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SUM CONNECTIVITY LEAP INDEX AND GEOMETRIC-ARTHETMIC LEAP INDEX OF CERTAIN WINDMILL GRAPHS

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Abstract: We propose the sum connectivity leap and geometric-arithmetic leap indices of a graph. In this paper, the sum connectivity leap and geometric-arithmetic leap indices of Kulli cycle windmill graph, Kulli path windmill graph, Dutch windmill graph and French windmill graph are computed.
Keywords: sum connectivity leap index, geometric-arithmetic leap index, windmill graph.
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1. INTRODUCTION

We are concerned only with finite, connected, undirected graphs having no loops and multiple edges. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. For a vertex v, the degree $d(v)$ is the number of edges incident to v. The distance $d(u,v)$ between any two vertices u and v is equal to the length of a shortest path connecting u and v. For a positive integer k, the open k-neighborhood $N_k(v)$ of a vertex v in G is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k-distance degree $d_k(v)$ of v in G is defined as the number of k neighbors of v in G. Clearly $d_k(v) = |N_k(v)|$. We refer to [1], for any terminology or notation not given here.

In [2], Naji et al. introduced the first and second leap Zagreb indices, defined as

$$LM_1(G) = \sum_{u \in V(G)} d_2^2(u), \qquad LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v).$$

In [3], Kulli proposed a new version of the first leap Zagreb index, defined as

$$LM_1^*(G) = \sum_{uv \in E(G)} \left[d_2(u) + d_2(v)\right]$$

We introduce the sum connectivity leap index and geometric-arithmetic leap index of a graph G, defined as

$$SL(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_2(u) + d_2(v)}} \qquad (1)$$

$$GAL(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_2(u)d_2(v)}}{d_2(u) + d_2(v)}. \qquad (2)$$

Very recently, some new leap indices were proposed and studied in [4, 5, 6]. In recent years, some new connectivity indices were introduced and studied such as sum connectivity index [7], sum connectivity Gourava index [8], sum connectivity Revan index [9], geometric-arithmetic reverse and sum connectivity reverse indices [10], connectivity Banhatti indices [11].

In this paper, we compute the sum connectivity leap index and geometric-arithmetic leap index of four types of windmill graphs.

2. Results for Kulli Cycle Windmill Graphs

Definition 1 [12]. The Kulli cycle windmill graph C_{n+1}^m is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex K_1 in common. This graph is presented in Figure 1.

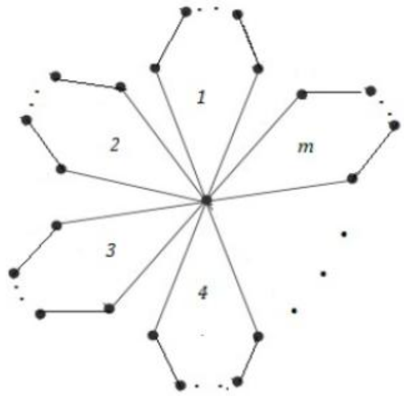

Figure 1. The graph C_{n+1}^{m}

Lemma 1. Let C_{n+1}^{m} be a Kulli cycle windmill graph with $mn+1$ vertices and $2mn$ edges, $m \geq 2$, $n \geq 5$. Then there are two types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(C_{n+1}^{m}) \mid d_2(u) = 0.\ d_2(v) = mn - 2\}$, $\quad |E_1| = mn.$

$E_2 = \{uv \in E(C_{n+1}^{m}) \mid d_2(u) = d_2(v) = mn - 2\}$, $\quad |E_2| = mn.$

Theorem 2. The sum connectivity leap index of a Kulli cycle windmill graph C_{n+1}^{m}, $m \geq 2$, $n \geq 5$ is given by

$$SL\left(C_{n+1}^{m}\right) = \frac{\left(\sqrt{2}+1\right)mn}{\sqrt{2(mn-2)}}.$$

