} = \min\{1, \mu_A(mx)\} = \mu_A(mx)$
Therefore, $\mu_A(mx^{-1}) \geq \mu_A(mx)$, for all x in G.
ii) $\nu_A(mx^{-1}) = \nu_A(mex^{-1}) \leq \max\{\nu_A(me), \nu_A(mx)\} = \max\{0, \nu_A(mx)\} = \nu_A(mx)$.

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Therefore, $\nu_A (mx^{-1}) \leq \nu_A (mx)$, for all x in G.
iii) $\mu_A (mxy) = \mu_A (m x(y^{-1})^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\} \geq \min \{ \mu_A(mx), \mu_A (my)\}$
Therefore, $\mu_A (mxy) \geq \min \{\mu_A (mx), \mu_A (my)\}$ , for all x and y in G.
iv) $\nu_A (mxy) = \nu_A ( mx(y^{-1})^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my^{-1})\} \leq \max \{ \nu_A (mx), \nu_A (my)\}$
Therefore, $\nu_A (mxy) \leq \max \{\nu_A (mx) , \nu_A (my)\}$ for all x and y in G.
v) $\mu_A (mx \vee my) = \mu_A (mx \vee m(y^{-1})^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\} \geq \min \{ \mu_A(mx), \mu_A (my)\}$
Therefore, $\mu_A (mx \vee my) \geq \min \{\mu_A (mx), \mu_A (my)\}$ , for all x and y in G.
vi) $\mu_A (mx \wedge my) = \mu_A (mx \wedge m(y^{-1})^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\} \geq \min \{ \mu_A(mx), \mu_A (my)\}$
Therefore, $\mu_A (mx \wedge my) \geq \min \{\mu_A (mx), \mu_A (my)\}$ , for all x and y in G.
vii) $\nu_A (mx \vee my) = \nu_A ( mx \vee m(y^{-1})^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my^{-1})\} \leq \max \{ \nu_A (mx), \nu_A (my)\}$
Therefore, $\mu_A (mx \vee my) \leq \max \{\mu_A (mx), \mu_A (my)\}$ , for all x and y in G.
viii) $\nu_A (mx \wedge my) = \nu_A ( mx \wedge m(y^{-1})^{-1}) \leq \max \{ \nu_A (mx), \nu_A (my^{-1})\} \leq \max \{ \nu_A (mx), \nu_A (my)\}$
Therefore, $\mu_A (mx \wedge my) \leq \max \{\mu_A (mx), \mu_A (my)\}$ , for all x and y in G.
Hence A is an IFLOMG of G.Hence A is an intuitionistic L-fuzzy M-subgroup of a M-group G.

**3.5 Theorem:** If A is an IFLOMG of a M-group (G, ·), then $H = \{mx / x\epsilon\ G : \mu_A(mx) = 1, \nu_A (mx) = 0\}$ is either empty or is a M-subgroup of a M-group G.

**Proof:** If no element satisfies this condition, then H is empty. If m x and m y in H, then
$\mu_A ( mxy^{-1}) \geq \min \{\mu_A(mx), \mu_A(my^{-1})\} \geq \min \{ \mu_A (mx), \mu_A (my)\} = 1$.
Therefore, $\mu_A ( mxy^{-1}) = 1$, for all x and y in G.
And, $\nu_A (m xy^{-1}) \leq \max \{\nu_A (mx), \nu_A (my^{-1})\} \leq \max\{ \nu(mx), \nu(my)\}=0$.
Therefore, $\nu_A ( mxy^{-1}) = 0$, for all x and y in G. We get $mxy^{-1}$ in H.
Therefore, H is a M-subgroup of a M-group G.
Hence H is either empty or is a M-subgroup of M-group G.

**3.6 Theorem:** If A is an IFLOMG of a M-group (G, ·), then $H= \{ mx\epsilon\ G: \mu_A (mx) = \mu_A (me)$ and $\nu_A (mx) = \nu_A (me) \}$ is either empty or is a M subgroup of a M-group G.

**Proof:** If no element satisfies this condition, then H is empty. If mx and my satisfies this condition, then $\mu_A (m x^{-1}) = \mu_A (mx) = \mu_A (me)$, $\nu_A ( m x^{-1}) = \nu_A (mx) = \mu_A (me)$.
Therefore, $\mu_A ( mx^{-1}) = \mu_A (me)$ and $\nu_A (m x^{-1}) = \nu_A (m e)$.Hence $mx^{-1}$ in H.
Now, $\mu_A ( mxy^{-1}) \geq \min \{\mu_A (mx), \mu_A (my^{-1})\}= \min \{\mu_A (mx) , \mu_A (my)\}$
$= \min \{ \mu_A (me), \mu_A (me)\} = \mu_A (me)$.
Therefore, $\mu_A ( mxy^{-1}) \geq \mu_A (me)$ , for all x and y in G---(1).
And, $\mu_A (me) = \mu_A (m (xy^{-1})(xy^{-1})^{-1}) \geq \min\{ \mu_A (m xy^{-1}) , \mu_A (m( xy^{-1})^{-1})\}$
$= \min \{\mu_A (mxy^{-1}), \mu_A (mxy^{-1})\}= \mu_A (mxy^{-1})$.
Therefore, $\mu_A (me) = \mu_A (mxy^{-1})$, for all x and y in G----(2).
From (1) and (2), we get $\mu_A (me) = \mu_A (mxy^{-1})$.
Now, $\nu_A ( mxy^{-1}) \geq \min \{\nu_A (mx), \nu_A (my^{-1})\}= \min \{\nu_A (mx) , \nu_A (my)\}$
$= \min \{ \nu_A (me), \nu_A (me)\} = \nu_A (me)$.
Therefore, $\nu_A ( mxy^{-1}) \geq \nu_A (me)$ , for all x and y in G ---(1).
And, $\nu_A (me) = \nu_A (m (xy^{-1})(xy^{-1})^{-1}) \geq \min\{ \nu_A (m xy^{-1}) , \nu_A (m( xy^{-1})^{-1})\}$
$= \min \{\nu_A (mxy^{-1}), \nu_A (mxy^{-1})\}= \nu_A (mxy^{-1})$.
Therefore, $\nu_A (me) = \nu_A (mxy^{-1})$, for all x and y in G ----(2).
From (1) and (2), we get $\nu_A (me) = \nu_A (mxy^{-1})$.
Hence H is either empty or is a M-subgroup of a M-group G.

**3.7 Theorem:** Let (G, ·) be a M-group. If A is an IFLOMG of G, then
$\mu_A (mxy) = \min \{\mu_A (mx), \mu_A (my)\}$ and $\nu_A (mxy) = \max \{\nu_A (mx), \nu_A (my)\}$ with $\mu_A (mx) \neq \mu_A (my)$ and $\nu_A(mx) \neq \nu_A(my)$, for each x and y in G.
**Proof:** Let x and y belongs to G.
Assume that $\mu_A (mx) > \mu_A (my)$ and $\nu_A(mx) < \nu_A(my)$, for each x and y in G.
Now, $\mu_A(my) = \mu_A ( mx^{-1}x y ) \geq \min\{ \mu_A (mx^{-1} ), \mu_A (m x y )\} \geq \min \{ \mu_A (mx) , \mu_A (mxy)\}$
$= \mu_A (m xy) \geq \min\{ \mu_A (mx), \mu_A (my)\}= \mu_A (my)$.
Therefore,$\mu_A (m xy) = \mu_A (my) = \min \{ \mu_A (mx), \mu_A (my)\}$ for all x and y in G.
Assume that $\nu_A(mx) < \nu_A(my)$, for each x and y in G.
Now, $\nu_A (my) = \nu_A ( mx^{-1}x y ) \leq \max\{ \nu_A (mx^{-1} ), \nu_A (m x y )\} \leq \max \{ \nu_A (mx) , \nu_A (mxy)\}$
$= \nu_A (m xy) \leq \max\{ \nu_A (mx), \nu_A (my)\}= \nu_A (my)$.
Therefore, $\nu_A (m xy) = \nu_A (my) = \max \{ \nu_A (mx), \nu_A (my)\}$ for all x and y in G.

