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GENERAL MULTIPLICATIVE ve-DEGREE INDICES OF DOMINATING OXIDE AND REGULAR TRIANGULATE OXIDE NETWORKS

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Abstract: Recently, the ve-degree concept is defined in Graph Theory. In this paper, we propose the general first and second multiplicative ve-degree indices and the first and second multiplicative hyper-ve-degree indices of a molecular graph. Furthermore we compute the multiplicative ve-degree indices, the multiplicative hyper-ve-degree indices and general first and second multiplicative ve-degree indices for dominating oxide and regular triangulate oxide networks.

Keywords: multiplicative ve-degree indices, multiplicative hyper-ve-degree indices, dominating oxide network, regular triangulate oxide network.

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*90.

1. INTRODUCTION

All graphs considered here are finite, simple connected graphs. Let G be a connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Let S_v denote the sum of the degrees of all vertices adjacent to a vertex v. For convenience, we call S_v as sum degree. The set of all vertices which adjacent to v is called the open neighborhood of v and denoted by $N(v)$. The closed neighborhood set of v is the set $N[v] = N(v) \cup \{v\}$.

Chemical Graph Theory has an important effect on the development of Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have quantitative structure activity (QSAR) and quantitative structure property QSPR) study [1].

Recently, Chellali et al. [2] defined the ve-degree concept in Graph Theory as follows:

Definition . The ve-degree $d_{ve}(v)$ of a vertex v in a connected graph G is the number of different edges that incident to any vertex from the closed neighborhood of v.

Recently, Sahil and Ediz [3] defined the ve-Narumi-Katayama index of a graph G and it is defined as

$$NK_{ve}(G) = \prod_{v\in V(G)} d_{ve}(v).$$

We now introduce the first and second multiplicative ve-degree indices, first multiplicative alpha ve-degree index, first and second multiplicative hyper-ve-degree indices and general first and second multiplicative ve-degree indices of a graph as follows:

The first and second multiplicative ve-degree indices of a graph G are defined as

$$VeII_1(G) = \prod_{uv\in E(G)} \left(d_{ve}(u) + d_{ve}(v)\right) \quad \text{and} \quad VeII_2(G) = \prod_{uv\in E(G)} d_{ve}(u)d_{ve}(v).$$

The first multiplicative alpha ve-degree index of a graph G is defined as

$$VeII_1^*(G) = \prod_{v\in V(G)} d_{ve}(v)^2.$$

The first and second multiplicative hyper-ve-degree indices of a graph G are defined as

$$HVeII_1(G) = \prod_{uv\in E(G)} \left(d_{ve}(u) + d_{ve}(v)\right)^2 \quad \text{and} \quad HVeII_2(G) = \prod_{uv\in E(G)} \left(d_{ve}(u)d_{ve}(v)\right)^2.$$

The general first and second multiplicative ve-degree indices of a graph G are defined as

$$VeII_1^a(G) = \prod_{uv\in E(G)} \left[d_{ve}(u) + d_{ve}(v)\right]^a, \tag{1}$$

and $$VeII_2^a(G) = \prod_{uv \in E(G)} \left[d_{ve}(u) d_{ve}(v) \right]^a. \quad (2)$$

Recently some ve-degree topological indices were studied, for example, in [4, 5, 6, 7].

We consider the families of dominating oxide networks and regular triangulate oxide networks [8, 9]. In this paper, we compute the first and second multiplicative ve-degree, the first and second multiplicative hyper-ve-degree, general first and second multiplicative ve-degree indices for dominating oxide networks (*DOX*) and regular triangulate oxide networks (*RTOX*).

2. RESULTS FOR DOMINATING OXIDE NETWORKS

The family of dominating oxide networks is symbolized by *DOX*(*n*). The molecular structure of a dominating oxide network is presented in Figure 1.