Proof: From equation (1) and by Lemma 1, we have

$$SL\left(C_{n+1}^{m}\right) = \sum_{uv \in E\left(C_{n+1}^{m}\right)} \frac{1}{\sqrt{d_2(u)+d_2(v)}}$$

$$= mn\frac{1}{\sqrt{0+mn-2}} + mn\frac{1}{\sqrt{(mn-2)+(mn-2)}} = \frac{\left(\sqrt{2}+1\right)mn}{\sqrt{2(mn-2)}}$$

Theorem 3. The geometric-arithmetic leap index of a Kulli cycle windmill graph C_{n+1}^{m}, $m \geq 2$, $n \geq 5$ is given by

$$GAL\left(C_{n+1}^{m}\right) = mn.$$

Proof: From equation (2) and by Lemma 1, we obtain

$$GAL\left(C_{n+1}^{m}\right) = \sum_{uv \in E\left(C_{n+1}^{m}\right)} \frac{2\sqrt{d_2(u)d_2(v)}}{d_2(u)+d_2(v)} = mn\frac{2\sqrt{0\times(mn-2)}}{0+mn-2} + mn\frac{2\sqrt{(mn-2)(mn-2)}}{(mn-2)+(mn-2)} = mn$$

3. Results for Kulli Path Windmill Graphs

Definition 2[13]. The Kulli path windmill graph P_{n+1}^{m} is the graph obtained by taking m copies of the graph $K_1 + P_n$ with a vertex K_1 in common. This graph is depicted in Figure 2.

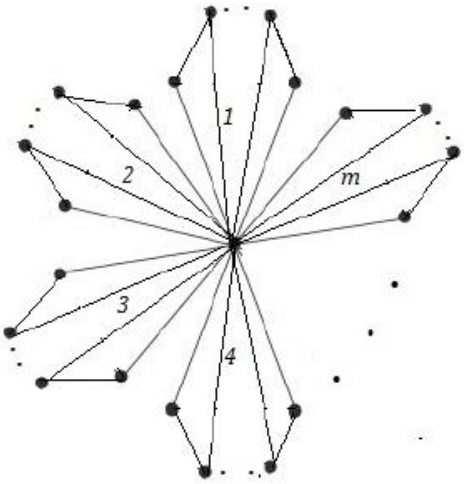

Figure 2. The graph P_{n+1}^m

Lemma 4. Let P_{n+1}^m be a Kulli path windmill graph with $mn+1$ vertices and $2mn - m$ edges, $m \geq 2$, $n \geq 5$. Then there are four types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(P_{n+1}^m) \mid d_2(u) = 0, d_2(v) = mn - 2\}$, $|E_1| = 2m$.

$E_2 = \{uv \in E(P_{n+1}^m) \mid d_2(u) = 0, d_2(v) = mn - 3\}$, $|E_2| = m(n - 2)$.

$E_3 = \{uv \in E(P_{n+1}^m) \mid d_2(u) = mn - 2, d_2(v) = mn - 3\}$, $|E_3| = 2m$.

$E_4 = \{uv \in E(P_{n+1}^m) \mid d_2(u) = d_2(v) = mn - 3\}$, $|E_4| = m(n - 3)$.

Theorem 5. The sum connectivity leap index of a Kulli path windmill graph P_{n+1}^m, $m \geq 2$, $n \geq 5$ is given by

$$SL(P_{n+1}^m) = \frac{2m}{\sqrt{mn-2}} + \frac{m(n-2)}{\sqrt{mn-3}} + \frac{2m}{\sqrt{2mn-5}} + \frac{m(n-3)}{\sqrt{2mn-6}}.$$

Proof: From equation (1) and by Lemma 4, we have

$$SL(P_{n+1}^m) = \sum_{uv \in E(P_{n+1}^m)} \frac{1}{\sqrt{d_2(u) + d_2(v)}}$$

$$= \frac{2m}{\sqrt{0 + mn - 2}} + \frac{m(n-2)}{\sqrt{0 + mn - 3}} + \frac{2m}{\sqrt{mn - 2 + mn - 3}} + \frac{m(n-3)}{\sqrt{mn - 3 + mn - 3}}$$

$$= \frac{2m}{\sqrt{mn-2}} + \frac{m(n-2)}{\sqrt{mn-3}} + \frac{2m}{\sqrt{2mn-5}} + \frac{m(n-3)}{\sqrt{2mn-6}}.$$