**3.8 Theorem:** If A and B are two IFLOMG of a M-group (G, ·), then their intersection $A\cap B$ is an IFLOMG of G.
**Proof:** Let x and y belong to G,
$A = \{< x, \mu_A (x), \nu_A(x) > / x\epsilon\ G \}$ and

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$B = \{< x, \mu_B(x), \nu_B(x) > / x \in G \}$.
$A \cap B = \{ <x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > / x \in G \}$.
i) $\mu_{A\cap B}(mxy) = \mu_A(mxy) \wedge \mu_B(mxy)$
$\geq \min\{ \mu_A(mx), \mu_A(my) \} \wedge \min\{ \mu_B(mx), \mu_B(my) \}$
$\geq \min\{ \mu_A(mx) \wedge \mu_B(mx), \{ \mu_A(my) \wedge \mu_B(my) \}$
$= \min\{ \mu_{A\cap B}(mx), \mu_{A\cap B}(my)\}$
ii) $\mu_{A\cap B}(mx^{-1}) = \mu_A(mx^{-1}) \wedge \mu_B(mx^{-1}) \geq \mu_A(mx) \wedge \mu_B(mx) = \mu_{A\cap B}(mx)$
iii) $\nu_{A\cap B}(mxy) = \nu_A(mxy) \vee \nu_B(mxy)$
$\leq \max\{ \nu_A(mx), \nu_A(my) \} \vee \max\{ \nu_B(mx), \nu_B(my) \}$
$\leq \max\{ \nu_A(mx) \vee \nu_B(mx), \{ \nu_A(my) \vee \nu_B(my) \}$
$= \max\{ \nu_{A\cap B}(mx), \nu_{A\cap B}(my)\}$
iv) $\nu_{A\cap B}(mx^{-1}) = \nu_A(mx^{-1}) \vee \nu_B(mx^{-1}) \geq \nu_A(mx) \vee \nu_B(mx) = \nu_{A\cup B}(mx)$
v) $\mu_{A\cap B}(mx \vee my) = \mu_A(mx \vee my) \wedge \mu_B(mx \vee my)$
$\geq \min\{ \mu_A(mx), \mu_A(my) \} \wedge \min\{ \mu_B(mx), \mu_B(my) \}$
$\geq \min\{ \mu_A(mx) \wedge \mu_B(mx), \{ \mu_A(my) \wedge \mu_B(my) \}$
$= \min\{ \mu_{A\cap B}(mx), \mu_{A\cap B}(my)\}$
vi) $\mu_{A\cap B}(mx \wedge my) = \mu_A(mx \wedge my) \wedge \mu_B(mx \wedge my)$
$\geq \min\{ \mu_A(mx), \mu_A(my) \} \wedge \min\{ \mu_B(mx), \mu_B(my) \}$
$\geq \min\{ \mu_A(mx) \wedge \mu_B(mx), \{ \mu_A(my) \wedge \mu_B(my) \}$
$= \min\{ \mu_{A\cap B}(mx), \mu_{A\cap B}(my)\}$
vii) $\nu_{A\cap B}(mx \vee my) = \nu_A(mx \vee my) \vee \nu_B(mx \vee my)$
$\leq \max\{ \nu_A(mx), \nu_A(my) \} \vee \max\{ \nu_B(mx), \nu_B(my) \}$
$\leq \max\{ \nu_A(mx) \vee \nu_B(mx), \{ \nu_A(my) \vee \nu_B(my) \}$
$= \max\{ \nu_{A\cap B}(mx), \nu_{A\cap B}(my)\}$
viii) $\nu_{A\cap B}(mx \wedge my) = \nu_A(mx \wedge my) \vee \nu_B(mx \wedge my)$
$\leq \max\{ \nu_A(mx), \nu_A(my) \} \vee \max\{ \nu_B(mx), \nu_B(my) \}$
$\leq \max\{ \nu_A(mx) \vee \nu_B(mx), \{ \nu_A(my) \vee \nu_B(my) \}$
$= \max\{ \nu_{A\cap B}(mx), \nu_{A\cap B}(my)\}$
Hence $A \cap B$ is an IFLOMG of a M-group G.

**3.9 Theorem:** If $\{A_i\}$ is a family of IFLOMG of G then $\cap A_i$ is a IFLOMG of G where
$\cap A_i = \{ < x, \wedge \mu_{Ai}(x), \vee \nu_{Ai}(x) / x \in G \}$.

**Proof:**

i) $\cap \mu_{Ai}(mxy) = \wedge \mu_{Ai}(mxy)$
$\geq \wedge \min\{ \mu_{Ai}(mx), \mu_{Ai}(my) \}$
$= \min\{ \wedge \mu_{Ai}(mx), \wedge \mu_{Ai}(my) \}$
$= \min\{\cap \mu_{Ai}(mx), \cap \mu_{Ai}(my)\}$
ii) $\cap \mu_{Ai}(mx^{-1}) = \wedge \mu_{Ai}(mx^{-1}) \geq \wedge \mu_{Ai}(mx) = \cap \mu_{Ai}(mx)$
iii) $\cap \nu_{Ai}(mxy) = \vee \nu_{Ai}(mxy)$
$\leq \vee \max\{ \nu_{Ai}(mx), \nu_{Ai}(my) \}$
$= \max\{ \vee \nu_{Ai}(mx), \vee \nu_{Ai}(my) \}$
$= \max\{ \cap \nu_{Ai}(mx), \cap \nu_{Ai}(my)\}$
iv) $\cap \nu_{Ai}(mx^{-1}) = \vee \nu_{Ai}(mx^{-1}) \geq \vee \cap \nu_{Ai}(mx) = \cap \nu_{Ai}(mx)$
v) $\cap \mu_{Ai}(mx \vee my) = \wedge \mu_{Ai}(mx \vee my)$
$\geq \wedge \min\{ \mu_{Ai}(mx), \mu_{Ai}(my) \}$
$= \min\{ \wedge \mu_{Ai}(mx), \wedge \mu_{Ai}(my) \}$
$= \min\{\cap \mu_{Ai}(mx), \cap \mu_{Ai}(my)\}$

vi) $\cap \mu_{Ai}(mx \wedge my) = \wedge \mu_{Ai}(mx \wedge my)$
$\geq \wedge \min\{ \mu_{Ai}(mx), \mu_{Ai}(my) \}$
$= \min\{ \wedge \mu_{Ai}(mx), \wedge \mu_{Ai}(my) \}$
$= \min\{\cap \mu_{Ai}(mx), \cap \mu_{Ai}(my)\}$

vii) $\cap \nu_{Ai}(mx \vee my) = \vee \nu_{Ai}(mx \vee my)$
$\leq \vee \max\{ \nu_{Ai}(mx), \nu_{Ai}(my) \}$
$= \max\{ \vee \nu_{Ai}(mx), \vee \nu_{Ai}(my) \}$
$= \max\{ \cap \nu_{Ai}(mx), \cap \nu_{Ai}(my)\}$

viii) $\cap \nu_{Ai}(mx \wedge my) = \vee \nu_{Ai}(mx \wedge my)$
$\leq \vee \max\{ \nu_{Ai}(mx), \nu_{Ai}(my) \}$
$= \max\{ \vee \nu_{Ai}(mx), \vee \nu_{Ai}(my) \}$

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$= \max\{ \cap \nu_{Ai} ( mx) , \cap \nu_{Ai} (my)\}$
Hence $\cap A_i$ is an IFLOMG of a M-group G.