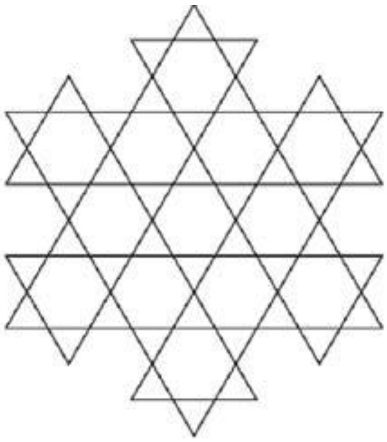

Figure 1. The structure of a dominating oxide network

In [8], Ediz obtained the partition of the edges with respect to their sum degree of end vertices of dominating oxide networks in Table 1.

Table 1

(S_u, S_v)	(8, 12)	(8, 14)	(12, 12)	(12, 14)	(14, 16)	(16, 16)
Number of edges	$12n$	$12n-12$	6	$12n-12$	$24n-24$	$54n^2-114n+60$

Also he obtained the ve-degree partition of the end vertices of edges for dominating oxide networks in Table 2. Table

2. The ve-degree of the end vertices of edges for *DOX* networks

$(d_{ve}(u), d_{ve}(v))$	(7, 10)	(7, 12)	(10, 10)	(10, 12)	(12, 14)	(14, 14)
Number of edges	$12n$	$12n-12$	6	$12n-12$	$24n-24$	$54n^2-114n+60$

Theorem 1. The general first multiplicative ve-degree index of a dominating oxide network *DOX*(*n*) is

$$VeII_1^a(DOX(n)) = 17^{12na} \times 19^{(12n-12)a} \times 20^{6a} \times 22^{(12n-12)a} \times 26^{(24n-24)a} \times 28^{(54n^2-114n+60)a}. \quad (3)$$

Proof: Let *G* be the molecular graph of a dominating oxide network *DOX*(*n*). By using equation (1) and Table 2, we deduce

$$VeII_1^a(DOX(n)) = \prod_{uv \in E(G)} \left[d_{ve}(u) + d_{ve}(v) \right]^a$$

$$= \left[(7+10)^a\right]^{12n} \times \left[(7+12)^a\right]^{12n-12} \times \left[(10+10)^a\right]^{6} \times \left[(10+12)^a\right]^{12n-12}$$

$$\times \left[(12+14)^a\right]^{24n-24} \times \left[(14+14)^a\right]^{54n^2-114n+60}$$

$$= 17^{12na} \times 19^{(12n-12)a} \times 20^{6a} \times 22^{(12n-12)a} \times 26^{(24n-24)a} \times 28^{(54n^2-114n+60)a}.$$

We establish the following results by Theorem 1.

Corollary 1.1. Let $DOX(n)$ be the family of dominating oxide networks. Then the first multiplicative ve-degree index of $DOX(n)$ is

$$VeII_1(DOX(n)) = 17^{12n} \times 19^{12n-12} \times 20^{6} \times 22^{12n-12} \times 26^{24n-24} \times 28^{54n^2-114n+60}.$$

Proof: Put a = 1 in equation (3), we get the desired result.

Corollary 1.2. Let $DOX(n)$ be the family of dominating oxide networks. Then the first multiplicative hyper-ve-degree index of $DOX(n)$ is

$$HVeII_1(DOX(n)) = 17^{24n} \times 19^{24n-24} \times 20^{12} \times 22^{24n-24} \times 26^{48n-48} \times 28^{108n^2-228n+120}.$$

Proof: Put a = 2 in equation (3), we get the desired result.

In the following theorem, we compute the general second multiplicative ve-degree index of $DOX(n)$.

