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ONE MODULO THREE HARMONIC MEAN LABELING OF SOME CYCLE-RELATED GRAPHS

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Abstract : Let $G = (V, E)$ be a graph with p vertices and q edges.A function $f: V(G) \to \{1,3, \dots 3q-2, 3q\}$ is called one modulo three harmonic mean labeling of G if f is injective and the induced function $f^*: E(G) \to \{1,4, \dots ,3q-2\}$ defined as $f^*(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ $\forall u, v \in E(G)$ is bijective.
A graph that admits one modulo three harmonic mean labeling is called one modulo three harmonic mean graph.
In this paper we prove the graph $T_n \odot K_1$, $A(T_n)$ K_1, $M(P_n)$,
C_n^{+t} are one modulo three harmonic mean graphs.

Keywords: one modulo three harmonic mean labeling , one modulo three harmonic mean graph.
Mathematics Subject Classification: 05C78

1 INTRODUCTION

We begin with finite, simple and undirected graph G=(V,E)with p vertices and q edges.For standard terminology and notations related to graph labeling,we refer to Gallian[1], S.S.Sandhya,S.Somasundaram and R.Ponraj introduced the concept of harmonic mean labeling in [2]and studied their behaviour in [3]. C.David Raj,S.S.Sandhya and C.Jayasekaran [4]introduced the concept of one modulo three harmonic mean graphs.We provide following definitions necessary for investigations.

Definition 1.1 If the vertices of a graph are assigned values subject to certain conditions then it is known as graph labeling.

Most of the graph labeling will have three common characteristics.

- A set of numbers from which vertex labels are chosen
- A rule that assigns a value to each edge
- A condition that this value has to satisfy.

Definition 1.2 Let $G = (V, E)$ be a graph with p vertices and q edges.A functionf is called one modulo three harmonic mean labeling of G if $f: V(G) \to \{1,3, \dots 3q-2, 3q\}$ is injective and the induced function $f^*: E(G) \to \{1,4, \dots ,3q-2\}$ defined as

$f^*(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$ $\forall$ u, v ∈ E(G) is bijective.A graph that admits one modulo three harmonic mean labeling is called one modulo three harmonic mean graph. The Corona G=G_1 G_2 of two graphsG_1andG_2isthegraph obtainedbytakingonecopyofG_1(withpvertices) and p copiesof G_2 and then joining the i^th vertexof G_1 to everyvertexof i^th copyof G_2 . C_n^+t denotestheclassofgraphsformedbyaddingasinglependantedgetotverticesofacycleoflengthn. ATriangularsnake T_n isobtainedfromapath u_1,u_2, ... ,u_n byjoining u_i and u_i+1 to a newvertex v_i ,for 1 i n-1 ;that is every edge of a path is replaced by a cycle

C_3 AnAlternateTriangularsnake A(T_n) isobtainedfromapath u_1,u_2 , ..., u_n byjoining u_i and u_i+1 alternativelytoanewvertex v_i ,for 1 i n-1 ; that is every alternateedge of a pathisreplaced by a cycle C_3 TheMiddlegraph M(G) ofagraphGisthegraphwhosevertexsetis V(G) E(G) andinwhichtwoelementsareadjacentiffeitherbothareadjacentedgesinGoroneoftheelementsisavertexandotherisanedgein T_n K_1 isonemodulothreeharmonicmeanlabeledgraph. Proof: Let u_1,u_2 , ..., u_n beverticesofpathP_n. Let, v_i bevertexjoinedtou_iandu_i+1; for$1 \leq i \leq n-1$;

w_i be vertex joined to v_i; for1 i n-1andz_ibevertexjoinedtou_i; for$1 \leq i \leq n$.