**3.10 Theorem:** If A is an IFLOMG of a M-group G, then (i) $\mu_A( mxy) = \mu_A(myx)$ if and only if $\mu_A (mx) = \mu_A (my^{-1}x\, my)$ (ii) $\nu_A (m\, xy) = \nu_A (m\, yx)$ if and only if $\nu_A (mx) = \nu_A (m\, y^{-1}x\, my)$, for x and y in G.
**Proof:** Let x and y be in G.
Assume that $\mu_A (m\, xy) = \mu_A (m\, yx)$, we have $\mu_A (m\, y^{-1}x\, m\, y ) = \mu_A ( my^{-1} m\, xy )$
$= \mu_A ( my^{-1} my\, x ) = \mu_A (mex) = \mu_A (mx)$.
Therefore, $\mu_A ( mx) = \mu_A (m\, y^{-1}x\, my )$, for all x and y in G.
Conversely, assume that $\mu_A (m\, x) = \mu_A (m\, y^{-1}x\, my )$, we have $\mu_A (m\, xy ) = \mu_A (m\, xy\, e)$
$= \mu_A (m\, xy\, me ) = \mu_A ( m\, xy\, mxx^{-1} ) = \mu_A ( m\, x\, y\, x\, m\, x^{-1} ) = \mu_A (yx)$.
Therefore, $\mu_A ( m\, xy) = \mu_A (m\, yx)$, for all x and y in G.
Now, we assume that $\nu_A (xy) = \nu_A (yx)$, we have $\nu_A (m\, y^{-1}x\, m\, y ) = \nu_A ( my^{-1} m\, xy )$
$= \nu_A ( my^{-1} m\, yx ) = \nu_A ( me\, x ) = \nu_A (m\, x) = \nu_A ( m\, x)$. Therefore, $\nu_A (mx) = \nu_A ( my^{-1}x\, my )$
, for all x and y in G.
Conversely, we assume that $\nu_A (m\, x) = \nu_A (m\, y^{-1}x\, my )$, we have $\nu_A (m\, xy ) = \nu_A (m\, xy\, e)$
$= \nu_A (m\, xy\, me ) = \nu_A ( m\, xy\, mxx^{-1} ) = \nu_A ( m\, x\, y\, x\, m\, x^{-1} ) = \nu_A (yx)$.
for all x and y in G .Hence (ii) is proved.

**3.11 Theorem:** Let A be an IFLOMG of a M-group G. If $\mu_A (mx) < \mu_A (my )$ and $\nu_A (mx) > \nu_A (my)$, for some x and y in G, then (i) $\mu_A (mxy) = \mu_A (mx) = \mu_A (myx)$,
(ii) $\nu_A (mxy) = \nu_A (mx) = \nu_A (myx)$, for all x and y in G .

**Proof:** Let A be an IFLOMG of a M group G.
Also we have $\mu_A (mx) < \mu_A (my)$ and $\nu_A (mx) > \nu_A(my)$, for some x and y in G,
$\mu_A(m\, xy) \geq \min \{\mu_A (mx) , \mu_A (my)\}$ (as A is an IFLOMG of G) $= \mu_A (mx)$ ; and
$\mu_A (mx) = \mu_A (mxyy^{-1}) \geq \min \{ \mu_A (mxy) ,\mu_A (my^{-1}) \} \geq \min\{\mu_A (mxy), \mu_A (my)\} = \mu_A(mxy)$.
Therefore, $\mu_A (mxy) = \mu_A (mx)$, for all x and y in G.
And, $\mu_A (myx) \geq \min\{ \mu_A (my), \mu_A (mx)\}$ (as A is an ILFMSG of G) $= \mu_A (mx)$ ;
and $\mu_A (mx) = \mu_A (my^{-1}yx ) \geq \min \{\mu_A (my^{-1}) , \mu_A (myx)\} \geq \min\{ \mu_A (my), \mu_A (myx)\}$, as A is an IFLOMG of G $= \mu_A (myx)$.
Therefore, $\mu_A(m\, yx) = \mu_A(m\, x)$, for all x and y in G.
Hence $\mu_A (m\, xy) = \mu_A (mx) = \mu_A (myx)$, for all x and y in G. Thus (i) is proved.
Now, $\nu_A (m\, xy) \leq \max \{\nu_A (mx), \nu_A (my)\}$, as A is an IFLOMG of G $= \nu_A (mx)$ ;
And $\nu_A (mx) = \nu_A ( mxyy^{-1} ) \leq \max \{\nu_A (mxy), \nu_A (my^{-1})\} \leq \max\{\nu_A (mxy),\nu_A (my)\}$, as A is an IFLOMG of G $= \nu_A(mxy)$.
Therefore, $\nu_A ( m\, xy) = \nu_A (x)$, for all x and y in G.
And, $\nu_A (m\, yx) \leq \max \{\nu_A (my),\nu_A (mx)\}$ as A is an IFLOMG of G $= \nu_A (mx)$ ; and
$\nu_A (mx) = \nu_A (my^{-1} yx ) \leq \max \{\nu_A (my^{-1}), \nu_A (m\, yx)\} \leq \max \{\nu_A (my), \nu_A (m\, yx)\}$ as A is an IFLOMG of G$= \nu_A (myx)$.
Therefore, $\nu_A ( m\, yx) = \nu_A ( mx)$, for all x and y in G.
Hence $\nu_A (m\, xy) = \nu_A (m\, x) = \nu_A (m\, yx)$, for all x and y in G. Thus (ii) is proved.

**3.12 Theorem:** Let A be an IFLOMG of a M-group G such that Im $\mu_A = \{ \alpha \}$ and Im $\nu_A = \{\beta\}$, where $\alpha$ and $\beta$ in L. If $A = B \sqcup C$, where B and C are IFLOMG of G, then either $B \subseteq C$ or $C \subseteq B$.

**Proof: Case (i) :** Let $A = B \sqcup C = \{< x , \mu_B (x) \vee \mu_C (x) , \mu_B (x) \wedge \mu_C (x) > / x \in G \}$,
$B = \{< x, \mu_B(x), \nu_B (x) >/ x \in G \}$ and $C = \{ <x, \mu_C(x), \nu_C (x)> /x \in G \}$.
Assume that $\mu_B (mx) > \mu_C (mx)$ and $\mu_B (my) < \mu_C (my)$, for some x and y in G.
Then, $\alpha = \mu_A (mx) = \mu_{B \sqcup C}(mx) = \mu_B (mx) \vee \mu_C (mx) = \mu_B (mx) > \mu_C (mx)$.
Therefore, $\alpha > \mu_C (mx)$, for all x in G.
And,$\alpha = \mu_A (my) = \mu_{B \sqcup C} (my) = \mu_B (my) \vee \mu_C(my) = \mu_C (my) > \mu_B(my)$.
Therefore, $\alpha > \mu_B (my)$, for all y in G.So that, $\mu_C (my) > \mu_C (mx)$ and $\mu_B (mx) > \mu_B (my)$.
Hence $\mu_B (mxy) = \mu_B (my)$ and $\mu_C (mxy) = \mu_C (mx)$.
But then, $\alpha = \mu_A (mxy) = \mu_{B \sqcup C} (mxy) = \mu_B(mxy) \vee \mu_C (mxy) = \mu_B (my) \vee \mu_C (mx) < \alpha$ --------------(1).
**Case (ii):** Assume that $\nu_B (mx) < \nu_C (mx)$ and $\nu_B (my) > \nu_C (my)$, for some x and y in G.
Then, $\beta = \nu_A (mx) = \nu_{B \sqcup C} (mx) = \nu_B (mx) \wedge \nu_C (mx) = \nu_B (mx) < \nu_C (mx)$.
Therefore, $\beta < \nu_C (mx)$, for all x in G.
And,$\beta = \nu_A (my) = \nu_{B \sqcup C} (my) = \nu_B (my) \wedge \nu_C (my) = \nu_C (my) < \nu_B (my)$.
Therefore, $\beta < \nu_B(my)$, for all x in G. So that, $\nu_C (my) < \nu_C (mx)$ and $\nu_B (mx) < \nu_B(my)$.
Hence $\nu_B (mxy) = \nu_B (my)$ and $\nu_C (mxy) = \nu_C(mx)$.
But then, $\beta = \nu_A (mxy) = \nu_{B \sqcup C} (mxy) = \nu_B (mxy) \wedge \nu_C (mxy) = \nu_B (my) \wedge \nu_C(mx) > \beta$ --------------(2).
It is a contradiction by (1) and (2).

