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1-movable Perfect Domination in Graphs

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Abstract

A nonempty subset S of $V(G)$ is a *1-movable perfect dominating set* of G if $S = V(G)$ or $S \subset V(G)$ is a perfect dominating set of G and for every $v \in S$, there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a perfect dominating set of G. The smallest cardinality of a 1-movable perfect dominating set of G is called 1-*movable perfect domination number* of G, denoted by $\gamma^1_{mp}(G)$. A 1-movable perfect dominating set of G with cardinality equal to $\gamma^1_{mp}(G)$ is called γ^1_{mp}*-set* of G. This paper characterizes of the 1-movable perfect dominating sets in the join and corona of two connected graphs.

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1 Introduction

Let $G = (V(G), E(G))$ be a graph and $v \in V(G)$. The *open neighborhood* of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$. If $S \subseteq V(G)$, then the *open neighborhood* of S is the set $N_G(S) = N(S) = \cup_{v\in S} N_G(v)$.The *join* of two graphs G and H denoted by $G + H$ is the graph with vertex-set $V(G + H) = V(G) \dot{\cup} V(H)$ and edge-set $E(G + H) = E(G) \dot{\cup} E(H) \cup \{uv : u \in V(G), v \in V(H)\}$. The *corona* of two graphs G and H, denoted by $G \circ H$, is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the ith vertex of G to every vertex in the ith copy of H. For every $v \in V(G)$, we denote by H^v the copy of H whose vertices are joined or attached to the vertex v.

A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G)\setminus S$, there exists $u \in S$ such that $uv \in E(G)$. A nonempty subset S of $V(G)$ is a *perfect* *dominating set* of G if S is a dominating set of G and every vertex $v \in V(G) \setminus S$ is adjacent to exactly one vertex in S. The *perfect domination number* of G denoted by $\gamma_p(G)$, is the smallest cardinality of a perfect dominating set of G.

A nonempty set $S \subseteq V(G)$ is a 1-*movable dominating set* of G if S is a dominating set of G and for every $v \in S$, $S \setminus \{v\}$ is a dominating set of G or there exists a vertex $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. The 1-*movable domination number* of a graph G, denoted by $\gamma_m^1(G)$, is the smallest cardinality of a 1-movable dominating set of G. A 1-movable dominating set of G with cardinality equal to $\gamma_m^1(G)$ is called γ_m^1-*set* of G.

A nonempty set $S \subseteq V(G)$ is a 1-*movable perfect dominating set* of G if $S = V(G)$ or $S \subset V(G)$ is a perfect dominating set of G and for every $v \in S$ there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a perfect dominating set of G. The smallest cardinality of a 1-movable perfect dominating set of G is called 1-*movable perfect domination number* of G, denoted by $\gamma_{mp}^1(G)$. A 1-movable perfect dominating set of G with cardinality equal to $\gamma_{mp}^1(G)$ is called γ_{mp}^1-*set* of G.

This paper presents some results of 1-movable perfect dominating sets in the join and corona of graphs.

2 Results

Remark 2.1 *For every connected graph G of order $n \geq 2$, $1 \leq \gamma_{mp}^1(G) \leq n$.*

Theorem 2.2 *Let G be a connected nontrivial graph. Then $\gamma_{mp}^1(G) = 1$ if and only $G = K_2$ or $G \cong K_2 + H$ for some graph H.*

Proof: Suppose that $\gamma_{mp}^1(G) = 1$. Then G has a 1-movable perfect dominating set say S with $|S| = 1$. If $|V(G)| = 2$, then $G = K_2$. Suppose $|V(G)| \geq 3$. Let $S = \{a\}$ for some $a \in V(G)$. Since S is a 1-movable perfect dominating set of G, there exists $u \in (V(G) \setminus S) \cap N_G(a)$ such that $(S \setminus \{a\}) \cup \{u\} = \{u\}$ is a perfect dominating set of G. So, $ua \in E(G)$. Take $V(K_2) = \{u, a\}$ and $\langle V(G) \setminus V(K_2) \rangle = H$. Then $G = K_2 + H$.

For the converse, suppose first that $G = K_2$. Clearly $S = \{a\} \subseteq V(K_2)$ is a 1-movable perfect dominating set of K_2. Suppose that $G = K_2 + H$. Let $V(K_2) = \{a, b\}$ and $S = \{a\}$ for some $a, b \in G$. Then S is a dominating set of G. Let $y \in V(G) \setminus S$. Then $|N_G(y) \cap lS| = 1$. Then S is a perfect dominating set of G. Now, $S \setminus \{a\} \cup \{b\} = \{b\}$ is a dominating set of G. Let $w \in V(G) \setminus \{b\}$. Then $|N_G(w) \cap \{b\}| = 1$. Hence, $S \setminus \{a\} \cup \{b\} = \{b\}$ is a perfect dominating set of G. Since $|S| = 1$, S is a γ_{mp}^1-set of G. Thus, $\gamma_{mp}^1(G) = |S| = 1$. □