Theorem 6. The geometric-arithmetic leap index of a Kulli path windmill graph P_{n+1}^m, $m \geq 2$, $n \geq 5$ is given by

$$GAL(P_{n+1}^m) = \frac{4m\sqrt{(mn-2)(mn-3)}}{2mn-5} + m(n-3).$$

Proof: From equation (2) and by Lemma 4, we have

$$GAL(P_{n+1}^m) = \sum_{uv \in E(P_{n+1}^m)} \frac{2\sqrt{d_2(u) d_2(v)}}{d_2(u) + d_2(v)}$$

$$= 2m \frac{2\sqrt{0 \times (mn-2)}}{0 + mn - 2} + 2m(m-2) \frac{2\sqrt{0 \times (mn-3)}}{0 + mn - 3}$$

$$+ 2m \frac{2\sqrt{(mn-2)(mn-3)}}{mn - 2 + mn - 3} + m(n-3) \frac{2\sqrt{(mn-3)(mn-3)}}{mn - 3 + mn - 3}$$

$$= \frac{4m\sqrt{(mn-2)(mn-3)}}{2mn-5} + m(n-3).$$

4. Results for Dutch Windmill Graphs

Definition 3. The Dutch windmill graph D_n^m $m \geq 2$, $n \geq 5$ is the graph obtained by taking m copies of the cycle C_n with a vertex in common.

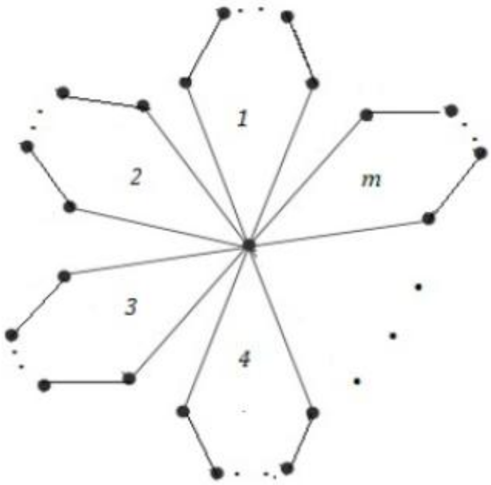

Figure 3. The graph D_n^m

Lemma 7. Let D_n^m be a Dutch windmill graph with $1+ m(n-1)$ vertices and mn edges, $m \geq 2$, $n \geq 5$. Then there are three types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(D_n^m) \mid d_2(u) = d_2(v) = 2m\}$, $|E_1| = 2m$.

$E_2 = \{uv \in E(D_n^m) \mid d_2(u) = 2m, d_2(v) = 2\}$, $|E_2| = 2m$.

$E_3 = \{uv \in E(D_n^m) \mid d_2(u) = d_2(v) = 2\}$, $|E_3| = m(n-4)$.

Theorem 8. The sum connectivity leap index of a Dutch windmill graph D_n^m, $m \geq 2$, $n \geq 5$ is given by

$$SL(D_n^m) = \sqrt{m} + \frac{\sqrt{2}m}{\sqrt{m+1}} + \frac{1}{2}m(n-4).$$

Proof: From equation (1) and by Lemma 7, we have

$$SL(D_n^m) = \sum_{uv \in E(D_n^m)} \frac{1}{\sqrt{d_2(u) + d_2(v)}}$$

$$= \frac{2m}{\sqrt{2m+2m}} + \frac{2m}{\sqrt{2m+2}} + \frac{m(n-4)}{\sqrt{2+2}} = \sqrt{m} + \frac{\sqrt{2}m}{\sqrt{m+1}} + \frac{1}{2}m(n-4).$$