Therefore, either $B \subseteq C$ or $C \subseteq B$ is true.

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**3.13 Theorem:** If A and B are IFLOMG of the M-groups G and H, respectively, then AxB is an IFLOMG of GxH.
**Proof:** Let A and B be IFLOMG of the M-groups G and H respectively. Let $x_1$ and $x_2$ be in G,
$y_1$ and $y_2$ be in H.
Then ( $x_1$, $y_1$ ) and ( $x_2$, $y_2$ ) are in GxH. Now,
i)$\mu_{AxB}$ [ m($x_1$, $y_1$) ($x_2$, $y_2$) ] = $\mu_{AxB}$ ( m$x_1x_2$, m$y_1y_2$ ) = $\mu_A$ (m$x_1x_2$) ∧ $\mu_B$ ( m$y_1y_2$)
≥ min {$\mu_A$(m$x_1$), $\mu_A$(m$x_2$)}∧ min {$\mu_B$(m$y_1$), $\mu_B$(m$y_2$)}
= min {$\mu_A$(m$x_1$) ∧ $\mu_B$(m$y_1$), $\mu_A$(m$x_2$) ∧ $\mu_B$(m$y_2$)}
= min{ $\mu_{AxB}$ [ m($x_1$, $y_1$) ], $\mu_{AxB}$ [ m($x_2$, $y_2$)]
Therefore, $\mu_{AxB}$ [ m($x_1$, $y_1$) ($x_2$, $y_2$) ] ≥ min{ $\mu_{AxB}$ [ m($x_1$, $y_1$) ], $\mu_{AxB}$ [ m($x_2$, $y_2$)]
For all $x_1$ and $x_2$ in G, $y_1$ and $y_2$ in H. Now,
For all $x_1$ and $x_2$ in G, $y_1$ and $y_2$ in H
ii) $\mu_{AxB}$ [ m($x_1$, $y_1)^{-1}$] = $\mu_{AxB}$ (m$x_1^{-1}$, m$y_1^{-1}$ ) = $\mu_A$ (m$x_1^{-1}$) ∧ $\mu_A$ ( m$y_1^{-1}$) ≥ $\mu_A$ (m$x_1$) ∧ $\mu_A$ ( m$y_1$) = $\mu_{AxB}$ [ m($x_1$, $y_1$)]
iii) $\mu_{AxB}$ [ m($x_1$, $y_1$) v m ($x_2$, $y_2$) ] = $\mu_{AxB}$ ( m$x_1$ v m$x_2$, m$y_1$ v m$y_2$ )
=$\mu_A$ ( m$x_1$ v m$x_2$) ∧ $\mu_A$ ( m$y_1$ v m$y_2$ )
≥ min {$\mu_A$(m$x_1$), $\mu_A$(m$x_2$)}∧ min {$\mu_B$(m$y_1$), $\mu_B$(m$y_2$)}
= min {$\mu_A$(m$x_1$) ∧ $\mu_B$(m$y_1$), $\mu_A$(m$x_2$) ∧ $\mu_B$(m$y_2$)}
= min{ $\mu_{AxB}$ [ m($x_1$, $y_1$) ], $\mu_{AxB}$ [ m($x_2$, $y_2$)]
iv) $\mu_{AxB}$ [ m($x_1$, $y_1$) ∧ m ($x_2$, $y_2$) ] = $\mu_{AxB}$ ( m$x_1$ ∧ m$x_2$, m$y_1$ ∧ m$y_2$ )
=$\mu_A$ ( m$x_1$ ∧ m$x_2$) ∧ $\mu_A$ ( m$y_1$ ∧ m$y_2$ )
≥ min {$\mu_A$(m$x_1$), $\mu_A$(m$x_2$)}∧ min {$\mu_B$(m$y_1$), $\mu_B$(m$y_2$)}
= min {$\mu_A$(m$x_1$) ∧ $\mu_B$(m$y_1$), $\mu_A$(m$x_2$) ∧ $\mu_B$(m$y_2$)}
= min{ $\mu_{AxB}$ [ m($x_1$, $y_1$) ], $\mu_{AxB}$ [ m($x_2$, $y_2$)]
v) $\nu_{AxB}$ [ m($x_1$, $y_1$) ($x_2$, $y_2$) ] = $\nu_{AxB}$ ( m$x_1x_2$, m$y_1y_2$ ) = $\nu_A$ (m$x_1x_2$) ∧ $\nu_B$ ( m$y_1y_2$)
≤ max {$\nu_A$(m$x_1$), $\nu_A$(m$x_2$)}v min {$\nu_B$(m$y_1$), $\nu_B$(m$y_2$)}
= max {$\nu_A$(m$x_1$) v $\nu_B$(m$y_1$), $\nu_A$(m$x_2$) v $\nu_B$(m$y_2$)}
= max{ $\nu_{AxB}$ [ m($x_1$, $y_1$) ], $\nu_{AxB}$ [ m($x_2$, $y_2$)]
Therefore, $\nu_{AxB}$ [ m($x_1$, $y_1$) ($x_2$, $y_2$) ] ≤ max{ $\nu_{AxB}$ [ m($x_1$, $y_1$) ], $\nu_{AxB}$ [ m($x_2$, $y_2$)]
For all $x_1$ and $x_2$ in G, $y_1$ and $y_2$ in H.
vi) $\nu_{AxB}$ [ m($x_1$, $y_1)^{-1}$] = $\nu_{AxB}$ (m$x_1^{-1}$, m$y_1^{-1}$ ) = $\nu_A$ (m$x_1^{-1}$) v $\nu_A$ ( m$y_1^{-1}$) ≤ $\nu_A$ (m$x_1$) v $\nu_A$ ( m$y_1$) = $\nu_{AxB}$ [ m($x_1$, $y_1$)]
vii) $\nu_{AxB}$ [ m($x_1$, $y_1$) v m ($x_2$, $y_2$) ] = $\nu_{AxB}$ ( m$x_1$ v m$x_2$, m$y_1$ v m$y_2$ )
= $\nu_A$ ( m$x_1$ v m$x_2$) v $\nu_A$ ( m$y_1$ v m$y_2$ )
≤ max {$\nu_A$(m$x_1$), $\nu_A$(m$x_2$)} v max {$\nu_B$(m$y_1$), $\nu_B$(m$y_2$)}
= max {$\nu_A$(m$x_1$) v $\mu_B$(m$y_1$), $\nu_A$(m$x_2$) v $\nu_B$(m$y_2$)}
= max{ $\nu_{AxB}$ [ m($x_1$, $y_1$) ], $\nu_{AxB}$ [ m($x_2$, $y_2$)]
viii) $\nu_{AxB}$ [ m($x_1$, $y_1$) ∧ m ($x_2$, $y_2$) ] = $\nu_{AxB}$ ( m$x_1$ ∧ m$x_2$, m$y_1$ ∧ m$y_2$ )
= $\nu_A$ ( m$x_1$ ∧ m$x_2$) v $\nu_A$ ( m$y_1$ ∧ m$y_2$ )
≤ max {$\nu_A$(m$x_1$), $\nu_A$(m$x_2$)} v max {$\nu_B$(m$y_1$), $\nu_B$(m$y_2$)}
= max {$\nu_A$(m$x_1$) v $\nu_B$(m$y_1$), $\nu_A$(m$x_2$) v $\nu_B$(m$y_2$)}
= max{ $\nu_{AxB}$ [ m($x_1$, $y_1$) ], $\nu_{AxB}$ [ m($x_2$, $y_2$)]
Therefore AxB is an IFLOMG of GxH.

