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Hydro Magnetic Effects on the Flow of Couple Stress Fluid through an Extended Porous Channel

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Abstract:-This work examine hall and ion-slip effects on the steady state flow of an incompressible and electrically conducting couple stress fluid through an extended porous channel with walls suction and injection. Solutions have been made numerically for the constitutive equations governing the steady state flow of an incompressible and electrically conducting couple stress fluid through a porous channel under the influence of uniform applied magnetic field assuming uniform suction at the upper plate and uniform injection at the lower plate by considering current distribution across the porous channel and the corresponding effect. The obtained magneto hydro dynamic (*MHD*) momentum equation governing the flow is non-linear, coupled system of higher order differential equation and solved numerically using the so called Quasi-Linearization (Q-L) technique. Furthermore variations of shearing stress and velocity profiles around the extended porous channel with respect to couple stress fluid parameter, hall parameter; ion-slip parameter, suction Reynolds and Hartmann numbers have all been well calculated. Eventually, the variations and relationships of these dimensionless parameters on the magneto hydro dynamic (*MHD*) flow have been depicted graphically. For computational verification of relationships of parameters we used visual FORTRAN version 95.

Key words: - Steady Hydro Magnetic Flow, Magneto-Couple Stress Fluid, Quasi-Linearization, Suction, Injection, Hall and Ion-slip Effects.

1. Introduction

In recent times almost of all studies concerning *MHD* flows of an electrically conducting fluid M.D.Raisinghania in 2003 [6] through porous channel(s) the presence of magnetic field is of importance in various areas of fields of technology and engineering such as *MHD* power generation, *MHD*-flow meters, *MHD*-micro pumps, various *MHD* working systems those are used on reducing the rate of aerodynamic heat transfer, different magnetic field controlling units of modern mechanical working machineries and so on. This model is chosen because of its relative mathematical simplicity when compared with other models developed for couple stress fluid flow problems and currently it is widely used one. Therefore the study of such most attractive phenomena is being timely and hence very important for technological improvements.

On the other hand in the development of category of non-Newtonian fluids, the couple stress fluid as an electrically conducting special non-Newtonian fluid has been highly attracted most scientists, mathematicians, and engineers for the last few years. The couple stress fluid theory developed by V.K.Stokes in 1966 [12] represents the simplest generalization of the classical viscous fluid theory that sustains couple stresses and the body couples. The important feature of these fluids is that the stress tensor is not symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. The fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, such as blood, lubricants containing small amount of polymer additive, electro-rheological fluids and synthetic fluids are examples of these fluids. The flow of a couple stress fluid between two parallel horizontal stationary plates due to fluid injection through the lower porous plate is considered by Kabadi.A in 1987 [5]. Recently, D.Srinivasacharya and S.Mekonnen in 2008 [1] discussed hall and ion-slip effects on the flow of a micro polar fluid between parallel Plates. Ke-Qin Zhua and Yong-Li Chen in 2008 [14] analyzed Couette-Poiseuille Flow of Bingham Fluids between Two Porous Parallel Plates with Slip Conditions. Again Darhasayanam Srinivasacharya and S.Mekonnen in 2009 [4] examined hydro magnetic effects on the flow of a micropolar fluid in a diverging channel. R.N.Jat and Santosh Chaudhary in 2010 [9] studied the flow and heat transfer for an electrically conducting fluid past a continuously moving plate with variable surface temperature in the presence of a uniform transverse magnetic field. D.Srinivasacharya and K.Kaladhar in 2012 [3] studied hall and ion-slip effects on electrically conducting couple stress fluid flow between two circular cylinders in the presence of a temperature dependent heat source and the homotopy analysis method is employed to solve the non linear governing flow problem. A significant work by Darbhasayanam Srinivasacharya, N.Srinivasacharyulu and Odelu Ojjela in 2012 [2] studied the steady state flow of incompressible couple stress fluid flow between parallel porous plates maintained at constant but different temperatures with the assumption that there is a constant suction at upper plate and a constant injection at the lower plate. Odelu Ojjela in 2012 [7] considered the flow of a viscous fluid between two parallel porous plates subjected to the periodic oscillation without neglecting non linear terms. Furthermore M.Veera Krishna, Syamala Sarojini and Shankar.C.Uma in 2012 [13] studied analytical study of unsteady magneto hydro dynamic flow of a couple stress fluid through a porous medium between parallel plates under the influence of pulsation pressure gradient. However, many of the problems outlined above dealt with flow and heat transfer of couple stress fluid between porous plates with expanding and contracting walls. It is also seen unsteady flow through channels by considering rotation on couple stress fluid, and influence of traverse magnetic field, *MHD* flows of other type of conducting non-Newtonian fluids. The aim of this paper is to examine the steady state hydromagnetic flow of an incompressible and electrically conducting couple stress fluid through a porous channel under the influence of uniform applied magnetic field assuming uniform suction at the upper plate and uniform injection at the lower plate with the consideration of hydro magnetic effects of Hartman number, hall and ion-slip parameters on the flow.

In general an attempt has been made to demonstrate the flow of a steady, electrically conducting and incompressible couple stress fluid through parallel porous plates (porous channel) under the influence of applied uniform magnetic field suited perpendicular to the walls of the porous plates. Furthermore, the flow analysis has been developed for high values of Reynolds number ($\mathrm{Re} = \rho U_0 h \mu^{-1}$) and for relatively low values of Hartmann number ($Ha = B_0 h\sqrt{\sigma/\mu}$) or magnetic Reynolds number. Solutions can be examined numerically for the constitutive equations under the influence of applied uniform magnetic field situated perpendicular to the plane of flow, current distribution across the porous channel and the corresponding effect. The hydro magnetic flow governing momentum equation in the presence of applied uniform magnetic field ignoring gravitational field effects, considering hall and ion-slip current effects is coupled, nonlinear higher order differential equation which cannot be analytically solved and can be solved numerically. Thus we engaged here to apply a special numerical method called Quasi-Linearization. Also an attempt has been made to examine the magnetic field effects and effects of different parameters, such as; material constant responsible for the couple stress fluid property, suction Reynolds and Hartmann numbers along with hall and ion-slip parameters on the *MHD* flow of electrically conducting couple stress fluid through a porous channel with walls suction and injection.

Finally, to describe the general fluid dynamical aspects of *MHD*, suppose that the fluid is incompressible and electrically conducting and is in the presence of an arbitrary magnetic field. The magnetic field then interacts with the fluid by means of body force and body couple per unit mass. If gravitational effects are not present, then a regular magneto-fluid dynamics assumption is $\rho \bar{f} = \rho_e E + \bar{J} \times \bar{B}$, where ρ_e is the free charge density. Since, the electric force density $\rho_e E$ is smaller than the Lorentz force or the electromagnetic force term $\bar{J} \times \bar{B}$ i.e. $\rho_e E \ll \bar{J} \times \bar{B}$ so that it can be neglected [6]. Hence, the fluid dynamical aspects of magneto hydro dynamic or hydro magnetic flows are handled by adding an electromagnetic force term to the momentum equation of the fluid. In other words, the fluid dynamical aspects of hydro magnetic flows are handled by adding the electromagnetic force term $\bar{J} \times \bar{B}$ to the non-hydro magnetic flow governing momentum equation.