Corollary 2.3 *For every complete graph of order $n \geq 2$, $\gamma_{mp}^1(K_n) = 1$.*

Theorem 2.4 *Let G and H be connected nontrivial graphs. Then $S \subseteq V(G+H)$ is a 1-movable perfect dominating set of $G+H$ if and only if one of the following holds:*

(i) $S = V(G+H)$

(ii) $|S| = 1$ *and either S is a 1-movable perfect dominating set of G S is a dominating set of G and there exists $y \in V(H)$ which dominates H.*

(iii) $|S| = 1$ *and either S is a 1-movable perfect dominating set of H or S is a dominating set of H and there exists $a \in V(G)$ which dominates G.*

Proof: Suppose that S is a 1-movable perfect dominating set of $G+H$. If $S = V(G+H)$, then (i) holds. Suppose $S \subset V(G+H)$. Since S is a perfect dominating set of $G+H$, $S \subseteq V(G)$ or $S \subseteq V(H)$. Suppose first that $S \subseteq V(G)$. Then S is a dominating set of G. Since S is a perfect dominating set, $|S| = 1$. Let $S = \{v\}$ for some $v \in V(G)$. Since S is a 1-movable perfect dominating set of $G+H$, there exists $u \in (V(G+H) \setminus S) \cap N_{G+H}(v)$ such that $(S \setminus \{v\}) \cup \{u\} = \{u\}$ is a dominating set of $G+H$. If $u \in (V(G) \setminus S) \cap N_G(v)$, then $S \setminus \{v\} \cup \{u\}$ is a dominating set of G. Hence, S is a 1-movable dominating set of G. Suppose $u \notin (V(G) \setminus S) \cap N_G(v)$. Then $u \in V(H)$. Take $y = u$. Hence, $S \setminus \{v\} \cup \{y\} = \{y\}$ is a dominating set of H. Thus, (ii) holds. Similarly, (iii) holds if $S \subseteq V(H)$.

For the converse, suppose (i) holds. By definition, S is a 1-movable perfect dominating set of G. Suppose (ii) holds. Suppose first that S is a 1-movable dominating set of G and $|S| = 1$. Let $S\{a\}$ for some $a \in V(G)$. Then S is a dominating set of $G+H$. Let $y \in V(G+H) \setminus S$. Then $|N_{G+H}(y) \cap S| = 1$. Thus, S is a perfect dominating set of of $G+H$. Since is S is a 1-movable perfect dominating set of $G+H$, there exists $b \in v(G) \setminus S) \cap N_G(a)$ such that $S \setminus \{a\} \cup \{b\} = \{b\}$ is a dominating set of G. Hence,$S \setminus \{a\} \cup \{b\}$ is a dominating set of $G+H$. Let $w \in V(G+H) \setminus S \setminus \{a\} \cup \{b\}$. Then $|N_{G+H}(w) \cap S \setminus \{a\} \cup \{b\}| = 1$. Thus, $S \setminus \{a\} \cup \{b\}$ is a dominating set of $G+H$. Suppose S is not a 1-movable dominating set of $G+H$. By assumption, there exists $y \in V(H)$ which dominates H. Thus, $S \setminus \{a\} \cup \{y\} = \{y\}$ is a dominating set of H and hence of $G+H$. Let $q \in V(G+H) \setminus S \setminus \{a\} \cup \{y\}$. Then $|N_{G+H}(q) \cap [S \setminus \{a\} \cup \{y\}]| = 1$. Hence, $S \setminus \{a\} \cup \{y\}$ is a perfect dominating set of $G+H$. Therefore, S is a 1-movable perfect domnating set of $G+H$. Similarly, if (iii) holds then S is a 1-movable perfect dominating set of $G+H$. □

Corollary 2.5 *Let G and H be connected nontrivial graphs. Then*

$$\gamma^1_{mp}(G+H) = \begin{cases} 1, \text{ if } \gamma(G) = 1 = \gamma(H) \text{ or } \gamma^1_m(G) = 1 \text{ or } \gamma^1_m(H) = 1 \\ |V(G+H)|, \text{ otherwise} \end{cases}$$

Theorem 2.6 *Let H be a connected nontrivial graph. Then $S \subseteq V(K_1+H)$ is a 1-movable perfect dominating set of K_1+H if and only if one of the following holds:*

(i) $S = V(K_1 + H)$.

(ii) $S = V(K_1)$ *and there exists* $y \in V(H)$ *which dominates* H.

(iii) S *is a dominating set in* H *with* $|S| = 1$.

