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Some forms of α - generalized – δ - compactness

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Abstract: The aim of this paper is to introduce and investigate the concept of $\alpha G\delta O$-compact and $\alpha G\delta O$-connected spaces which are stronger than αGO-compact [5] and αGO-connected [5] respectively. Some properties of these spaces are studied.

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INTRODUCTION

The concept of α -open sets and generalized open sets play an important role in researches of generalizations of compactness and connectedness in topological spaces. By using these sets many authors introduced and studied various types of generalizations of compactness and connectedness. In 1991, Balachandran and et. al [3] introduced the notions of GO-compact and GO-connected. Some properties of αGO-compact and αGO-connected spaces were studied in [5]. In the present paper, we introduce the notions of $\alpha G\delta O$-compact and $\alpha G\delta O$-connected as spaces from a set X satisfying some minimal conditions into a topological space and investigate their properties and relationships between them and other related generalized forms of spaces.

PRELIMINARIES

Throughout the present paper, spaces mean topological spaces (X, τ)(or simply X) on which no separation axioms are assumed unless explicitly stated. The closure (resp. the interior, the complement) of A for a space X are denoted by cl(A) (resp. int(A) , X-A). Some definitions and results which will be needed in this paper are recalled in the following stated.

DEFINITION 2.1:
A subset A of a space X is said to be:
(i) regular open [12] if A= int(cl(A)),
(ii) α -open [11] if $A \subseteq$ int(cl(int(A))),
(iii) δ -open [13] if it is the union of regular open sets.

The complement of a regular open (resp. δ -open , α -open) set is said to be regular closed (resp. δ -closed, α -closed). The intersection of all regular closed (resp. δ -closed, α -closed) sets containing A is called the regular closure [12] (resp. δ -closure [13], α -closure [4]) of A and is denoted by r-cl(A) (resp. $cl_\delta(A)$, α -cl(A)). The family of all regular open (resp. δ -open, α -open) sets in a space (X, τ) is denoted by RO(X, τ) (resp. τ^δ , τ^α).

The union of all regular open (resp. δ -open, α -open) sets contained in A is called the regular interior [12] (resp. δ - int(A)[13], α -interior [4]) of A and is denoted by r-int(A) (resp. $int_\delta(A)$, α -int(A)). It is known that $\tau^\delta \subseteq \tau \subseteq \tau^\alpha$ and τ^α , τ^δ [11, 13] forms a topology on X.

Definition 2.2. A subset A of a space (X, τ) is said to be:
(i) a generalized closed (briefly, g-closed) [2,8] set if cl(A) $\subseteq$ U ,whenever A $\subseteq$ U and U is open,
(ii) a generalized open (briefly, g-open) [2,8] set if its complement X-A is g-closed.

Definition 2.3. A mapping f: $(X, \tau)\rightarrow(Y, \sigma)$ is called:
(i) generalized-continuous (briefly, g-cont.) [1] if $f^{-1}(V)$ is generalized-closed set in (X, τ), for each closed set V in (Y, σ).
(ii) δ-open [10] if $f(V)$ is δ-open in (Y, σ), for each open set V in (X, τ),
(iii) pre-α-closed [6] if $f(V)$ is α-closed in (Y, σ), for each α-closed set V in (X, τ).

Definition 2.4. a space (X, τ) is said to be:
(i) α-compact [9] if each α-open cover of X has a finite subcover,
(ii) GO-compact [3] if every g-open cover of X has a finite subcover,
(iii) GαO-compact [5] if every gα-open cover of X has a finite subcover,
(iv) αGO-compact [5] if every αg-open cover of X has a finite subcover,
(v) α-connected [7] if X cannot be written as a disjoint union of two non-empty α-open sets,
(vi) GO-connected [3] if X cannot be written as a disjoint union of two non-empty g-open sets,
(vii) GαO-connected [5] if X cannot be written as a disjoint union of two non-empty gα-open sets,
(viii) αGO-connected [5] if X cannot be written as a disjoint union of two non-empty αg-open sets,
(ix) α-space [11] if every α-closed set is closed.