**3.14 Theorem:** Let an IFLOMG A of a M-group G be conjugate to an IFLOMG M of G and an IFLOMG B of a M-group H be conjugate to an IFLOMG N of H. Then an IFLOMG AxB of a M-group GxH is conjugate to an IFLOMG MxN of GxH.
**Proof:** Let A and B be IFLOMG of the M-groups G and H respectively. Let x, $x^{-1}$ and f be in G and y, $y^{-1}$ and g be in H.
Then (x, y) ,( $x^{-1}$, $y^{-1}$ ) and (f, g) are in GxH.
Now, $\mu_{AxB}$ ( f, g ) = $\mu_A$( f) $\mu_B$( g)
= $\mu_M$(mx f m$x^{-1}$) $\mu_N$( my g m$y^{-1}$ )
= $\mu_{MxN}$ ( mx f m$x^{-1}$, my g m$y^{-1}$ )
= $\mu_{MxN}$[ (mx, my)(f, g)( m$x^{-1}$, m$y^{-1}$) ]
= $\mu_{MxN}$[ (mx, my) (f, g)( mx, my $)^{-1}$ ].
Therefore, $\mu_{AxB}$ ( f, g ) = $\mu_{MxN}$[ (mx, my) (f, g)(mx, my $)^{-1}$ ], for all x, $x^{-1}$ and f in G and y, $y^{-1}$ and g in H.
And, $\nu_{AxB}$ ( f, g ) = $\nu_A$(f) $\nu_B$(g) = $\nu_M$( mxf m$x^{-1}$) $\nu_N$( myg m$y^{-1}$ )
= $\nu_{MxN}$ ( mxf $x^{-1}$, myg m$y^{-1}$ )
= $\nu_{MxN}$ [ (mx, my)(f, g)(m$x^{-1}$, m$y^{-1}$) ]
= $\nu_{MxN}$ [ (mx, my) (f, g)(mx, my $)^{-1}$ ].
Therefore, $\nu_{AxB}$ ( f, g ) = $\nu_{MxN}$[ (mx, my) (f, g)(mx, my $)^{-1}$ ], for all x, $x^{-1}$ and f in G and y, $y^{-1}$ and g in H. Hence an IFLOMG AxB of a M-group GxH is conjugate to an IFLOMG MxN of GxH.

**3.15 Theorem:** Let A and B be IFLOMG of the M-groups G and H, respectively. Suppose that e and e' are the identity element of G and H, respectively. If AxB is an IFLOMG of GxH, then at least one of the following two statements must hold.
(i) $\mu_B$(me' ) ≥ $\mu_A$(mx) and $\nu_B$(me' ) ≤ $\nu_A$(mx), for all x in G,
(ii) $\mu_A$(me ) ≥ $\mu_B$(my) and $\nu_A$(me ) ≤ $\nu_B$(my), for all y in H.

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**Proof:** Let AxB is an intuitionistic L-fuzzy M-subgroup of GxH.
By contraposition, suppose that none of the statements (i) and (ii) holds.
Then we can find a in G and b in H such that $\mu_B$(me' ) < $\mu_A$(ma) and $\nu_B$(me' ) > $\nu_A$(ma),
$\mu_A$(me ) < $\mu_B$(mb) and $\nu_A$(me ) > $\nu_B$(mb),
We have, $\mu_{Ax\,B}$ ( ma, mb ) = $\mu_A$ (ma) ∧ $\mu_B$ (mb) > $\mu_A$ (me) ∧ $\mu_B$ (me' ) = $\mu_A$ (me, me' ).
And, $\nu_{AxB}$ ( ma, mb ) = $\nu_A$(ma) v $\nu_B$(mb) < $\nu_A$(me) v $\nu_B$(me' ) = $\nu_{AxB}$ (me, me' ).
Thus AxB is not an IFLOMG of GxH.
Hence either $\mu_B$ (me' ) ≥ $\mu_A$(mx) and $\nu_B$ (me' ) ≤ $\nu_A$(mx), for all x in G or $\mu_A$(me ) ≥ $\mu_B$(my) and $\nu_A$(me ) ≤ $\nu_B$(my), for all y in H.

**3.16 Theorem:** Let A be an IFLO subset of a M group G and V be the strongest intuitionistic L-fuzzy relation of G. Then A is an IFLOMG of G if and only if V is an IFLOMG of GxG.