Proof: Let $V(K_1) = \{x\}$ and suppose S is a 1-movable perfect dominating set of $K_1 + H$. If $S = V(K_1 + H)$, then (i) holds. Suppose $S \neq V(K_1 + H)$. Since S is a perfect dominating set of $K_1 + H$, $S = V(K_1)$ or $S \subseteq V(H)$. Suppose first that $S = V(K_1)$. Since S is a 1-movable perfect dominating set of $K_1 + H$ there exists $y \in V((K_1 + H) \setminus S) \cap N_{K_1+H}(x)$ such that $S \setminus \{x\} \cup \{y\} = \{y\}$ is a perfect dominating set of $K_1 + H$. Since $y \in V(H)$, $\{y\}$ is a dominating set of H. Hence (ii) holds. Suppose $S \neq V(K_1)$. Then $S \subseteq V(H)$ and S is a dominating set of H. Since S is a perfect dominating set of $K_1 + H$, $|S| = 1$. Hence (iii) holds.

For the converse, suppose (i) holds. By definition, S is a 1-movable perfect dominating set of G. Suppose (ii) holds. Then S is a dominating set of $K_1 + H$. Let $w \in V(H)$. Then $|N_{K_1+H}(w) \cap S| = 1$. Hence,$S$ is a perfect dominating set of $K_1 + H$. Now, $S \setminus \{x\} \cup \{y\} = \{y\}$ is a dominating set of H and hence of K_1+H. Let $q \in V(K_1+H) \setminus S \setminus \{x\} \cup \{y\}$. Then $|N_{K_1+H}(q) \cap S \setminus \{x\} \cup \{y\}| = 1$. Hence, $S \setminus \{x\} \cup \{y\}$ is a perfect dominating set of H and hence of $K_1 + H$. Thus, S is a 1-movable perfect dominating set of $K_1 + H$. Suppose (iii) holds. Let $S = \{y\}$ for some $y \in V(H)$. Then S is a dominating set of $K_1 + H$. Let $z \in V(K_1 + H) \setminus S$. Then $|N_{K_1+H}((z) \cap S| = 1$. Hence, S is a perfect dominating set of $K_1 + H$. Now, $S \setminus \{y\} \cup \{x\} = \{x\}$ is a dominating set of $K_1 + H$. Let $p \in V(H)$. Then $|N_{K_1+H}(p) \cap S \setminus \{y\} \cup \{x\}| = 1$. Hence, $S \setminus \{y\} \cup \{x\} = \{x\}$ is a perfect dominating set of $K_1 + H$.Therefore, S is a 1-movable perfect dominating set of $K_1 + H$. □

Corollary 2.7 *Let H be a connected nontrivial graph.*

$$\gamma^1_{mp}(K_1 + H) = \begin{cases} 1, \ \textit{if}\, \gamma(H) = 1 \\ |V(K_1 + H)|, \ \textit{otherwise} \end{cases}$$

Theorem 2.8 *Let G and H be connected nontrivial graphs. Then $C \subseteq V(G \circ H)$ is a 1-movable perfect dominating set of $G + H$ if and only if $C = V(G \circ H)$ or $C = \bigcup_{v \in V(G)} D_v$ with $|D_v| = 1$ and D_v is a 1-movable dominating set in $H^v$for all $v \in V(G)$.*

Proof: Suppose that C is a 1-movable perfect dominating set of G. Suppose $C = V(G \circ H)$. Then were are done. Suppose $C \neq V(G \circ H)$. Let $x \in C$ and suppose $x \in C \cap V(G)$. Then there exists $y \in N(x)$ such that $|N(y) \cap C| = 1$. Thus $N_{G \circ H}(z) \cap C = \varnothing$ for all $z \in V(H^y)$. This contradicts the assumption. Hence $x \in D_v$ for all $v \in V(G)$. Thus, $C = \bigcup_{v \in V(G)} D_v$. Since C is a perfect dominating set, $|N_G(w) \cap D_v| = 1$ for all $w \in V(G)$. This means that $|D_v| = 1$. Since C is a 1-movable perfect dominating set of $G \circ H$, D_v is a 1-movable dominating set of H^v for all $v \in V(G)$.

For the converse, Suppose that $C = V(G \circ H)$. By definition, C is a 1-movable perfect dominating set of $G \circ H$. Suppose that $C = \bigcup_{v \in V(G)} D_v$, where $|D_v| = 1$ and D_v is a 1-movable dominating set of H^v for each $v \in V(G)$. Clearly, C is a perfect dominating set of $G \circ H$. Let $v \in C$. Since $|D_v| = 1$ for all $v \in V(G)$, $C \setminus \{v\} \cup \{u\}$ are perfect dominating sets of $G \circ H$ for some $u \in V((G \circ H) \setminus C) \cap N(v)$. Therefore, C is a 1-movable perfect dominating set of $G \circ H$. □

Corollary 2.9 *For every connected nontrivial graphs G and H,*

$$\gamma^1_{mp}(G \circ H) = \begin{cases} 1, \ \textit{if}\, \gamma^1_m(H) = 1 \\ |V(G \circ H)|, \ \textit{otherwise} \end{cases}$$

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