Main Results

Definition 3.1. A subset A of a topological space (X, τ) is said to be:
(i) an α-generalized δ-closed (briefly, $\alpha g\delta$-closed) set if α-cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is δ-open,
(ii) an α-generalized δ-open (briefly, $\alpha g\delta$-open) set if its complement X-A is $\alpha g\delta$-closed,
(iii) an α-generalized δ-closed relative to B if $\alpha - cl_B(A) \subseteq B\cap G$, whenever $A \subseteq B\cap G$ and G is a δ-open set in X,
(iv) an α-generalized δ-neighbourhood (briefly, $\alpha g\delta$-nbd) of a point $x \in X$ if there exists an $\alpha g\delta$-open set U such that $x \in U \subseteq A$.

The family of all α-generalized δ-closed (resp. α-generalized δ-open) sets of a space (X, τ) will be denoted by $\alpha g\delta C(X,\tau)$ (resp. $\alpha g\delta O(X,\tau)$).

Definition 3.2. (i) The intersection of all $\alpha g\delta$-closed sets containing A is called the $\alpha g\delta$-closure of A and will be denoted by $\alpha g\delta$-cl(A),
(ii) The union of all $\alpha g\delta$-open sets contained in A is called the $\alpha g\delta$-interior of A and will be denoted by $\alpha g\delta$-int(A).

Theorem 3.1. For a subset A of A space X, then the following statements are equivalent:
(i) A is $\alpha g\delta$-closed,
(ii) For every δ-open set G containing A, there exists an α-closed set F such that

$A \subseteq F \subseteq G$.

Proof. (i)→(ii). Let A be an $\alpha g\delta$ -closed set and G be a δ-open set containing A. Then α -cl(A) $\subseteq$ G .If we put α -cl(A)=F . Hence, $A \subseteq F \subseteq G$.
(ii)→(i). Let $A \subseteq G$ and G be a δ-open set .Then by hypothesis, there exists an α -closed set F such that $A \subseteq F \subseteq G$. Therefore, α -cl (A) $\subseteq$ G. So, A is $\alpha g\delta$ -closed.

Lemma 3.1. If A and B are $\alpha g\delta$ -closed sets, then $A \cup B$ also $\alpha g\delta$ -closed set.

Proof. Let A, B be two $\alpha g\delta$ -closed sets. Then $A \subseteq F \subseteq G$ and $B \subseteq E \subseteq U$ where F, E are α -closed and G, U are δ -open sets. Hence, $A \cup B \subseteq F \cup E \subseteq G \cup U$. But, $F \cup E$ is α - closed and $G \cup U$ is δ -open sets then by Theorem 3.1. $A \cup B$ is $\alpha g\delta$ -closed .

Remark 3.1. If A and B are $\alpha g\delta$ -open sets, then $A \cap B$ is $\alpha g\delta$ -open set.

Lemma 3.2. Suppose that $A \subseteq Y \subseteq X$. Then A is an $\alpha g\delta$ -closed (resp. $\alpha g\delta$ -open) set relative to Y iff A is $\alpha g\delta$ -closed (resp. $\alpha g\delta$ -open) relative to X.

Proof. Firstly. Let $A \subseteq G$ and G be a δ-open in X. Since $A \subseteq Y$, then $A \subseteq Y \cap G$ but, A is $\alpha g\delta$ -closed set relative to Y, hence $\alpha\text{-}cl_Y(A) \subseteq Y \cap G$ this implies that $Y \cap$ α -cl(A) $\subseteq Y \cap G$. Hence, α -cl(A) $\subseteq$ G. Therefore, A is $\alpha g\delta$ -closed relative to X.
Secondly. If $A \subseteq Y \cap G$ and G is a δ-open in X, then $A \subseteq G$ but, A is $\alpha g\delta$ -closed in X hence α -cl(A) $\subseteq$ G and therefore, $Y \cap$ α -cl(A) $\subseteq Y \cap G$. So, A is $\alpha g\delta$ -closed relative to Y.