Next we use the significant consequences of fluid dynamical aspects of *MHD* for the formulation of our problem, by considering the effects of hall and ion-slip parameters on the *MHD* flow with suction and injection.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

To investigate hydromagnetic effects on the flow of couple stress fluid through a porous channel with walls suction and injection. For this consider a steady state, incompressible and electrically conducting couple stress fluid flow through a horizontal extended porous channel at distance '*h*'. Choose the Cartesian coordinate system such that the origin is at the middle of the plates, y-axis is perpendicular to the plates, x – axis is in the flow direction and the two planes are infinitely extended in the x and z directions. A constant pressure gradient is applied along the direction of the x-axis and the flow is subjected to a uniform magnetic field (B_0) perpendicular to the flow direction as shown in Fig.1 below.

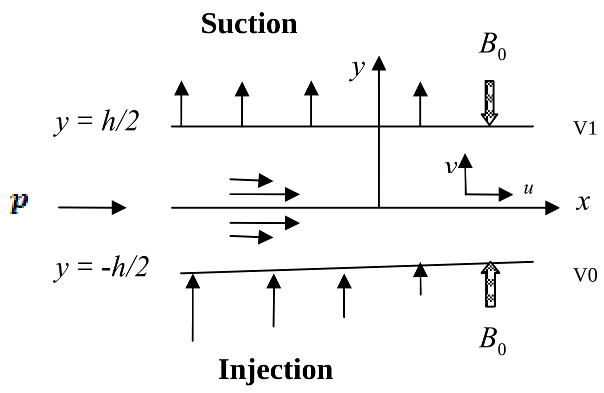

Fig.1 Formulation of the problem.

The velocity and the magnetic field vectors are respectively given in the form as:

$$\vec{q} = u(x, y)\hat{i} + v(x, y)\hat{j}\text{, and } \vec{B} = B_0\, \hat{j}. \tag{1}$$

Where $\vec{B}$ the magnetic field intensity.

The governing equation of *MHD* incompressible, electrically conducting couple stress fluid flow (when body force and body moments are absent) is given in the form:

$$div\,\vec{q} = 0 \tag{2}$$

$$\rho\left(\frac{d\vec{q}}{dt}\right) = \rho\left(\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}.\vec{\nabla}\right)\vec{q}\right)$$

$$= -gradp - \mu curlcurl\,\vec{q} - \eta curlcurlcurlcurl\,\vec{q} + \vec{J}\times\vec{B}. \tag{3}$$

The Maxwell's equations those are given below in order to relate the intensity of electric field $\vec{E}$, intensity of magnetic field $\vec{B}$, current density $\vec{J}$, and velocity of the fluid $\vec{q}$ ignoring Hall and ion-slip effects as:

$$\vec{\nabla}\times\vec{E} = 0\text{, (as } \frac{\partial \vec{B}}{\partial t} = 0 \text{ for steady motion)}$$

$$\vec{\nabla}\times\vec{B} = 0,\ \vec{\nabla}\times\vec{B} = \mu_e\vec{J},\ \vec{\nabla}\times\vec{J} = 0\text{, and}$$

$$\vec{J} = \sigma(\vec{q}\times\vec{B}) \quad \text{(Assuming } \vec{E} = 0\text{)} \tag{4}$$

where μ_e is the magnetic permeability and σ is the electrical conductivity of the fluid.

Since the fluid moves steadily (i.e. $\frac{\partial \vec{q}}{\partial t} = 0$), and $u = u(y)$, and $v = v(x)$ may be verified using equation (2). Then

$$\vec{q}.\vec{\nabla} = \left(u\hat{i} + v\hat{j}\right).\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}\right) = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \tag{5}$$

$$\left(\vec{q}.\vec{\nabla}\right)\vec{q} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(u\hat{i} + v\hat{j}\right)$$

$$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\hat{i} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\hat{j}. \tag{6}$$

$$grad p = \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j}. \qquad (7)$$

Also we have:

$$-\mu curl curl \vec{q} = \mu\left\{\frac{\partial^2 u}{\partial y^2}\hat{i} + \frac{\partial^2 v}{\partial x^2}\hat{j}\right\}, \text{ and}$$

$$-\eta curl curl curl curl \vec{q} = -\eta\left\{\frac{\partial^4 u}{\partial y^4}\hat{i} + \frac{\partial^4 v}{\partial x^4}\hat{j}\right\} \qquad (8)$$

Substitute the results of equations (6) through (8) in to (3), then we get

$$\rho\left[\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\hat{i} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)\hat{j}\right]$$
$$= -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j}\right) + \mu\left\{\frac{\partial^2 u}{\partial y^2}\hat{i} + \frac{\partial^2 v}{\partial x^2}\hat{j}\right\} - \eta\left\{\frac{\partial^4 u}{\partial y^4}\hat{i} + \frac{\partial^4 v}{\partial x^4}\hat{j}\right\} + \vec{J}\times\vec{B}.$$

It follows, by comparing coefficients of unit vectors in the respective directions of Cartesian coordinates that:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4} + \vec{J}\times\vec{B} \text{ (i}^{th}\text{ -component)} \qquad (9)$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\frac{\partial^2 v}{\partial x^2} - \eta\frac{\partial^4 v}{\partial x^4} + \vec{J}\times\vec{B} \text{ (j}^{th}\text{ component).} \qquad (10)$$

Now the current density which retains Hall current and ion-slip current terms is given by:

$$\vec{J} = \sigma\left\{\vec{q}\times\vec{B} - \beta\left(\vec{J}\times\vec{B}\right) + \frac{\beta B_i}{B_0}\left(\vec{J}\times\vec{B}\right)\times\vec{B}\right\}. \qquad (11)$$

Here $\vec{E} = 0$, by assumption that the induced magnetic field is very small.

Where, $\vec{J}$ is the current density, σ is the electrical conductivity, β is the Hall factor, and B_i is the ion-slip parameter.