Theorem 9. The geometric-arithmetic leap index of a Dutch windmill graph D_n^m, $m \geq 2$, $n \geq 5$ is given by

$$GAL(D_n^m) = mn - 2m + \frac{4m^{3/2}}{m+1}.$$

Proof: From equation (2) and by Lemma 7, we obtain

$$GAL(D_n^m) = \sum_{uv \in E(D_n^m)} \frac{2\sqrt{d_2(u)d_2(v)}}{d_2(u) + d_2(v)}$$

$$= \frac{2m2\sqrt{(2m)(2m)}}{2m+2m} + \frac{2m2\sqrt{2m(2)}}{2m+2} + \frac{m(n-4)2\sqrt{2 \times 2}}{2+2}$$

$$= mn - 2m + \frac{4m^{3/2}}{m+1}.$$

5. Results for French Windmill Graphs

Definition 4. The French windmill graph F_n^m is the graph obtained by taking $m \geq 2$ copies of the graph K_n, $n \geq 2$ with a vertex in common.

This graph is shown in Figure 4. For more information about windmill graph, see [14].

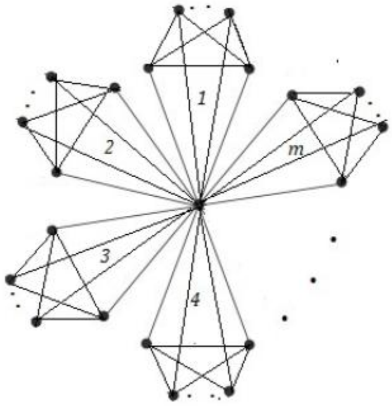

Figure 4. The graph F_n^m

Lemma 10. Let F_n^m be a French windmill graph with $1+m(n-1)$ vertices and $\frac{1}{2}mn(n-1)$ edges, $m \geq 2$, $n \geq 2$. Then there are two types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(F_n^m) \mid d_2(u) = 0, d_2(v) = (n-1)(m-1)\}$, $|E_1| = m(n-1)$.

$E_2 = \{uv \in E(F_n^m) \mid d_2(u) = d_2(v) = (n-1)(m-1)\}$, $|E_2| = \frac{1}{2}m(n-1)(n-2)$.

Theorem 11. The sum connectivity leap index of a French windmill graph F_n^m, $m \geq 2$, $n \geq 2$ is given by

$$SL(F_n^m) = m\sqrt{\frac{n-1}{m-1}}\left(1 + \frac{n-2}{2\sqrt{2}}\right)$$

Proof: By using equation (1) and by Lemma 10, we derive

$$SL(F_n^m) = \sum_{uv \in E(F_n^m)} \frac{1}{\sqrt{d_2(u) + d_2(v)}}$$

$$= \frac{m(n-1)}{\sqrt{0 + (n-1)(m-1)}} + \frac{m(n-1)(n-2)}{2\sqrt{(n-1)(m-1) + (n-1)(m-1)}}$$

$$= m\sqrt{\frac{n-1}{m-1}}\left(1 + \frac{n-2}{2\sqrt{2}}\right)$$

Theorem 12. The geometric-arithmetic leap index of a French windmill graph F_n^m, $m \geq 2$, $n \geq 2$ is given by

$$GAL(F_n^m) = \frac{1}{2}m(n-1)(n-2).$$

Proof: By using equation (2) and by Lemma 10, we deduce

$$GAL(F_n^m) = \sum_{uv \in E(F_n^m)} \frac{2\sqrt{d_2(u)d_2(v)}}{d_2(u) + d_2(v)}$$

$$= m(n-1)\frac{2\sqrt{0 \times (n-1)(m-1)}}{0 + (n-1)(m-1)} + \frac{m(n-1)(n-2)}{2}\frac{2\sqrt{(n-1)(m-1)(n-1)(m-1)}}{(n-1)(m-1) + (n-1)(m-1)}$$

$$= \frac{1}{2}m(n-1)(n-2).$$

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