Definition 3.3. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called:
(i) α - generalized δ - continuous (briefly, $\alpha g\delta$ -cont.) if $f^{-1}(V)$ is $\alpha g\delta$ -open in (X, τ), for each open set V in (Y, σ),
(ii) α - generalized δ - irresolute (briefly, $\alpha g\delta$ -irr.) if $f^{-1}(V)$ is $\alpha g\delta$ -open in (X, τ), for each $\alpha g\delta$ -open set V in (Y, σ).

Lemma 3.3. The restriction mapping $f_A : (A, \tau_A) \to (Y, \sigma)$ of $\alpha g\delta$ -continuous mapping $f : (X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$ - continuous if $A \in \alpha g\delta C(X, \tau)$.

Proof. Let $V \subseteq Y$ be a closed set. Then $f^{-1}(V) \subseteq X$ is $\alpha g\delta$ -closed. By Remark 2.1. $f^{-1}(V) \cap A$ is $\alpha g\delta$ -closed. But $f^{-1}(V) \cap A \quad (f_A)^{-1}(V)$. Therefore, f_A is $\alpha g\delta$ -continuous.

Definition 3.4. A mapping f: $(X, \tau) \to (Y, \sigma)$ is called:
(i) $\alpha g\delta$-closed if f(V) is $\alpha g\delta$-closed in (Y, σ), for each closed set V in (X, τ),
(ii) pre-$\alpha g\delta$-closed if f(V) is $\alpha g\delta$-closed in (Y, σ), for each $\alpha g\delta$-closed set V in (X, τ),
(iii) pre-$\alpha g\delta$-open if f(V) is $\alpha g\delta$-open in (Y, σ), for each $\alpha g\delta$-open set V in (X, τ),
(iv) α - generalized δC - homeomorphism if f is bijective , α - generalized δ - irresolute and pre - $\alpha g\delta$ -open.

Definition 3.5. a space (X, τ) is said to be:
(i) $\alpha\delta T_{1/2}$ -space if every $\alpha g\delta$ -closed set is α -closed,
(ii) $\alpha\delta T_b$ -space if every $\alpha g\delta$ -closed set is closed,

(iii) $\alpha g\delta$-regular if for each closed set F of X and each point $x \in$ X-F , there exist disjoint $\alpha g\delta$-open sets U and V such that F $\subseteq$ U and $x \in$ V,

Lemma 3.4. Let (Y, σ) be an α-space. If $f:(X, \tau) \to (Y, \sigma)$ is bijective $\alpha g\delta$-cont and δ-open mapping. Then the inverse image of each $\alpha g\delta$-closed is $\alpha g\delta$-closed.

Proof. Let B be an $\alpha g\delta$-closed set in Y and suppose that $f^{-1}(B) \subseteq$ U, where U is δ-open set in X. Then f(U) is δ-open,. Then $B \subseteq f(U)$, where f is bijective. Hence, α-cl(B) $\subseteq$ f(U) and therefore, $f^{-1}(\alpha\text{-cl}(B)) \subseteq$ U. Since, α-cl(B) is closed in α-space (Y, σ) and f is $\alpha g\delta$-cont, then $\alpha\text{-cl}(f^{-1}(\alpha\text{-cl}(B))) \subseteq U$ this implies that $\alpha\text{-cl}(f^{-1}(B)) \subseteq U$ and hence, $f^{-1}(B)$ is $\alpha g\delta$-closed set in X.

Theorem 3.2. Let X be a space. Then the following are equivalent:
(i) X is $\alpha g\delta$-regular,
(ii) For each set F $\subseteq$ X and p $\in$ X-F there exists an $\alpha g\delta$-open set U such that
p $\in$ U $\subseteq$ $\alpha g\delta$-cl(U) $\subseteq$ X-F.