**Proof:** Suppose that A is an IFLOMG of G. Then for any x = ($x_1$, $x_2$) and y = ($y_1$ , $y_2$) are in GxG.
i) $\mu_V$ ( mx $y^{-1}$ ) = $\mu_V$ [m($x_1$, $x_2$) ($y_1$, $y_2$ )$^{-1}$]
= $\mu_V$ ( m$x_1y_1^{-1}$ ,m $x_2y_2^{-1}$ )
= min {$\mu_A$(m$x_1y_1^{-1}$) , $\mu_A$ (m$x_2y_2^{-1}$)}
≥ min { min [$\mu_A$ (m$x_1$), $\mu_A$ (m$y_1$)], min [ $\mu_A$ (m$x_2$), $\mu_A$ (m$y_2$)] }
= min{ [ $\mu_A$ (m$x_1$), $\mu_A$ (m$x_2$)], min [ $\mu_A$ (m$y_1$), $\mu_A$ (m$y_2$)] }
= min { $\mu_V$ (m$x_1$, m$x_2$) , $\mu_V$ (m$y_1$, m$y_2$)
= min{$\mu_V$ (mx), $\mu_V$ (my)}
Therefore, $\mu_V$ ( mx $y^{-1}$ ) ≥ max {$\mu_V$ (mx), $\mu_V$ (my)}, for all x and y in GxG.
ii) $\nu_V$ ( mx $y^{-1}$ ) = $\nu_V$ [m($x_1$, $x_2$) ($y_1$, $y_2$ )$^{-1}$]
= $\nu_V$ ( m$x_1y_1^{-1}$ ,m $x_2y_2^{-1}$ )
= max {$\nu_A$(m$x_1y_1^{-1}$) , $\nu_A$ (m$x_2y_2^{-1}$)}
≤ max { max [$\nu_A$ (m$x_1$), $\nu_A$ (m$y_1$)],max[ $\nu_A$ (m$x_2$), $\nu_A$ (m$y_2$)] }
= max {max [ $\nu_A$ (m$x_1$), $\nu_A$ (m$x_2$)],max[ $\nu_A$ (m$y_1$), $\nu_A$ (m$y_2$)] }
= max { $\nu_V$ (m$x_1$, m$x_2$) , $\nu_V$ (m$y_1$, m$y_2$)
= max{$\nu_V$ (mx), $\nu_V$ (my)}
Therefore, $\nu_V$ ( mx $y^{-1}$ ) ≤ max {$\nu_V$ (mx), $\nu_V$ (my)}, for all x and y in GxG.
iii) $\mu_V$ ( mx v m $y^{-1}$ ) = $\mu_V$ [m($x_1$, $x_2$) v m($y_1$, $y_2$ )$^{-1}$]
= $\mu_V$ ( m$x_1$ v m$y_1^{-1}$ , m$x_2$ v m$y_2^{-1}$ )
= min {$\mu_A$(m$x_1$ v m$y_1^{-1}$) , $\mu_A$ (m$x_2$ v m$y_2^{-1}$)}
≥ min { min [$\mu_A$ (m$x_1$), $\mu_A$ (m$y_1$)], min [ $\mu_A$ (m$x_2$), $\mu_A$ (m$y_2$)] }
= min{ [ $\mu_A$ (m$x_1$), $\mu_A$ (m$x_2$)], min [ $\mu_A$ (m$y_1$), $\mu_A$ (m$y_2$)] }
= min { $\mu_V$ (m$x_1$,m$x_2$) , $\mu_V$ (m$y_1$, m$y_2$)
= min{$\mu_V$ (mx), $\mu_V$ (my)}
iv) $\mu_V$ (m x ∧ my $^{-1}$ ) = $\mu_V$ [m($x_1$, $x_2$) ∧ m($y_1$, $y_2$ )$^{-1}$]
= $\mu_V$ ( m$x_1$ ∧ m$y_1^{-1}$ ,m $x_2$ ∧ m$y_2^{-1}$ )
= min {$\mu_A$(m$x_1$ ∧ m$y_1^{-1}$) , $\mu_A$ (m$x_2$ ∧ m$y_2^{-1}$)}
≥ min { min [$\mu_A$ (m$x_1$), $\mu_A$ (m$y_1$)], min [ $\mu_A$ (m$x_2$), $\mu_A$ (m$y_2$)] }
= min{ [ $\mu_A$ (m$x_1$), $\mu_A$ (m$x_2$)], min [ $\mu_A$ (m$y_1$), $\mu_A$ (m$y_2$)] }
= min { $\mu_V$ (m$x_1$, m$x_2$) , $\mu_V$ (m$y_1$, m$y_2$)
= min{$\mu_V$ (mx), $\mu_V$ (my)}
v) $\nu_V$ ( mx v my $^{-1}$ ) = $\nu_V$ [m($x_1$, $x_2$) v m($y_1$, $y_2$ )$^{-1}$]
= $\nu_V$ (m $x_1y_1^{-1}$ ,m $x_2y_2^{-1}$ )
= max {$\nu_A$(m$x_1$ v m$y_1^{-1}$) , $\nu_A$ (m$x_2$ v m$y_2^{-1}$)}
≤ max { max [$\nu_A$ (m$x_1$), $\nu_A$ (m$y_1$)],max[ $\nu_A$ (m$x_2$), $\nu_A$ (m$y_2$)] }
= max {max [ $\nu_A$ (m$x_1$), $\nu_A$ (m$x_2$)],max[ $\nu_A$ (m$y_1$), $\nu_A$ (m$y_2$)] }
= max { $\nu_V$ (m$x_1$, m$x_2$) , $\nu_V$ (m$y_1$,m$y_2$)
= max{$\nu_V$ (mx), $\nu_V$ (my)}
vi) $\nu_V$ ( mx ∧ my $^{-1}$ ) = $\nu_V$ [m($x_1$, $x_2$) ∧ m($y_1$, $y_2$ )$^{-1}$]
= $\nu_V$ ( m$x_1$ ∧ m$y_1^{-1}$ ,m $x_2$ ∧ m$y_2^{-1}$ )
= max {$\nu_A$(m$x_1$ ∧ m$y_1^{-1}$) , $\nu_A$ (m$x_2$ ∧ m$y_2^{-1}$)}
≤ max { max [$\nu_A$ (m$x_1$), $\nu_A$ (m$y_1$)],max[ $\nu_A$ (m$x_2$), $\nu_A$ (m$y_2$)] }
= max {max [ $\nu_A$ ($x_1$), $\nu_A$ ($x_2$)],max[ $\nu_A$ ($y_1$), $\nu_A$ ($y_2$)] }
= max { $\nu_V$ ($x_1$, $x_2$) , $\nu_V$ ($y_1$, $y_2$)
= max{$\nu_V$ (x), $\nu_V$ (y)}
This proves that V is an IFLOM subgroup of GxG.
Conversely, assume that V is an IFLOMG of GxG, then for any x = ($x_1$, $x_2$) and y = ($y_1$, $y_2$) are in GxG, we have
i)min { $\mu_A$( $x_1$ $y_1^{-1}$ ) , $\mu_A$ ( $x_2$ $y_2^{-1}$ )}= $\mu_V$( $x_1$ $y_1^{-1}$ , $x_2$ $y_2^{-1}$)
= $\mu_V$ [ ( $x_1$, $x_2$ ) ( $y_1$ ,$y_2$ ) $^{-1}$ ]
= $\mu_V$ ( x$y^{-1}$) ≥ min { $\mu_V$ ( x) , $\mu_V$ ( y)}
= min {$\mu_V$ ( $x_1$, $x_2$ ) , $\mu_V$( $y_1$, $y_2$ )}
=min { min [ $\mu_A$ ( $x_1$), $\mu_A$ ( $x_2$)] , min [ $\mu_A$ ( $y_1$), $\mu_A$ ( $y_2$)]}
If we put $x_2$ = $y_2$ = 0,