Now

$$\vec{q}\times\vec{B} = \left(u\hat{i} + v\hat{j}\right)\times B_0\hat{j} = uB_0\hat{k}. \qquad (12)$$

Let $\vec{J} = J_1\hat{i} + J_2\hat{j} + J_3\hat{k}$, then

$$\vec{J}\times\vec{B} = \left(J_1\hat{i} + J_2\hat{j} + J_3\hat{k}\right)\times B_0\hat{j}$$
$$= -B_0J_3\hat{i} + B_0J_1\hat{k}. \qquad (13)$$

Again using (13), we have

$$\left(\vec{J}\times\vec{B}\right)\times\vec{B} = -B_0^{\ 2}J_1\hat{i} - B_0^{\ 2}J_3\hat{k}. \qquad (14)$$

Using the values of equations (12) through (14) obtained above in equation (11), we have

$$J_1\hat{i} + J_2\hat{j} + J_3\hat{k} = \sigma\left\{uB_0\hat{k} + \beta B_0J_3\hat{i} - \beta B_0J_1\hat{k} - \beta B_iB_0J_1\hat{i} - \beta B_iB_0J_3\hat{k}\right\}.$$
$$= (\sigma\beta B_0J_3 - \sigma\beta B_iB_0J_1)\hat{i} + (\sigma B_0u - \sigma\beta B_0J_1 - \sigma\beta B_iB_0J_3)\hat{k}. \qquad (15)$$

Comparing the coefficients of the unit vectors of (15) in x-, y- and z- directions respectively and we obtain

$$J_2 = 0, \qquad (16a)$$
$$J_1 = \sigma\beta B_0J_3 - \sigma\beta B_iB_0J_1, \text{ and} \qquad (16b)$$
$$J_3 = \sigma B_0u - \sigma\beta B_0J_1 - \sigma\beta B_iB_0J_3. \qquad (16c)$$

From equation (16b) we get

$$J_3 = \frac{(1+\sigma\beta B_iB_0)J_1}{\sigma\beta B_0} \quad \text{or} \quad J_3 = \frac{(1+B_iB_h)J_1}{B_h}. \qquad (17)$$

From equation (16c) we get

$$J_1 = \frac{\sigma B_0 B_h}{{B_h}^2 + (1 + B_i B_h)^2} u, \tag{18}$$

and
$$J_3 = \left[\frac{(1 + B_i B_h)}{{B_h}^2 + (1 + B_i B_h)^2} \right] \sigma B_0 u \ \text{[using (17) and (18)]} \tag{19}$$

where $B_h = \sigma\beta\ B_0$ is the Hall parameter.

From equations (18), (19) and (13), we obtain

$$\vec{J} \times \vec{B} =$$
$$-\sigma {B_0}^2 u \left[\frac{(1 + B_i B_h)}{{B_h}^2 + (1 + B_i B_h)^2} \right] \hat{i} + \left[\frac{\sigma {B_0}^2 B_h}{{B_h}^2 + (1 + B_i B_h)^2} u \right] \hat{k}\ . \tag{20}$$

Now using (20), equations (9) and (10) may be written as:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial x} \right) = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4}$$
$$- \sigma {B_0}^2 u \left[\frac{(1 + B_i B_h)}{{B_h}^2 + (1 + B_i B_h)^2} \right]\ , \tag{21a}$$

and
$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} - \eta \frac{\partial^4 v}{\partial x^4} \tag{21b}$$

or
$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\vec{\nabla}^2 u \right) - \eta \left(\vec{\nabla}^4 u \right)$$
$$- \sigma {B_0}^2 u \left[\frac{(1 + B_i B_h)}{{B_h}^2 + (1 + B_i B_h)^2} \right], \tag{22a}$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\vec{\nabla}^2 v \right) - \eta \left(\vec{\nabla}^4 v \right) \tag{22b}$$

We introduce the stream function ψ through

$$u = \frac{\partial \psi}{\partial y}\ \text{, and}\ v = -\frac{\partial \psi}{\partial x}.$$

Putting in to equations (21a) through (22b) above we get

$$\rho \left[\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right] = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left[\mu \left(\vec{\nabla}^2 \psi \right) - \eta \left(\vec{\nabla}^4 \psi \right) \right]$$
$$- \sigma {B_0}^2 \frac{\partial \psi}{\partial y} \left[\frac{(1 + B_i B_h)}{{B_h}^2 + (1 + B_i B_h)^2} \right] \text{, and} \tag{23a}$$

$$-\rho \left[\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y \partial x} \right] = -\frac{\partial p}{\partial y} - \frac{\partial}{\partial x} \left[\mu \left(\vec{\nabla}^2 \psi \right) - \eta \left(\vec{\nabla}^4 \psi \right) \right]. \tag{23b}$$

Where $\vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator.

Introducing $\lambda = \frac{y}{h}$, then equations (23a) and (23b) reduced to the form:

$$-\frac{h^2}{\rho} \frac{\partial p}{\partial x} = \left[\frac{\partial \psi}{\partial \lambda} \frac{\partial^2 \psi}{\partial x \partial \lambda} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial \lambda^2} \right] - \frac{h}{\rho} \frac{\partial}{\partial \lambda} \left[\mu \left(\vec{\nabla}_1^{\,2} \psi \right) - \eta \left(\vec{\nabla}_1^{\,4} \psi \right) \right]$$

$$+\frac{\sigma B_0{}^2 h}{\rho}\frac{\partial\psi}{\partial\lambda}\left[\frac{(1+B_iB_h)}{B_h{}^2+(1+B_iB_h)^2}\right], \qquad (24a)$$

and $$\frac{1}{\rho}\frac{\partial p}{\partial\lambda}=\left[\frac{\partial\psi}{\partial\lambda}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial\lambda\partial x}\right]-\frac{h}{\rho}\frac{\partial}{\partial x}\left[\mu\left(\vec{\nabla}_1{}^2\psi\right)-\eta\left(\vec{\nabla}_1{}^4\psi\right)\right] \qquad (24b)$$

or $$-\frac{h^2}{\rho}\frac{\partial p}{\partial x}=\left[\frac{\partial\psi}{\partial\lambda}\frac{\partial^2\psi}{\partial x\partial\lambda}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial\lambda^2}\right]-\frac{\mu}{\rho h}\frac{\partial^3\psi}{\partial\lambda^3}+\frac{\eta}{\rho h^3}\frac{\partial^5\psi}{\partial\lambda^5}$$

$$+\frac{\sigma B_0{}^2 h}{\rho}\frac{\partial\psi}{\partial\lambda}\left[\frac{(1+B_iB_h)}{B_h{}^2+(1+B_iB_h)^2}\right], \qquad (25a)$$

and $$\frac{1}{\rho}\frac{\partial p}{\partial\lambda}=\left[\frac{\partial\psi}{\partial\lambda}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial\lambda\partial x}\right]-\frac{\mu}{\rho h}\frac{\partial^3\psi}{\partial x\partial\lambda^2}+\frac{\eta}{\rho h^3}\frac{\partial^5\psi}{\partial x\partial\lambda^4} \qquad (25b)$$

Where $\vec{\nabla}_1{}^2=\frac{\partial^2}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2}{\partial\lambda^2}$.

Following G.Sherstha and R.Terrill (1965), we take the stream function as:

$$\psi(x,\lambda)=h\left(\frac{U_0}{a}-\frac{v_1x}{h}\right)f(\lambda) \qquad (26)$$

where U_0 is entrance velocity, $a=1-\frac{v_0}{v_1}$ and $f(\lambda)$ is a function of λ to be determined.