Proof. (i)→(ii). Let X be an $\alpha g\delta$-regular space, F $\subseteq$ X and p $\in$ X- F. Then there exists disjoint $\alpha g\delta$-open sets U and V such that p $\in$ U and F $\subseteq$ V = X - $\alpha g\delta$-cl(U). This implies that $\alpha g\delta$- cl(U) $\subseteq$ X-F and hence, p $\in$ U $\subseteq$ $\alpha g\delta$- cl(U) $\subseteq$ X-F.

(ii)→(i). Let p $\in$ X and F $\subseteq$ X -{p} be a closed set such that p $\in$ U $\subseteq$ $\alpha g\delta$- cl(U) $\subseteq$ X-F. Then F $\subseteq$ X - $\alpha g\delta$-cl(U) which is an $\alpha g\delta$-open set and U $\cap$ (X - $\alpha g\delta$- cl(U)) = φ . Hence, (X, τ) is $\alpha g\delta$-regular space.

4. $\alpha G\delta O$ - compact spaces.

In this article we introduced the concepts of $\alpha G\delta O$-compact space and we studied the connection between it and other compactness. Further, we presented some properties on this space.

Definition 4.1. (i) Let B be a subset of X. A collection $\{A_i : i \in I\}$ of $\alpha g\delta$-open sets in a topological space (X, τ) is called an $\alpha g\delta$-open cover of B if $B \subset \cup\{A_i : i \in I\}$ holds.
(ii) A topological space (X, τ) is said to be $\alpha G\delta O$-compact, if every $\alpha g\delta$-open cover of X has a finite sub cover.
(iii) A subset B of (X, τ) is said to be $\alpha G\delta O$-compact relative to (X, τ) if for every cover of B by $\alpha g\delta$-open sets of (X, τ) has a finite sub cover.

The following observation studied the relation between this notion and the other of them.

Observation 4.1. For a space (X, τ) , every $\alpha G\delta O$-compact space is αGO-compact.

Proof. (i) Let $\{G_i : i \in I\}$ be an αg-open cover of X. Then $\{G_i : i \in I\}$ is an $\alpha g\delta$-open cover of X. Since, X is $\alpha G\delta O$-compact, then there exist a finite subcover $\{G_{i_1}, G_{i_2}, \ldots, G_{i_n}\}$ of X such that $X \subset \underset{K\ 1}{\overset{n}{\cup}} G_{i_K}$. Hence, X is αg-compact.

We show that the relations between $\alpha G\delta O$-compactness and other types of compactness in this diagram.

GO-compact → compact
↑
$\alpha G\delta O$-compact → α GO-compact ↑
↓
Gα-compact→ α-compact

Remark 4.1. α -compact space (X, τ) is $\alpha G\delta O$-compact if it is $\alpha\delta T_{1/2}$.

Theorem 4.1. For a space (X, τ), then the following statements are holds.
(i) an $\alpha g\delta$ -closed subset of $\alpha G\delta O$-compact space is $\alpha G\delta O$-compact relative to (X, τ),
(ii) A closed set of X is $\alpha G\delta O$-compact if and only if it is $\alpha G\delta O$-compact relative to (X, τ).

Proof. (i) Let A be an $\alpha g\delta$ -closed subset of $\alpha G\delta O$-compact space X. Then X-A is $\alpha g\delta$ -open in X. Let M= $\{G_i : i \in I\}$ be a cover of A by $\alpha g\delta$ -open set in X. Then {M, X-A} is $\alpha g\delta$ -open cover of X. Since, X is $\alpha G\delta O$-compact, then it has a finite subcover, say $\{G_1, G_2, \ldots, G_n\}$ and so, A is $\alpha G\delta O$-compact relative to (X, τ).
(ii) Obvious.