Theorem 2. The general second multiplicative ve-degree index of a dominating oxide network $DOX(n)$ is

$$VeII_2^a(DOX(n)) = 70^{12na} \times 84^{(12n-12)a} \times 100^{6a} \times 120^{(12n-12)a} \times 168^{(24n-24)a} \times 196^{(54n^2-114n+60)a}. \quad (4)$$

Proof: Let G be the molecular graph of a dominating oxide network $DOX(n)$. By using equation (2) and Table 2, we derive

$$VeII_2^a(DOX(n)) = \prod_{uv \in E(G)} \left[d_{ve}(u)d_{ve}(v)\right]^a$$

$$= \left[(7\times 10)^a\right]^{12n} \times \left[(7\times 12)^a\right]^{12n-12} \times \left[(10\times 10)^a\right]^{6} \times \left[(10\times 12)^a\right]^{12n-12}$$

$$\times \left[(12\times 14)^a\right]^{24n-24} \times \left[(14\times 14)^a\right]^{54n^2-114n+30}.$$

$$= 70^{12na} \times 84^{(12n-12)a} \times 100^{6a} \times 120^{(12n-12)a} \times 168^{(24n-24)a} \times 196^{(54n^2-114n+60)a}.$$

We obtain the following results by Theorem 2.

Corollary 2.1. The second multiplicative ve-degree index of a dominating oxide network $DOX(n)$ is

$$VeII_2(DOX(n)) = 70^{12n} \times 84^{12n-12} \times 100^{6} \times 120^{12n-12} \times 168^{24n-24} \times 196^{54n^2-114n+60}.$$

Proof: Put a = 1 in equation (4), we get the desired result.

Corollary 2.2. The second multiplicative hyper-ve-degree index of a dominating oxide network $DOX(n)$ is

$$HVeII_2(DOX(n)) = 70^{24n} \times 84^{24n-24} \times 100^{12} \times 120^{24n-24} \times 168^{48n-48} \times 196^{108n^2-228n+120}.$$

Proof: Put a = 2 in equation (4), we get the desired result.

3. RESULTS FOR REGULAR TRIANGULATE OXIDE NETWORKS *RTOX*(*n*)

The family of regular triangulate oxide networks is denoted by $RTOX(n)$, $n \geq 3$. The molecular structure of a regular triangulate oxide network is shown in Figure 2.

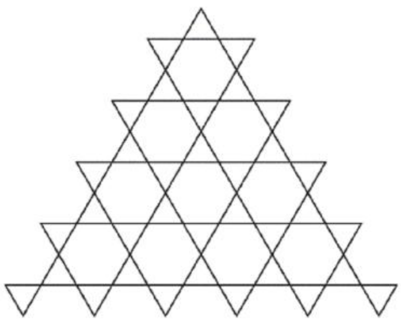

Figure 2. The structure of a regular triangulate oxide network

Ediz [8] obtained the partition of the edges with respect to their sum degree of end vertices of regular triangulate oxide networks in Table 3.

Table 3

(S_u, S_v)	(6,6)	(6,12)	(8,12)	(8,14)	(12,12)	(12,14)	(14,14)	(14,16)	(16, 16)
Number of edges	2	4	4	$6n-8$	1	6	$6n-9$	$6n-12$	$3n^2-12n+12$

Also he obtained the ve-degree partition of the end vertices of edges for regular triangulate oxide networks in Table 4.

Table 4

$(d_{ve}(u), d_{ve}(v))$	(5,5)	(5,10)	(7,10)	(7,12)	(10,10)	(10,12)	(12,12)	(12,14)	(14,14)
Number of edges	2	4	4	$6n-8$	1	6	$6n-9$	$6n-12$	$3n^2-12n+12$

In the following theorem, we compute the general first multiplicative ve-degree index of $RTOX(n)$.

Theorem 3. The general first multiplicative ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$VeII_1^a(RTOX(n)) = 10^{2a} \times 15^{4a} \times 17^{4a} \times 19^{(6n-8)a} \times 20^{a} \times 22^{6a} \times 24^{(6n-9)a} \times 26^{(6n-12)a} \times 28^{(3n^2-12n+12)a}. \quad (5)$$