Substituting (26) in to (24a) through (25b), we get

$$-\frac{h^2}{\rho}\frac{\partial p}{\partial x}=\left(\frac{U_0}{a}-\frac{v_1x}{h}\right)\left[hv_1\left(ff''-(f')^2\right)-\frac{\mu}{\rho}f'''+\right.$$

$$\left.\frac{\eta}{\mu h^2}f^V+\frac{\sigma B_0{}^2h^2}{\rho}\left\{\frac{(1+B_iB_h)}{B_h{}^2+(1+B_iB_h)^2}\right\}f'\right],$$

or $$-\frac{h^2}{\mu}\frac{\partial p}{\partial x}=\left(\frac{U_0}{a}-\frac{v_1x}{h}\right)\left[\frac{\rho v_1h}{\mu}\left(ff''-(f')^2\right)-f'''+\right.$$

$$\left.\frac{\eta}{\mu h^2}f^V+\frac{\sigma B_0{}^2h^2}{\mu}\left\{\frac{(1+B_iB_h)}{B_h{}^2+(1+B_iB_h)^2}\right\}f'\right],$$

$$\frac{1}{\rho}\frac{\partial p}{\partial\lambda}=-v_1{}^2ff'+\frac{\mu v_1}{\rho h}f''-\frac{\eta v_1}{\rho h^3}f^{IV}$$

and

or $$-\frac{h^2}{\mu}\frac{\partial p}{\partial x}=\left(\frac{U_0}{a}-\frac{v_1x}{h}\right)\left\{S_R\left(ff''-(f')^2\right)-f'''+\alpha^2f^V+k_1Ha^2f'\right\}, \qquad (27a)$$

and $$\frac{h}{\mu v_1}\frac{\partial p}{\partial\lambda}=f''-\alpha^2f^{IV}-S_Rff' \qquad (27b)$$

or $$-\frac{h}{\mu v_1}\frac{\partial p}{\partial\lambda}=\alpha^2f^{IV}-f''+S_Rff' \qquad (28a)$$

or $$-\frac{h^2}{\mu}\frac{\partial p}{\partial\lambda}=v_1h\alpha^2f^{IV}-v_1hf''+v_1hS_Rff' \qquad (28b)$$

where

$$k_1=\frac{(1+B_iB_h)}{B_h{}^2+(1+B_iB_h)^2},$$

$S_R = \dfrac{\rho h v_1}{\mu}$, is the Suction Reynolds number,

$Ha = B_0 h\sqrt{\sigma/\mu}$, is the Hartmann number and

$\alpha = \dfrac{1}{h}\sqrt{\eta/\mu}$, is the couple stress fluid parameter.

Eliminating the pressure from equations (27a) and (28b), we get

$$0 = \left(\frac{U_0}{a} - \frac{v_1 x}{h}\right)\left\{S_R\left(ff''' - f'f''\right) - f^{IV} + \alpha^2 f^{VI} + k_1 Ha^2 f''\right\}$$

or $$0 = d\left\{S_R\left(ff'' - (f')^2\right) - f''' + \alpha^2 f^{V} + k_1 Ha^2 f'\right\}.$$

Therefore,

$$k_1 Ha^2 f' - f''' + \alpha^2 f^{V} + S_R[ff'' - (f')^2] = k \tag{29}$$

where, k is a constant to be determined and Ha is Hartmann number.

The boundary conditions are the no slip condition given by:

$$u(x,h) = u(x,0) = 0 \tag{30a}$$

$$v(x,0) = v_0,\ v(x,h) = v_1 \tag{30b}$$

$$\left\{curl\,\vec{q}\right\}_{y=0} = 0, \text{ and } \left\{curl\,\vec{q}\right\}_{y=h} = 0. \tag{30c}$$

In terms of $f(\lambda)$, these boundary conditions are taking the form:

$$\frac{1}{h}\frac{\partial u}{\partial \lambda} = 0 \text{ or } \left(\frac{U_0}{a} - \frac{v_1 x}{h}\right) f'(\lambda) = 0 \text{ or } f'(\lambda) = 0 \text{ when } \lambda = 0.$$

i.e. $$f'(0) = 0.$$

Again

$$-\frac{\partial \psi}{\partial x} = v_0 \text{ on } y = 0$$

or $$-h\left(\frac{-v_1}{h}\right) f(\lambda) = v_1 f(\lambda) = v_0 \text{ on } \lambda = 0$$

or $$f(\lambda) = \frac{v_0}{v_1} \text{ on } \lambda = 0.$$

Similarly, using the boundary condition given in equations (30b) and (30c)

$$-\frac{\partial \psi}{\partial x} = v_1 \text{ on } y = h$$

or $$-h\left(\frac{-v_1}{h}\right) f(\lambda) = v_1 f(\lambda) = v_1 \text{ on } \lambda = 1.$$

i.e. $$f(\lambda) = 1 \text{ on } \lambda = 1 \text{ or } f(1) = 1.$$

Again, $$curl\,\vec{q} = 0 \text{ on } y = 0, h$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \text{ on } y = 0, h$$

$$\frac{1}{h^2}\frac{\partial^2 \psi}{\partial \lambda^2} + \frac{\partial^2 \psi}{\partial x^2} = \vec{\nabla}_1^{\,2}\, \psi = 0 \text{ on } \lambda = 0,1$$

Substitute in to equation (26) we get

$$\frac{\partial^2 \psi}{\partial x^2} = 0 \text{ on } \lambda = 0,1$$

or $$f''(\lambda) = 0 \text{ on } \lambda = 0,1. \text{ i.e.}$$

$f''(0)=0\,;\, f''(1)=0\,.$

Therefore, we have

$f(0)=1-a\,;\;\; f(1)=1$

$f'(0)=0\,;\;\; f'(1)=0$

$f''(0)=0\,;\, f''(1)=0\,.$

Differentiating (29) with respect to λ we get

$$S_R[f'f'''-f'f'']-f^{IV}+\alpha^2 f^{VI}+k_1 Ha^2 f''=0 \qquad (31)$$

Boundary conditions:

$$f(0)=1-a\,;\;\; f(1)=1 \qquad (32a)$$

$$f'(0)=0\,;\;\; f'(1)=0 \qquad (32b)$$

$$f''(0)=0\,;\, f''(1)=0\,. \qquad (32c)$$

3. Solution of the Problem

[Quasi-Linearization Method for solving m^th Order Nonlinear Two-point Boundary Value Problem]

Quasi-linearization method is one of the numerical methods helps us to solve higher order non-linear two-point boundary value problem. In particular, the governing *MHD* momentum equation is coupled non-linear higher order differential equation which is not analytically solved, and therefore can be solved by applying a numerical method through which the one that involves linearizing the entire non-linear terms about some specified conditions, and is termed as quasi-linearization method (or technique).