Theorem 4.2. Let A and B be two subsets of a space (X, τ) such that $A \subset B \subset X$ and B be an $\alpha g\delta$ -open set, then A is $\alpha G\delta O$-compact relative to the subspace B if and only if A is $\alpha G\delta O$-compact relative to (X, τ).

Proof. Firstly. Let $\{G_i : i \in I\}$ be a cover of A by $G_i \in \alpha g\delta O(X,\tau)$. Then by Remark 3.1 and B is $\alpha g\delta$ -open. Then $B \cap G_i \in \alpha g\delta O(X,\tau)$ and by Lemma 3.2, $B \cap G_i \in \alpha g\delta O(B, \tau_B)$,for each $i \in I$ which implies that $A \subset \cup\{B \cap G_i : i \in I\}$. Since, A is $\alpha G\delta O$-compact relative to B, then there exists a finite subset I_0 of I such that $A \subset \cup\{B \cap G_i : i \in I_0\}$. Hence, $A \subset \cup\{G_i : i \in I_0\}$ and therefore, A is $\alpha G\delta O$-compact relative to (X, τ). Secondly . Let $\{G_i : i \in I\}$ be a cover of A by $G_i \in \alpha g\delta O(B, \tau_B)$, then by Lemma 3.2. $G_i \in \alpha g\delta O(X,\tau)$, for each $i \in I$. Since, A is $\alpha G\delta O$-compact relative to (X, τ) then there exists a finite subset I_0 of I such that $A \subset \cup\{G_i : i \in I_0\}$ and therefore, $A \subset \cup\{B \cap G_i : i \in I_0\}$ which implies that A is $\alpha G\delta O$-compact relative to B.

Theorem 4.3. Let A and B be two subsets of a space (X, τ) .If A is $\alpha G\delta O$-compact relative to X and B is an $\alpha g\delta$ -closed set in (X, τ) , then $A \cap B$ is $\alpha G\delta O$-compact relative to X .

Proof. Let $\{G_i : i \in I\}$ be a cover of $A \cap B$ by $G_i \in \alpha g\delta O(X,\tau)$. Since, $(X - B) \in \alpha g\delta O(X, \tau)$, then $(X - B) \cup \{G_i : i \in I\}$ is cover of A which is $\alpha g\delta$ -open. But A is $\alpha G\delta O$-compact relative to X, then there exists a finite subset I_0 of I such that $A \subseteq \cup\{G_i : i \in I_0\} \cup (X - B)$. Hence, $A \cap B \subseteq \cup\{G_i : i \in I_0\}$ and so, $A \cap B$ is $\alpha G\delta O$-compact relative to X.

Theorem 4.4. Let $\{A_j : j \in J\}$ be a finite family of $\alpha G\delta O$-compact subsets relative to a space (X, τ), then $\cup\{A_j : j \in J\}$ is $\alpha G\delta O$-compact relative to X.

Proof. Let $\{G_i : i \in I\}$ be a cover of $\cup\{A_j : j \in J\}$ by $G_i \in \alpha g\delta O(X,\tau)$.Then $\{G_i : i \in I\}$ is a cover of A_j, for each $j \in J$ and hence , for each $j \in J$, there exists a finite subset I_j of I such that $A_j \subset \cup\{A_i : i \in I_j\}$. Therefore, $\cup\{A_j : j \in J\} \subset \{G_i : i \in I_0\}$, where $I_0 \quad \cup\{I_j : j \in J\}$ this shows that $\cup\{A_j : j \in J\}$ is $\alpha G\delta O$-compact relative to X .

Proposition 4.1. For a space (X, τ), Then:
(i)The image of $\alpha G\delta O$-compact space is compact under surjective g-continuous mapping,
(ii) If a mapping $f : (X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$ -irresolute and a subset B of X is $\alpha G\delta O$-compact relative to X, then the image f(B) is $\alpha G\delta O$-compact relative to Y.,

(iii) If a bijection mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\alpha g\delta$-cont and δ-open and a subset B of X is $\alpha G\delta O$-compact relative to (X,τ), then the image f(B) is $\alpha G\delta O$-compact relative to (Y,σ) if (Y,σ) be an α-space.