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We get, $\mu_A( x_1 y_1^{-1} ) \geq \min\{ \mu_A ( x_1), \mu_A ( y_1)\}$
ii) $\min \{ \mu_A( x_1 v y_1^{-1} ) , \mu_A ( x_2 v y_2^{-1} )\}= \mu_V( x_1 v y_1^{-1} , x_2 v y_2^{-1})$
$= \mu_V [ ( x_1, x_2 ) v ( y_1 ,y_2 )^{-1} ]$
$= \mu_V ( x v y^{-1}) \geq \min \{ \mu_V ( x) , \mu_V ( y)\}$
$= \min \{\mu_V ( x_1, x_2 ) , \mu_V( y_1, y_2 )\}$
$=\min \{ \min [ \mu_A ( x_1), \mu_A ( x_2)] , \min [ \mu_A ( y_1), \mu_A ( y_2)]\}$
If we put $x_2 = y_2 = 0$,
We get, $\mu_A( x_1 v y_1^{-1} ) \geq \min\{ \mu_A ( x_1), \mu_A ( y_1)\}$
iii) $\min \{ \mu_A( x_1 \wedge y_1^{-1} ) , \mu_A ( x_2 \wedge y_2^{-1} )\}= \mu_V( x_1\wedge y_1^{-1} , x_2 \wedge y_2^{-1})$
$= \mu_V [ ( x_1, x_2 )\wedge ( y_1 ,y_2 )^{-1} ]$
$= \mu_V ( x\wedge y^{-1}) \geq \min \{ \mu_V ( x) , \mu_V ( y)\}$
$= \min \{\mu_V ( x_1, x_2 ) , \mu_V( y_1, y_2 )\}$
$= \min \{ \min [ \mu_A ( x_1), \mu_A ( x_2)] , \min [ \mu_A ( y_1), \mu_A ( y_2)]\}$
If we put $x_2 = y_2 = 0$,
We get, $\mu_A( x_1 \wedge y_1^{-1} ) \geq \min\{ \mu_A ( x_1), \mu_A ( y_1)\}$
iv) $\max \{ \nu_A( x_1 y_1^{-1} ) , \nu_A ( x_2 y_2^{-1} )\}= \nu_V( x_1 y_1^{-1} , x_2 y_2^{-1})$
$= \nu_V [ ( x_1, x_2 ) ( y_1 ,y_2 )^{-1} ]$
$= \nu_V ( xy^{-1}) \geq \max \{ \nu_V ( x) , \nu_V ( y)\}$
$= \max \{\nu_V ( x_1, x_2 ) , \nu_V( y_1, y_2 )\}$
$= \max \{ \max [ \nu_A ( x_1), \nu_A ( x_2)] , \max [ \nu_A ( y_1),\nu_A ( y_2)]\}$
If we put $x_2 = y_2 = 0$,
We get, $\nu_A( x_1 y_1^{-1} ) \geq \max\{\nu_A ( x_1), \nu_A ( y_1)\}$
v) $\max \{ \nu_A( x_1 v y_1^{-1} ) , \nu_A ( x_2 v y_2^{-1} )\}= \nu_V( x_1 v y_1^{-1} , x_2 v y_2^{-1})$
$=\nu_V [ ( x_1, x_2 )v ( y_1 ,y_2 )^{-1} ]$
$= \nu_V ( x v y^{-1}) \geq \max \{ \nu_V ( x) , \nu_V ( y)\}$
$= \max \{\nu_V ( x_1, x_2 ) , \nu_V( y_1, y_2 )\}$
$=\max \{ \max [ \nu_A ( x_1), \nu_A ( x_2)] , \max [ \nu_A ( y_1),\nu_A ( y_2)]\}$
If we put $x_2 = y_2 = 0$,
We get, $\nu_A( x_1 v y_1^{-1} ) \geq \max\{\nu_A ( x_1), \nu_A ( y_1)\}$
vi) $\max \{ \nu_A( x_1\wedge y_1^{-1} ) , \nu_A ( x_2 \wedge y_2^{-1} )\}= \nu_V( x_1\wedge y_1^{-1} , x_2\wedge y_2^{-1})$
$=\nu_V [ ( x_1, x_2 ) \wedge ( y_1 ,y_2 )^{-1} ]$
$= \nu_V ( x \wedge y^{-1}) \geq \max \{ \nu_V ( x) , \nu_V ( y)\}$
$= \max \{\nu_V ( x_1, x_2 ) , \nu_V( y_1, y_2 )\}$
$=\max \{ \max [ \nu_A ( x_1), \nu_A ( x_2)] , \max [ \nu_A ( y_1),\nu_A ( y_2)]\}$
If we put $x_2 = y_2 = 0$,
We get, $\nu_A( x_1\wedge y_1^{-1} ) \geq \max\{\nu_A ( x_1), \nu_A ( y_1)\}$
Hence A is an IFLOMG of a M-group G.

## 4 – INTUITIONISTIC L-FUZZY M SUBGROUPS OF A M-GROUP UNDER HOMOMORPHISM AND ANTIHOMOMORPHISM:

**4.1 Theorem:** Let ( G, · ) and ( G', · ) be any two M-groups. The homomorphic image (pre-image) of an IFLOMG of G is an IFLOMG of G'.

**Proof:** Let ( G, · ) and ( G', · ) be any two groups and $f : G\rightarrow G'$ be a homomorphism.
That is $f(xy) = f(x)f(y)$, $f(mx) = mf(x)$, for all x and y in G and m in M.
Let V=f(A), where A is an IFLOMG of a M-group G.
We have to prove that V is an IFLOMG of G'.
Now, for f(x) and f(y) in G',
we have
i) $\mu_V (mf(x)f(y))= \mu_V ( f(mxy) )$,(as f is a homomorphism)
$= \mu_A (mxy)$
$\geq \min \{ \mu_A (mx), \mu_A (my)\}$,as A is an ILFMSG of G
$= \min \{ \mu_V f(mx), \mu_V f(my)\}$
$= \min \{ \mu_V mf(x), \mu_V mf(y)\}$
which implies that $\mu_V ( mf(x)f(y ) ) \geq \min \{ \mu_V mf(x) ,\mu_V mf(y) \}$, for all x and y in G.
For f(x) in G', we have,
ii) $\mu_V ( [ mf(x) ]^{-1} ) = \mu_V ( [ f(mx) ]^{-1} ) = \mu_V ( f(mx^{-1}) )$, as f is a homomorphism
$= \mu_A ( mx^{-1}) \geq \mu_A ( mx) = \mu_V f(mx) = \mu_V mf(x)$
which implies that $\mu_V [m f(x) ]^{-1} \geq \mu_V mf(x)$ , for all x in G.
iii) $\nu_V (mf(x)f(y))= \nu_V ( f(mxy) )$,(as f is a homomorphism)
$=\nu_A (mxy)$
$\leq \max \{ \nu_A (mx), \nu_A (my)\}$,as A is an ILFMSG of G
$\leq \max \{ \nu_V f(mx), \nu_V f(my)\} = \max \{ \nu_V mf(x), \nu_V mf(y)\}$
which implies that $\nu_V ( mf(x)f(y ) ) \leq \max \{ \nu_V mf(x) ,\nu_V mf(y) \}$, for all x and y in G.

For f(x) in G', we have,
iv)$\nu_V$ ( $[m f(x) ]^{-1}$ ) = $\nu_V$ ( $f(mx)^{-1}$ ) = $\nu_V$ ( $f(mx^{-1})$) = $\nu_A$ $(mx^{-1}) \leq \nu_A$ (mx)
= $\nu_V$ f(mx) = $\nu_V$ mf(x)
which implies that $\nu_V$ $[mf(x)]^{-1} \leq \nu_V$ mf(x) , for all x in G.
v) $\mu_V$ (mf(x) v mf(y)) = $\mu_V$ ( f(mx) v f(my) )
= $\mu_V$ ( f(mx v my) ) (as f is a homomorphism)
= $\mu_A$ (mx v my)
$\geq$ min{ $\mu_A$ (mx), $\mu_A$ (my)},as A is an ILFMSG of G
= min { $\mu_V$ f(mx), $\mu_V$ f(my)}
= min { $\mu_V$ mf(x), $\mu_V$ mf(y)}
which implies that $\mu_V$ ( mf(x) v mf(y ) ) $\geq$ min { $\mu_V$ mf(x) ,$\mu_V$ mf(y) }, for all x and y in G.
vi) $\mu_V$ (mf(x) $\wedge$ mf(y)) = $\mu_V$ ( f(mx) $\wedge$ f(my) )
= $\mu_V$ ( f(mx $\wedge$ my) ) (as f is a homomorphism)
= $\mu_A$ (mx $\wedge$ my)
$\geq$ min{ $\mu_A$ (mx), $\mu_A$ (my)},as A is an ILFMSG of G
= min { $\mu_V$ f(mx), $\mu_V$ f(my)}
= min { $\mu_V$ mf(x), $\mu_V$ mf(y)}
which implies that $\mu_V$ ( mf(x) $\wedge$ mf(y ) ) $\geq$ min { $\mu_V$ mf(x) ,$\mu_V$ mf(y) }, for all x and y in G.
vii) $\nu_V$ (mf(x) v mf(y))= $\nu_V$ ( f(mx) v f(my) )
= $\nu_V$ ( f(mx v my) ) (as f is a homomorphism)
= $\nu_A$ (mx v my)
$\leq$ max{ $\nu_A$ (mx), $\nu_A$ (my)},as A is an ILFMSG of G
= max { $\nu_V$ f(mx), $\nu_V$ f(my)}
= max { $\nu_V$ mf(x), $\nu_V$ mf(y)}
which implies that $\nu_V$ ( mf(x) v mf(y ) ) $\leq$ max { $\nu_V$ mf(x) ,$\nu_V$ mf(y) }, for all x and y in G.
viii) $\nu_V$ (mf(x) $\wedge$ mf(y))= $\nu_V$ ( f(mx) $\wedge$ f(my) )
= $\nu_V$ ( f(mx $\wedge$ my) ) (as f is a homomorphism)
= $\nu_A$ (mx $\wedge$ my)
$\leq$ max{ $\nu_A$ (mx), $\nu_A$ (my)},as A is an ILFMSG of G
= max { $\nu_V$ f(mx), $\nu_V$ f(my)}
= max { $\nu_V$ mf(x), $\nu_V$ mf(y)}
which implies that $\nu_V$ ( mf(x) v mf(y ) ) $\leq$ max { $\nu_V$ mf(x) ,$\nu_V$ mf(y) }, for all x and y in G.
Hence V is an IFLOMG of a M-group G'.