Proof: Let G be the graph of a regular triangulate oxide network ($RTOX(n)$). By using equation (1) and Table 4, we deduce

$$VeII_1^a(RTOX(n)) = \prod_{uv \in E(G)} \left[d_{ve}(u) + d_{ve}(v)\right]^a$$

$$= \left[(5+5)^a\right]^2 \times \left[(5+10)^a\right]^4 \times \left[(7+10)^a\right]^4 \times \left[(7+12)^a\right]^{6n-8} \times \left[(10+10)^a\right]^1 \times \left[(10+12)^a\right]^6$$

$$\times \left[(12+12)^a\right]^{6n-9} \times \left[(12+14)^a\right]^{6n-12} \times \left[(14+14)^a\right]^{3n^2-12n+12}.$$

$$= 10^{2a} \times 15^{4a} \times 17^{4a} \times 19^{(6n-8)a} \times 20^{a} \times 22^{6a} \times 24^{(6n-9)a} \times 26^{(6n-12)a} \times 28^{(3n^2-12n+12)a}.$$

We obtain the following results by Theorem 3.

Corollary 3.1. The first multiplicative ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$VeII_1(RTOX(n)) = 10^2 \times 15^4 \times 17^4 \times 20 \times 22^6 \times 19^{6n-8} \times 24^{6n-9} \times 26^{6n-12} \times 28^{3n^2-12n+12}.$$

Proof: Put a = 1 in equation (5), we get the desired result.

Corollary 3.2. The first multiplicative hyper-ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$HVeII_1(RTOX(n)) = 10^4 \times 15^8 \times 17^8 \times 20^2 \times 22^{12} \times 19^{12n-16} \times 24^{12n-18} \times 26^{12n-24} \times 28^{6n^2-24n+24}.$$

Proof: Put a = 2 in equation (5), we get the desired result.

In the following theorem, we determine the general second multiplicative ve-degree index of $RTOX(n)$.

Theorem 4. The general second multiplicative ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$VeII_2^a(RTOX(n)) = 25^{2a} \times 50^{4a} \times 70^{4a} \times 84^{(6n-8)a} \times 100^{a} \times 120^{6a}$$

$$\times 144^{(6n-9)a} \times 168^{(6n-12)a} \times 196^{(3n^2-12n+12)a}. \quad (6)$$

Proof: Let G be the molecular graph of a regular triangulate oxide network $RTOX(n)$. By using equation (2) and Table 4, we derive

$$VeII_2^a(RTOX(n)) = \prod_{uv \in E(G)} \left[d_{ve}(u) d_{ve}(v)\right]^a$$

$$= \left[(5\times5)^a\right]^2 \times \left[(5\times10)^a\right]^4 \times \left[(7\times10)^a\right]^4 \times \left[(7\times12)^a\right]^{6n-8} \times \left[(10\times10)^a\right]^1 \times \left[(10\times12)^a\right]^6$$

$$\times \left[(12\times12)^a\right]^{6n-9} \times \left[(12\times14)^a\right]^{6n-12} \times \left[(14\times14)^a\right]^{3n^2-12n+12}.$$

$$= 25^{2a} \times 50^{4a} \times 70^{4a} \times 84^{(6n-8)a} \times 100^{a} \times 120^{6a} \times 144^{(6n-9)a} \times 168^{(6n-12)a} \times 196^{(3n^2-12n+12)a}.$$

We obtain the following results by Theorem 4.

Corollary 4.1. The second multiplicative ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$VeII_2(RTOX(n)) = 25^2 \times 50^4 \times 70^4 \times 84^{6n-8} \times 100^1 \times 120^6 \times 144^{6n-9} \times 168^{6n-12} \times 196^{3n^2-12n+12}.$$

Proof: Put a = 1 in equation (6), we get the desired result.

Corollary 2.2. The second multiplicative hyper-ve-degree index of a regular triangulate oxide network $RTOX(n)$ is

$$HVeII_2(RTOX(n)) = 25^4 \times 50^8 \times 70^8 \times 84^{12n-16} \times 100^2 \times 120^{12} \times 144^{12n-18} \times 168^{12n-24} \times 196^{6n^2-24n+24}.$$

Proof: Put a = 2 in equation (6), we get the desired result.

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