Proof. (i) Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be g-continuous mapping from an $\alpha G\delta O$-compact space X on to a topological space Y. Let $\{G_i : i\in I\}$ be an open cover of Y. Then $\{f^{-1}(G_i) : i\in I\}$ is g-open cover of X and then $\{f^{-1}(G_i) : i\in I\}$ is $\alpha g\delta$-open cover of X. Since, X is $\alpha G\delta O$-compact, then there exists a finite sub cover $\{f^{-1}(G_1),..., f^{-1}(G_n)\}$. But f is surjective, then $\{G_1, G_2, \ldots, G_n\}$ is an open cover of Y and so, Y is compact.
(ii) clear and (iii) Obvious from Lemma 3.4.

Theorem 4.5. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a pre-$\alpha g\delta$-open mapping and Y be $\alpha G\delta O$- compact, then X is $\alpha G\delta O$-compact.

Proof. Let $\{G_i : i\in I\}$ be a cover of X by $G_i \in \alpha g\delta O(X,\tau)$, for each $i\in I$. Since, f is pre-$\alpha g\delta$-open, then $\{f(G_i) : i\in I\}$ is a cover of Y by $f(G_i)\in \alpha g\delta O(Y,\sigma)$, for each $i\in I$. Since, Y is $\alpha G\delta O$-compact, then there exists a finite subset I_0 of I such that $Y \quad \cup\{f(G_i) : i\in I_0\}$. Then $X \quad \cup\{G_i : i\in I_0\}$ and therefore, X is $\alpha G\delta O$-compact.

Theorem 4.6. $\alpha G\delta O$-compactness is a topological property.

Theorem 4.7. If (X,τ) is $\alpha g\delta$-regular and if A is $\alpha G\delta O$-compact, then A is $\alpha g\delta$-closed.

Proof. Suppose that $A\subseteq U$ and U be δ-open. Then by Thorem 3.2. there exists an $\alpha g\delta$-open set V such that $A\subseteq V\subseteq \alpha g\delta$-cl(V)$\subseteq$U and it follows that α-cl(A)$\subseteq$U.

Theorem 4.8. Let K be an $\alpha G\delta O$-compact set of an $\alpha g\delta$-regular space X and U be nbd. of K. Then there exists an $\alpha g\delta$-closed nbd V of X which contained in U.

Proof. Let U be an open nbd of K. Then X-U is closed nbd of K. Also, let $p\in K$, since, X is $\alpha g\delta$-regular, $p\notin$ X-U then there exists an $\alpha g\delta$-open nbd V_P such that $p\in V_P \subseteq \alpha g\delta$-cl($V_P$)$\subseteq$U. So, U = { V_P : $p \in K$ } is an $\alpha g\delta$-open cover of K , then there exists a finite sub cover $\{V_{P_1}, V_{P_2},\ldots, V_{P_m}\}$ such that

$$K \subseteq \bigcup_{i=1}^{m} V_{P_i} \subseteq \bigcup_{i=1}^{m} \alpha g\delta\text{-}cl(V_{P_i}) \subseteq U.$$ Hence, $K\subseteq V\subseteq U$.

Theorem 4.9. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be bijective, $\alpha g\delta$-irresolute and X be an $\alpha G\delta O$-compact space, Y be an $\alpha g\delta$-regular space. Then f is $\alpha g\delta$ C-homeomorphism.