Next we apply quasi-linearization technique to solve equation (31) together with the boundary conditions given in (32). In order to implement quasi-linearization technique, the equation (31) can be set as a system of equations as follows:

$$(f, f', f'', f''', f^{IV}, f^{V})=(x_1,x_2,x_3,x_4,x_5,x_6)$$

$$\frac{dx_i}{d\lambda}=F_i(x_1,x_2,x_3,x_4,x_5,x_6)=F_i(X)\,,\; i=1,2,...,6\,.$$

where $\dfrac{dx_1}{d\lambda}=F_1(X)=x_2\,,\; \dfrac{dx_2}{d\lambda}=F_2(X)=x_3\,,\; \dfrac{dx_3}{d\lambda}=F_3(X)=x_4\,,$

$\dfrac{dx_4}{d\lambda}=F_4(X)=x_5\,,\; \dfrac{dx_5}{d\lambda}=F_5(X)=x_6\,.$ Therefore,

$$F_6(X)=\frac{dx_6}{d\lambda}=\frac{1}{\alpha^2}\left\{-S_R(x_1x_4-x_2x_3)+x_5-k_1Ha^2x_3\right\}. \qquad (33)$$

The boundary conditions in terms of X are:

$$x_1(0)=1-a \qquad x_2(0)=x_3(0)=0 \qquad (34a)$$

$$x_1(1)=1 \qquad x_2(1)=x_3(1)=0 \qquad (34b)$$

The system of equations (33) is solved numerically subject to the boundary conditions (34) using quasi-linearization method (also known as generalized Newton's method) given by R.Bellman and R.Kalaba in 1965 [8]. Let $(x_i^{(r)}, i=1,2,...,6)$ be an approximate current solution and $(x_i^{(r+1)}, i=1,2,...,6)$ be an improved solution of (33).

By taking Taylor's series expansion around the current solution and neglecting the second and higher order derivative terms, the coupled first order system (33) is linearized as:

$$\frac{dx_i}{d\lambda}=x_{i+1} \text{ for } i=1,2,...,5$$

$$F_6(X)=\frac{dx_6}{d\lambda}=\frac{1}{\alpha^2}\left\{-S_R(x_1x_4-x_2x_3)+x_5-k_1Ha^2x_3\right\}, and$$

$$\frac{dx_i^{\,r+1}}{d\lambda}=x_{i+1}^{\gamma+1} \text{ for } i=1,2,...,5$$

$$\frac{dx_6^{\,r+1}}{d\lambda}=\frac{1}{\alpha^2}\left\{-S_R(x_1^{\,r}x_4^{\,r}-x_2^{\,r}x_3^{\,r})+x_5^{\,r}-k_1Ha^2x_3^{\,r}\right\}-S_Rx_4^{\,r}(x_1^{\,r+1}-x_1^{\,r})$$
$$+S_Rx_3^{\,r}(x_2^{\,r+1}-x_2^{\,r})+S_Rx_2^{\,r}(x_3^{\,r+1}-x_3^{\,r})-k_1Ha^2(x_3^{\,r+1}-x_3^{\,r})$$

$$-S_R x_1{}^r (x_4{}^{r+1} - x_4{}^r) + (x_5{}^{r+1} - x_5{}^r)$$

$$= \frac{1}{\alpha^2} \Big\{ S_R(-x_4{}^r x_1{}^{r+1} + x_3{}^r x_2{}^{r+1} + x_2{}^r x_3{}^{r+1} - x_1{}^r x_4{}^{r+1} - x_2{}^r x_3{}^r + x_1{}^r x_4{}^r)$$

$$- k_1 Ha^2 x_3{}^{r+1} + x_5{}^{r+1} \Big\}. \quad (35)$$

Boundary conditions:

$$x_1^{\gamma+1}(0) = 1 - a \qquad x_1^{\gamma+1}(1) = 1 \qquad (36a)$$

$$x_2^{\gamma+1}(0) = 0 \qquad x_2^{\gamma+1}(1) = 0 \qquad (36b)$$

$$x_3^{\gamma+1}(0) = 0 \qquad x_3^{\gamma+1}(1) = 0 \qquad (36c)$$

To solve for $(x_i^{(r+1)},\ i = 1,2,...,6)$ the solution to six separate initial value problems, denoted by: $x_i{}^{h1}(\lambda)$, $x_i{}^{h2}(\lambda)$, $x_i{}^{h3}(\lambda)$ [which are the solutions of the homogeneous system corresponding to (35)] and $x_i{}^{p}(\lambda)$ [which is the particular solution of (35)], with the following initial conditions are obtained by using a Runge-Kutta method.

$$x_4{}^{h1}(0) = 1,\ x_i{}^{h1}(0) = 0 \text{ for } i \neq 4 \qquad (37a)$$

$$x_5{}^{h2}(0) = 1,\ x_i{}^{h2}(0) = 0 \text{ for } i \neq 5 \qquad (37b)$$

$$x_6{}^{h3}(0) = 1,\ x_i{}^{h3}(0) = 0 \text{ for } i \neq 6 \qquad (37c)$$

$$x_1{}^{p}(0) = 1 - a,\ x_i{}^{p}(0) = 0 \text{ for } i \neq 1 \qquad (37d)$$

Since the differential equations are linear, the principle of superposition holds and the general solution may be written as:

$$x_i^{(r+1)}(\lambda) = C_1 x_i{}^{h1}(\lambda) + C_2 x_i{}^{h2}(\lambda) + C_3 x_i{}^{h3}(\lambda) + C_4 x_i{}^{h4}(\lambda) + C_5 x_i{}^{h5}(\lambda) + x_i{}^{p}(\lambda) \qquad (38)$$

Where C_1, C_2, C_3, C_4 and C_5 are the unknown constants and are determine by considering the boundary condition at $\lambda = 1$. This solution $(x_i^{(r+1)}, i = 1,2,...,6)$ is then compared with solution at the previous step $(x_i^{(r)}, i = 1,2,...,6)$ and further iteration is performed if the convergence has not been achieved or greater accuracy is desired.

4. Results and Discussion

- The coefficient of the nondimensional shear stress S_f at the walls is given by:

$$S_f = \frac{\mu \partial^2\psi / \partial\lambda^2}{1/2(h^2 \rho U_0{}^2)} \quad \text{at } \lambda = 0 \text{ and } \lambda = 1$$

$$= \frac{2}{\mathrm{Re}}\left(\frac{1}{a} - \frac{S_R x}{h\,\mathrm{Re}}\right) f''(\lambda) \text{ at } \lambda = 0 \text{ and } \lambda = 1.$$

Where *Re* is the Reynolds number and *h* is the height of the channel.