**4.2 Theorem:** Let ( G, · ) and ( G', · ) be any two M-groups. The anti homomorphic image (pre-image) of an IFLOMG of G is an IFLOMG of G'.
**Proof:** Let ( G, · ) and ( G', · ) be any two groups and f : G→G' be an anti homomorphism.
That is f(yx) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M.
Let V=f(A), where A is an IFLOMG of a M-group G.
We have to prove that V is an IFLOMG of G'.
Now, for f(x) and f(y) in G',
we have
i)$\mu_V$ (mf(x)f(y))= $\mu_V$ ( f(myx) ),(as f is a homomorphism)
= $\mu_A$ (myx)
$\geq$ min { $\mu_A$ (my), $\mu_A$ (mx)},as A is an ILFMSG of G
= min { $\mu_A$ (mx), $\mu_A$ (my)}
= min { $\mu_V$ f(mx), $\mu_V$ f(my)}
= min { $\mu_V$ mf(x), $\mu_V$ mf(y)}
which implies that $\mu_V$ ( mf(x)f(y ) ) $\geq$ min { $\mu_V$ mf(x) ,$\mu_V$ mf(y) }, for all x and y in G.
For f(x) in G', we have,
ii)$\mu_V$ ( $[ mf(x) ]^{-1}$ ) = $\mu_V$ ( $[ f(mx) ]^{-1}$ ) = $\mu_V$ ( $f(mx^{-1})$ ), as f is a homomorphism
= $\mu_A$ ( $mx^{-1}$) $\geq \mu_A$ ( mx) = $\mu_V$ f(mx) = $\mu_V$ mf(x)
which implies that $\mu_V$ $[m f(x) ]^{-1} \geq \mu_V$ mf(x) , for all x in G.
iii)$\nu_V$ (mf(x)f(y))= $\nu_V$ ( f(myx) ),(as f is a homomorphism)
=$\nu_A$ (myx)
$\leq$ max { $\nu_A$ (my), $\nu_A$ (mx)},as A is an ILFMSG of G
= max { $\nu_A$ (mx), $\nu_A$ (my)}
= max { $\nu_V$ f(mx), $\nu_V$ f(my)} = max { $\nu_V$ mf(x), $\nu_V$ mf(y)}
which implies that $\nu_V$ ( mf(x)f(y ) ) $\leq$ max { $\nu_V$ mf(x) ,$\nu_V$ mf(y) }, for all x and y in G.
For f(x) in G', we have,
iv)$\nu_V$ ( $[m f(x) ]^{-1}$ ) = $\nu_V$ ( $f(mx)^{-1}$ ) = $\nu_V$ ( $f(mx^{-1})$) = $\nu_A$ $(mx^{-1}) \leq \nu_A$ (mx)
= $\nu_V$ f(mx) = $\nu_V$ mf(x)
which implies that $\nu_V$ $[mf(x)]^{-1} \leq \nu_V$ mf(x) , for all x in G.

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v) $\mu_V$ (mf(x) v mf(y)) = $\mu_V$ ( f(mx) v f(my) )
= $\mu_V$ ( f(my v mx) ) (as f is a homomorphism)
= $\mu_V$ ( f(mx v my) )
= $\mu_A$ (mx v my)
$\geq$ min{ $\mu_A$ (mx), $\mu_A$ (my)},as A is an ILFMSG of G
= min { $\mu_V$ f(mx), $\mu_V$ f(my)}
= min { $\mu_V$ mf(x), $\mu_V$ mf(y)}
which implies that $\mu_V$ ( mf(x) v mf(y ) ) $\geq$ min { $\mu_V$ mf(x) ,$\mu_V$ mf(y) }, for all x and y in G.
vi) $\mu_V$ (mf(x) $\wedge$ mf(y)) = $\mu_V$ ( f(mx) $\wedge$ f(my) )
= $\mu_V$ ( f(my $\wedge$ mx) ) (as f is a homomorphism)
= $\mu_V$ ( f(mx $\wedge$ my) )
= $\mu_A$ (mx $\wedge$ my)
$\geq$ min{ $\mu_A$ (mx), $\mu_A$ (my)},as A is an ILFMSG of G
= min { $\mu_V$ f(mx), $\mu_V$ f(my)}
= min { $\mu_V$ mf(x), $\mu_V$ mf(y)}
which implies that $\mu_V$ ( mf(x) $\wedge$ mf(y ) ) $\geq$ min { $\mu_V$ mf(x) ,$\mu_V$ mf(y) }, for all x and y in G.
vii) $\nu_V$ (mf(x) v mf(y))= $\nu_V$ ( f(mx) v f(my) )
= $\nu_V$ ( f(my v mx) ) (as f is a homomorphism)
= $\nu_V$ ( f(mx v my) )
= $\nu_A$ (mx v my)
$\leq$ max{ $\nu_A$ (mx), $\nu_A$ (my)},as A is an ILFMSG of G
= max { $\nu_V$ f(mx), $\nu_V$ f(my)}
= max { $\nu_V$ mf(x), $\nu_V$ mf(y)}
which implies that $\nu_V$ ( mf(x) v mf(y ) ) $\leq$ max { $\nu_V$ mf(x) ,$\nu_V$ mf(y) }, for all x and y in G.
viii) $\nu_V$ (mf(x) $\wedge$ mf(y))= $\nu_V$ ( f(mx) $\wedge$ f(my) )
= $\nu_V$ ( f(my $\wedge$ mx) ) (as f is a homomorphism)
= $\nu_V$ ( f(mx $\wedge$ my) )
= $\nu_A$ (mx $\wedge$ my)
$\leq$ max{ $\nu_A$ (mx), $\nu_A$ (my)},as A is an ILFMSG of G
= max { $\nu_V$ f(mx), $\nu_V$ f(my)}
= max { $\nu_V$ mf(x), $\nu_V$ mf(y)}
which implies that $\nu_V$ ( mf(x) v mf(y ) ) $\leq$ max { $\nu_V$ mf(x) ,$\nu_V$ mf(y) }, for all x and y in G.
Hence V is an IFLOMG of a M-group G'.

## 5. CONCLUSION

In this paper, we define a new algebraic structure of Intuitionistic fuzzy lattice ordered M-groups of M-groups and Homomorphism and anti-homomorphism of Intuitionistic fuzzy lattice ordered M-groups of M-groups.

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