Proof. Let $A\subseteq X$ be an $\alpha g\delta$-open .Then by Theorem 4.1,(i) (X-A) is $\alpha G\delta O$-compact and therefore f(X-A) is $\alpha g\delta$-compact by using proposition 4.1,(ii). Hence, f(X-A) is $\alpha g\delta$-closed by Theorem 4.7. Then f(A) $\alpha g\delta$-open and therefore, f is pre-$\alpha g\delta$-pen Hence, f is $\alpha g\delta$ C- homeomorphism.

Definition 4.2. A family U of subsets of X is said to be a closed under finite intersections if for every A ,B $\in$ U ,A $\cap$ B $\in$ U hold.

Example 4.1. The family of $\alpha g\delta\, O(X,\tau)$ is closed under finite intersections.

Theorem 4.10. For a space (X,τ), then the following statements are equivalent:
(i) X is $\alpha G\delta O$-compact,
(ii) Any family of $\alpha g\delta$-closed subsets of (X,τ) with empty intersection has a finite

subfamily with empty intersection.
(iii) Any family of $\alpha g\delta$ -closed subsets of (X, τ) satisfying the finite intersection Property has a non-empty intersection.

Proof. (i)→(ii). Let $\{F_i : i \in I\}$ be a family of $\alpha g\delta$ -closed subsets of (X, τ) with $\bigcap_{i \; 1} F_i \;\; \varphi$. By DeMorgan's Low then $X - (\bigcap_{i \; 1} F_i) \;\; X$ this implies that $\bigcup_{i \; 1} (X - F_i) \;\; X$. So, the collection $\{(X - F_i) : i \in I\}$ is an $\alpha g\delta$ -open cover of X. Since, X is $\alpha G\delta O$-compact, then there exists a finite sub cover $(X - F_{i_1}), \ldots, (X - F_{i_m})$ of X such that $X \;\; (X - F_{i_1}) \cup \ldots \cup (X - F_{i_m}) \;\; \bigcup_{K \; 1}^{m} (X - F_{i_K})$, by DeMorgan's Low. $\bigcap_{K \; 1}^{m} F_{i_k} \;\; \varphi$.

(ii) → (i). Let $\{F_i : i \in I\}$ be a collection of $\alpha g\delta$ -open cover of (X, τ). Then $X \;\; \bigcup_{i \; 1} F_i$ By DeMorgan's Low $\bigcap_{i \; 1} (X - F_i) \;\; \varphi$. So, there exists a collection of an $\alpha g\delta$ -closed subsets $\{X - F_i : i \in I\}$ of X such that $\bigcap_{i \; 1} (X - F_i) \;\; \varphi$. Then by hypothesis, there exists a finite sub collection $(X - F_{i_1}), \ldots, (X - F_{i_m})$ such that $\bigcap_{K \; 1}^{m} (X - F_{i_k}) \;\; \varphi$

by DeMorgan. $\bigcap_{K \; 1}^{m} F_{i_k} \;\; X$. Hence, X is $\alpha G\delta O$-compact

(i) → (iii), (iii) → (i). Obvious from equivalently (ii) and (iii).

Corollary 4.1. Let X be an $\alpha G\delta O$-compact space and G be a family of subsets of X satisfying the finite intersection property. Then $\bigcap \{\alpha g\delta - cl(G_i) : G_i \in G\} \neq \varphi$.

Proof. Since, G satisfying the finite intersection property, then there exists a finite subfamily $\{G_{i_K} : K \;\; 1, 2, \ldots, m\}$ such that $\bigcap_{K \; 1}^{m} G_{i_K} \neq \varphi$ this implies that $\bigcap_{K \; 1}^{m} \alpha g\delta - cl(G_{i_K}) \neq \varphi$. Therefore, $\alpha g\delta$ -cl(G) satisfying the finite intersection property.

$\alpha G\delta O$ - connected spaces.

We define new types of spaces called $\alpha G\delta O$-connected space and we introduced the connection between it and other spaces. Some characterizations of this spaces are study.