- In Table-1 below the dimensionless skin friction coefficient is tabulated for different values of suction Reynolds number (S_R) with Ha=5, α = 0.5, $B_i = 5$, B_h = 0.3 and Re = 200 at the lower plate (λ = 0) where injection takes place and upper plate where suction takes place (λ = 1). From this table it can be observed that the skin friction coefficient is increasing when one move away from the lower to upper plate and again to the axis of the channel. In other words, the skin friction coefficient is decreasing at the lower plate whilst increasing at the upper plate of the porous channel, with suction Reynolds number.

Suction Reynolds Number (S_R)	Lower Plate with Constant Injection	Upper Plate with Constant Suction
0.0	0.59010	-0.37970
1.0	0.37061	-0.34555
3.0	0.15210	-0.25982
5.0	0.10926	-0.15906

Table-1: Variation of skin friction for various values of (S_R), Ha = 5, α = 0.5, *a* = 0.2, $B_i = 5$, B_h = 0.3 and *Re* = 200.

- Generally in Table 2 as shown below the non dimensional skin friction is tabulated for different values of Hartmann number (Ha) with S_R = 10, α = 0.5, a = 0.2, $B_i = 5$, B_h = 0.3 and Re = 200 at the lower and upper plates where injection and suction takes place. From this table it can be observed that the skin friction decreases at upper and lower plates as the Hartmann number (Ha) increases.

Hartmann number (H_a)	Lower plate with Constant Injection	Upper plate with Constant Suction
0.0	0.20849	0.16336
1.0	0.20561	0.16259
3.0	0.18137	0.15402
5.0	0.12118	0.12312

Table-2: Variation of skin friction for various values of (Ha),
S_R = 10, α = 0.5, a = 0.2, $B_i = 5$, B_h = 0.3 and Re = 200.

- In Table-3 as shown below the dimensionless skin friction coefficient is tabulated for different values of couple stress fluid parameter (α) with S_R = 10, Ha = 5, $B_i = 5$, B_h = 0.3 and Re = 200 at the lower and upper plates. From this table it can be observed that the skin friction coefficient is decreasing at the upper plate where suction takes place (i.e. λ = 1) whilst increasing at the lower plate where injection takes place (i.e. λ = 0) with couple stress fluid parameter (α).

Material constant (α)	Lower plate with Constant Injection	Upper plate with Constant Suction
0.70	0.16429	-0.11063
1.00	0.23672	-0.27391
3.00	0.24090	-0.45378
5.00	0.22716	-0.47124

Table-3: Variation of skin friction for various values of (α),
S_R = 10, Ha= 5, a = 0.2, $B_i = 5$, B_h = 0.3 and Re = 200.

Next the velocity components (u, v) for various values of S_R, Ha, α, B_i, B_h and 'a' are calculated correct to six places of decimal by taking the axial distance 0.2 and $a = 1 - v_0/v_1$ from equation (26) where v_0/v_1 is the injection-suction velocity ratio.

- Fig.2 shows that the variation of axial velocity (u) with λ for different values of suction Reynolds number (S_R) for Ha=5, α = 0.5, $a = 0.2$, B_i = 5, B_h = 0.3 and Re=200. The velocity increase initially near to the lower plate (injection takes place) achieving the maxima which shifts toward the plate λ = 1 (suction takes place) and decrease thereafter, with suction Reynolds number.
- Fig.3 depicts the effect of Hartman number Ha on the axial velocity component for S_R = 10, α = 0.5, $a = 0.2$, B_i = 5, B_h = 0.3 and Re=200. From this Fig.3 it can be observed that the effect of increasing values of the Hartmann number on the flow is to damp the velocity profile. The dampening is pronounced at the center of the channel, the more the Hartman number the velocity flattened. Hence this creates a stagnation point and consequently fluid pushed to the walls of the channel thereby increasing the velocity in the boundary layer.
- Fig.4 shows that the variation of the axial velocity (u) with λ for different values of couple stress fluid parameter α for S_R = 10, Ha= 5, $a = 0.2$, $B_i = 5$, B_h = 0.3 and Re=200. The effect of the couple stress parameter on axial velocity component has been presented in Figs.4. It can be observed that the axial velocity increases near the central plane as the value of α increases. However, this trend is reversed near walls. In general Fig.4 shows the effect of (α) on axial velocity (u) for the values of S_R = 10, Ha= 5, $a = 0.2$, $B_i = 5$, B_h = 0.3 and Re=200 and it can be observed that the axial velocity (u) decreases as the couple stress fluid parameter (α) increases. It can also be noted that the more the fluid is non viscous fluid ($\alpha \to 0$) greater the velocity increases at the center.
- Fig.5 demonstrates the variations of the axial velocity component (u) for different values of the parameter (a) with S_R = 10, Ha = 5, $\alpha = 0.5$, $B_i = 5$, B_h = 0.3 and Re = 200. From Fig.5 it can be observed the axial velocity (u) decreasing as the parameter (a) increases or as injection-suction ratio decreases.
- Fig.6 indicating that the velocity profiles for different values of ionic-slip parameter (B_i) with S_R = 10, Ha= 5, $\alpha = 0.5$, $a = 0.2$, B_h = 0.3 and Re=200. The velocity component increase as the ionic slip parameter near to the lower plate and near to the upper plate the velocity decrease (effect of suction) as B_i increases. As B_i increases the effective conductivity also increases, in turn, decreases the damping force on velocity, and hence the velocity increases.

- Fig.7 show the effect of Hall parameter (B_h) on velocity for S_R = 10, Ha= 5, $\alpha = 0.5$, $B_i = 5$ and Re=200. It can be seen from these figure that the Hall parameter increases the velocity near the lower plate. Inclusion of Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity.

4.1. List of Figures

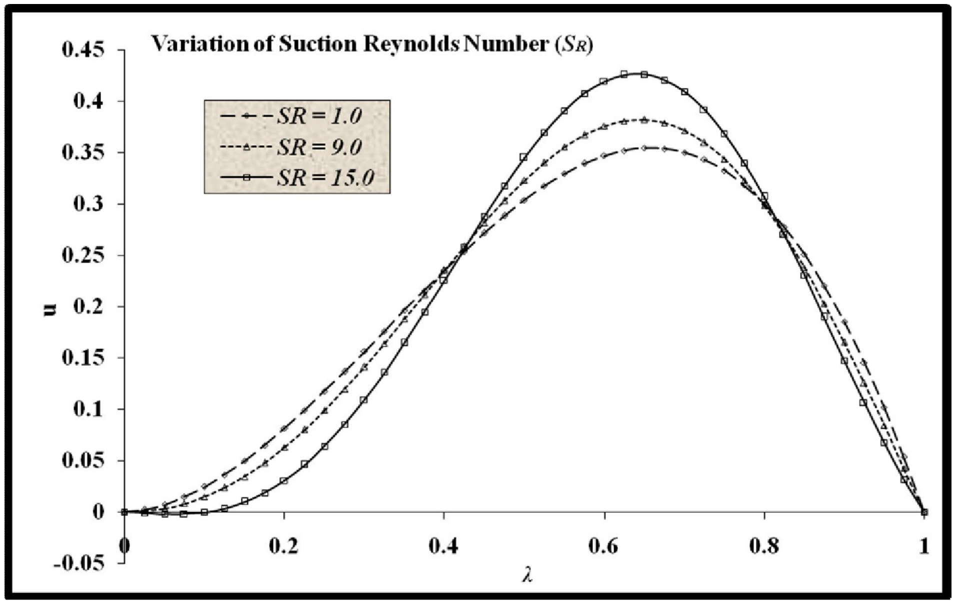

Fig.2 Illustration of non-dimensional axial velocity profile (u) with λ for different values of S_R , Re = 200, Ha= 5, α = 0.5, B_i = 5, B_h = 0.3, and a = 0.2.