Definition 5.1. (i) A topological space (X, τ) is called $\alpha G\delta O$-connected , if X cannot be written as a disjoint union of two non-empty $\alpha g\delta$ -open sets,
(ii) A subset Y of a space (X, τ) is $\alpha G\delta O$-connected if Y is $\alpha G\delta O$-connected as a subspace of X.

Observation 5.1. For a space (X, τ) every $\alpha G\delta O$-connected space is α GO-connected.

Proof. Let A, B be two α g-open subsets of X such that $X \;\; A \cup B$. Then A, B are $\alpha g\delta$ -open. But X is $\alpha G\delta O$-connected, then $X \neq A \cup B$ this implies that X is αGO-connected.

The converses of above lemma is not true this is shown by [5] and the following example.

Example 5.1. Let X= {a, b, c} with the topologies τ = {X, φ, {a, b}}. Then a space X is αGO-connected but not $\alpha G\delta O$-connected.

The following diagram shows how $\alpha G\delta O$-connected spaces are related to some similar types of connectedness.

GO-connected → connected
↑
$\alpha G\delta O$-connected → α GO-connected ↑
↓
G α -connected → α -connected

Remark 5.1. (i) An α -connected space X is $\alpha G\delta O$-connected if X is $\alpha\delta T_{1/2}$-space.
(ii) A connected space X is $\alpha G\delta O$-connected if X is $\alpha\delta T_b$ -space.

Theorem 5.1. For a space (X, τ), then the following statements are equivalent:
(i) X is $\alpha G\delta O$-connected,
(ii) The only subsets of X which are both $\alpha g\delta$ -open and $\alpha g\delta$ -closed are the empty sets φ and X.

Proof. (i)→(ii). Let G be an $\alpha g\delta$ -open and $\alpha g\delta$ -closed subset of X . Then X-G is both $\alpha g\delta$ -open and $\alpha g\delta$ -closed. Since, X is the union of disjoint $\alpha g\delta$ -open sets G and X-G on these must be empty , that is G = φ or G =X .
(ii)→(i). Suppose that $X \quad C \cup D$, where C and D are disjoint non empty $\alpha g\delta$ -open subsets of X, then C is both $\alpha g\delta$ -open and $\alpha g\delta$ -closed. By hypothesis C = φ or C =X. Therefore, X is $\alpha G\delta O$-connected.

Theorem 5.2. Let $f:(X, \tau) \to (Y, \sigma)$ be a surjective mapping. Then the following are hold.
(i) If, $f:(X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$ -continuous mapping and X is $\alpha G\delta O$-connected, then Y is connected,
(ii) If, $f:(X, \tau) \to (Y, \sigma)$ is $\alpha g\delta$ -irresolute mapping and X be $\alpha G\delta O$-connected, Then Y is $\alpha G\delta O$-connected.

Proof. (i) Assume that Y is not connected and let $Y \quad C \cup D$,where C and D are disjoints non-empty open subsets in Y. Since, f is $\alpha g\delta$ -continuous and surjective, then $X \quad f^{-1}(C) \cup f^{-1}(D)$, where, $f^{-1}(C)$ and $f^{-1}(D)$ are disjoints non-empty and $\alpha g\delta$ - open in X which is contradiction with X is $\alpha G\delta O$-connected. Hence, Y is connected.
(ii) Obvious.

Theorem 5.3. Let $f:(X, \tau) \to (Y, \sigma)$ be an $\alpha g\delta$ -continuous map. Then f(A) is connected subset of Y , for every $\alpha g\delta$ -closed and $\alpha G\delta O$-connected subset A of X.

Proof. The restriction f/A of f to A is $\alpha g\delta$ -continuous by Lemma 3.3. and Theorem 5.2(i). The image of the $\alpha G\delta O$-connected space (A, τ_A) under $f/A : (A, \tau_A) \to (f(A), \sigma_{f(A)})$ is connected. Thus $(f(A), \sigma_{f(A)})$ is connected. Therefore f(A) is a connected subset of Y.

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