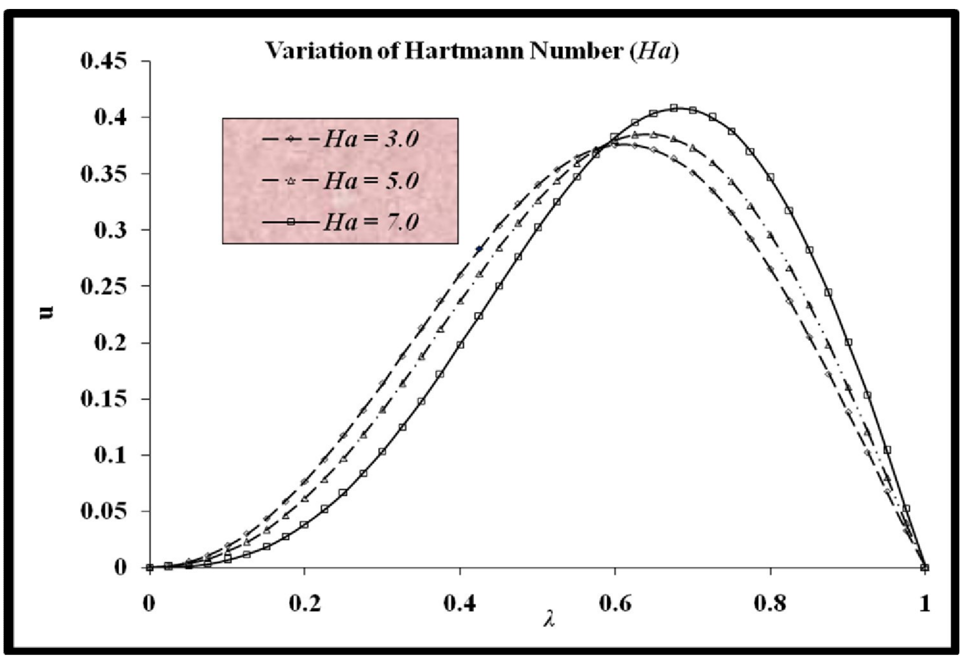

Fig.3 Illustration of non-dimensional axial velocity profile (u) with λ for different values of Hartmann number (Ha), Re = 200, S_R = 10, α = 0.5, B_i = 5, B_h = 0.3, and a = 0.2.

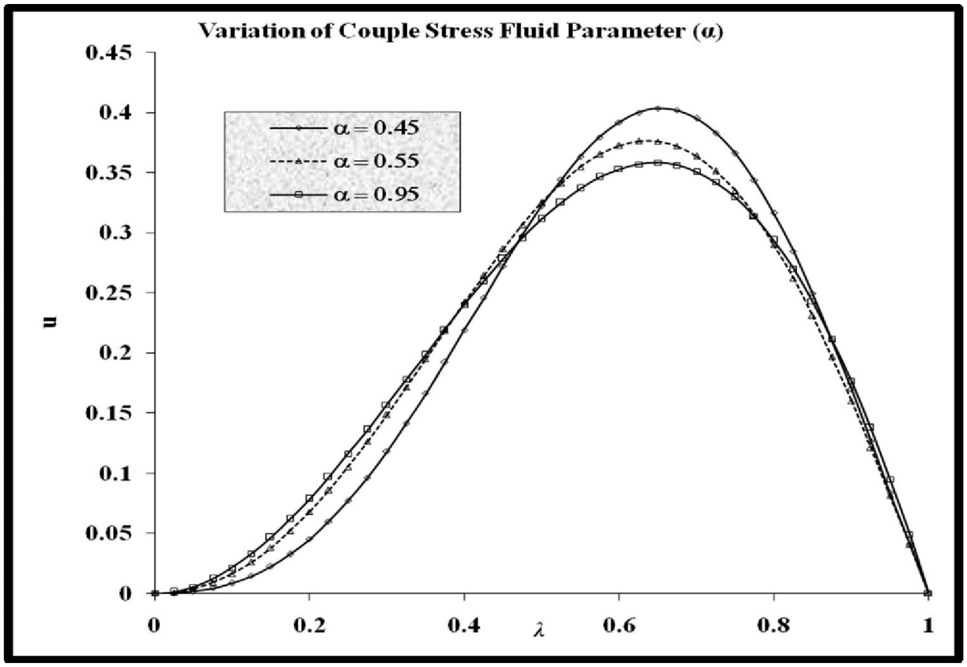

Fig.4 Illustration of non-dimensional axial velocity profile (u) with λ for different values of material constant (α), S_R = 10, Ha= 5, B_i = 5, a = 0.2, B_h = 0.3 and Re = 200.

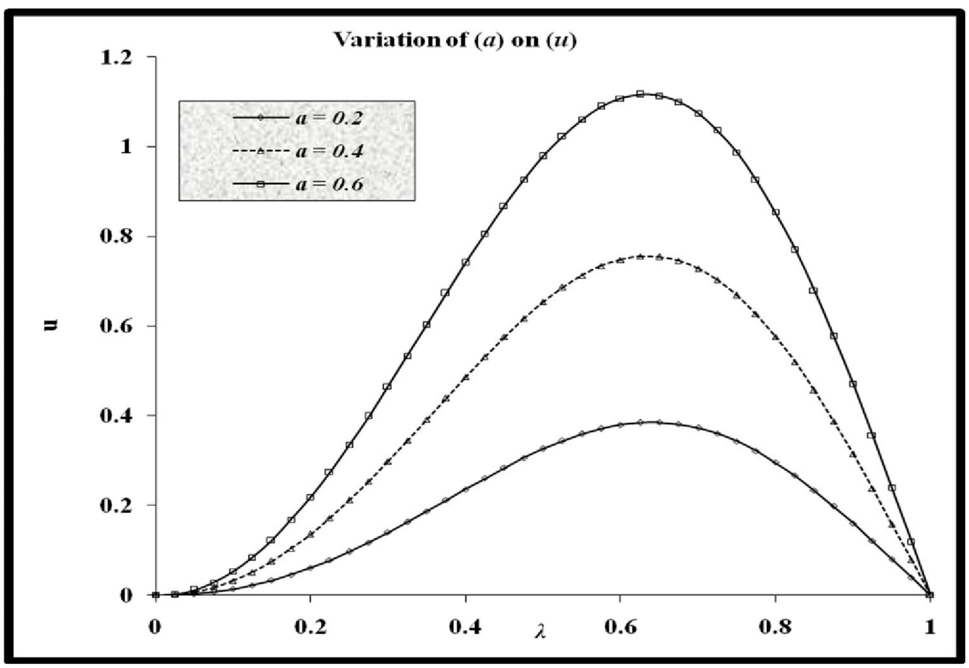

Fig.5 The variation of Dimensionless axial velocity profile (u) with λ for different values of the parameter (a), S_R = 10, Ha= 5, $\alpha = 0.5$, B_i = 5,, B_h = 0.3 and Re =200.

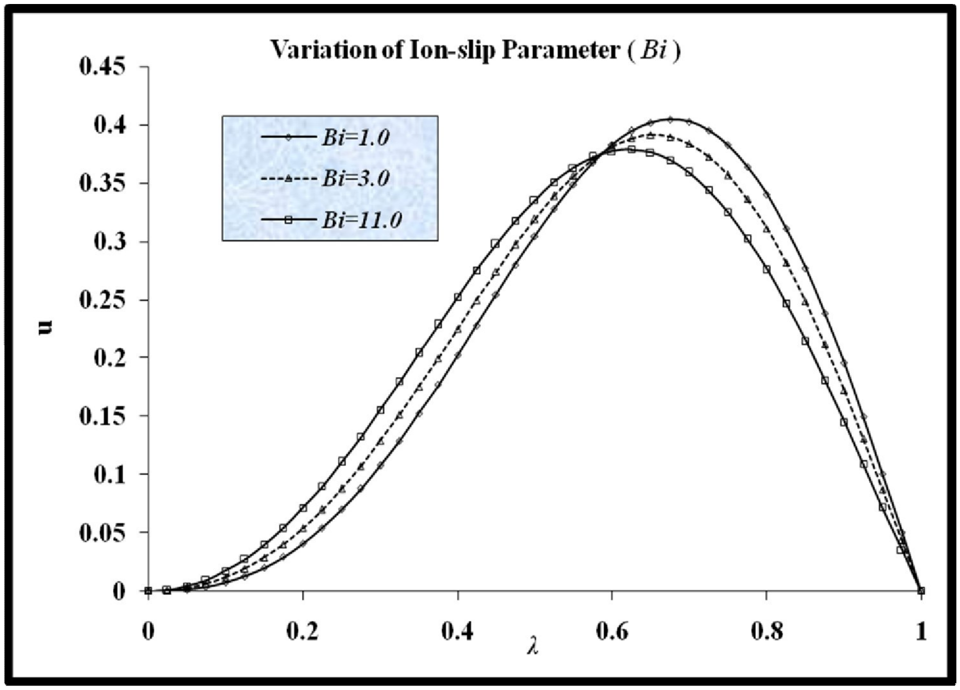

Fig.6 The variation of Dimensionless axial velocity profile (*u*) with λ for different values of ion-slip parameter (B_i), S_R = 10, Ha = 5, $\alpha = 0.5$, *a* = 0.2, B_h = 0.3 and *Re* =200.

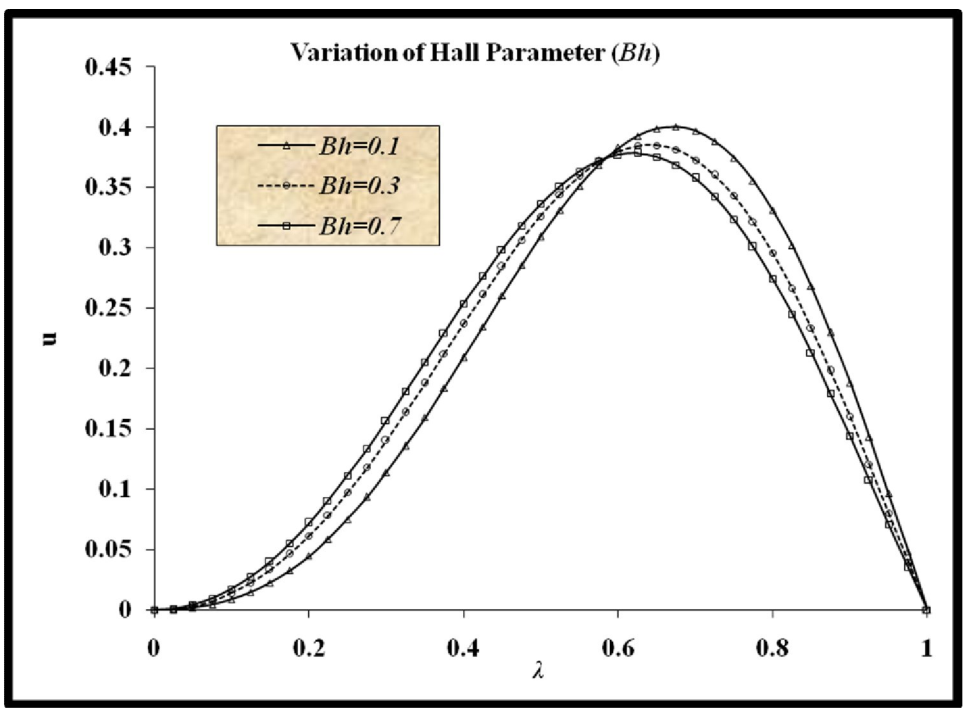

Fig.7 The variation of Dimensionless axial velocity profile (*u*) with λ for different values of Hall parameter (B_h), S_R = 10, $\alpha = 0.5$, Ha = 5, B_i = 5, *a* = 0.2 and *Re* =200.

5. Conclusion

The steady state incompressible and electrically conducting couple stress fluid flow through a porous channel with injection and suction under the influence of applied uniform magnetic field is studied. Increase in the expansion ratio increase in transverse velocity components and also the axial velocity component except the regions near the boundaries. The velocity decreases at the centerline and increases in the boundary layer with an increase the magnetic parameter. The micro-rotation increases with an increase in micro-concentration while it decreases with an increase in the magnetic field. The main results indicate the following findings.

- Increasing Hall and Ion slip parameters leads to an increase in the velocity around the center.
- The case couple stress fluid parameter $\alpha \to 0$ obtained results corresponds to the classical viscous fluid case. This provides a useful check.
- The axial velocity (*u*) decreasing as the parameter (a) increases or injection-suction velocity ratio decreases.
- Increasing the Hartman number the fluid velocity decreases.

The work presented in the paper can be extended to analyze the non-Newtonian behavior by modeling flow as power law fluid, Casson fluid, ferro fluids, anisotropic fluid etc. The results discussed in the paper may be extended to the blood flow in an artery with stenosis and/or the flow between clot and stenosis.

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