The resultant graph is $G = T_n \odot K_1$ with

$V(G) = \{u_i, z_i : 1 \leq i \leq n\} \cup \{v_i, w_i : 1 \leq i \leq n-1\}$

$E(G) = \{u_i z_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, \quad v_i w_i, \quad u_i v_i, \quad u_{i+1} v_i : 1 \leq i \leq n-1\}$

Then,

|V(G)|= p= 4n-2

|E(G)|= q= 5n-4

Define $f: V(G) \rightarrow \{1,2,\dots,3q-2,3q\}$ as follows:

$$
\begin{aligned}
f(u_1) &= 10 \\
f(u_2) &= 22 \\
f(u_i) &= 15i-14 \quad 3 \leq i \leq n \\
f(v_1) &= 6 \\
f(v_i) &= 15i-6 \quad 2 \leq i \leq n-1 \\
f(w_1) &= 3 \\
f(w_2) &= 15 \\
f(w_i) &= 15i-3 \quad 3 \leq i \leq n-1 \\
f(z_1) &= 1 \\
f(z_2) &= 12 \\
f(z_i) &= 15i-12 \quad 3 \leq i \leq n
\end{aligned}
$$

f induces a function $f^*: E(G) \rightarrow \{1,4,\dots,3q-2\}$ defined as follows:

$$
\begin{aligned}
f^*(u_i z_i) &= 15i-14 \quad 1 \leq i \leq n \\
f^*(u_1 v_1) &= 7 \\
f^*(u_2 v_2) &= 22 \\
f^*(u_i v_i) &= 15i-11 \quad 3 \leq i \leq n-1 \\
f^*(u_2 v_1) &= 10 \\
f^*(u_i v_{i-1}) &= 15i-17 \quad 3 \leq i \leq n \\
f^*(v_1 w_1) &= 4 \\
f^*(v_2 w_2) &= 19 \\
f^*(u_i w_i) &= 15i-5 \quad 3 \leq i \leq n-1 \\
f^*(u_1 u_2) &= 13 \\
f^*(u_2 u_3) &= 25 \\
f^*(u_i u_{i+1}) &= 15i-8 \quad 3 \leq i \leq n-1
\end{aligned}
$$

We observe that,

|V(G)|= p = 4n-2

|E(G)|= q = 5n-4

Hence f is a one modulo three harmonic mean labeling of G.

Example 1.3 One modulo three harmonic mean labeling of $T_6 \odot K_1$ is shown in figure 1.

Figure 1:

Theorem 1.4 $A(T_n) \odot K_1, n \geq 3$ is one modulo three harmonic mean labeled graph.

Proof: Let $G = A(T_n) \odot K_1$ be the graph with p vertices and q edges

We consider the following cases:

Case 1: The first triangle starts from the first vertex of path P_n

Subcase 11: n is even

Let $u_{1,}$ u_2, ... ,u_n be vertices of path P_n.Let

v_i be vertex joined to u_{2i-1} and u_{2i}, for 1 i n2;w_ibevertexjoinedtov_i, for $1 \leq i \leq \frac{n}{2}$ and

z_i be vertex joined to u_i, for 1 i n. Thus,V(G)={u_i,z_i:1 i n} { w_i,v_i:1 i n2}E(G)={u_iu_i+1:1 i n-1} {u_iz_i:1 i n }

{u_2iv_i , u_2i-1v_i ,v_iw_i :1 i n2}Then,|V(G)|=p= 3n

|E(G)|= q= 7n-22Definef:V(G) { 1,3,...,3q-2,3q}asfollows:lll f(u_1) =4

f(u_2) =19

f(u_2i-1) =21(i-1) 2 i n2

f(u_2i) =21i-3 2 i n2

f(z_1) =1

f(z_2i-1) =21i-18 2 i n2

f(z_2i) =21i-6 1 i n2

f(w_1) =3

f(w_i) =21i-9 2 i n2

f(v_1) =10

f(v_1) =21i-12 2 i n2

finducesafunctionf^*: E(G) {1,4, ..., 3q-2}definedasfollows:lll f^*(u_1z_1) = 1

f^*(u_2iz_2i) = 21i-5 1 i n2

f^*(u_2i-1z_2i-1) = 21i-20 2 i n2

f^*(u_1u_2) = 10

f^*(u_2iu_2i+1) = 21i-2 1 i n-22

f^*(u_2i-1u_2i) = 21i-14 2 i n2

f^*(u_1v_1) = 7

f^*(u_2i-1v_i) = 21i-17 2 i n2

f^*(u_2iv_i) = 21i-8 1 i n2

f^*(v_1w_1) = 4

f^*(v_iw_i) = 21i-11 2 i n2

We observe that, |V(G)| $= p = 3n - 2$ |E(G)| $= q =$ 7n-22 Hence, f is a one modulo three harmonic mean labeling of G. Subcase12: n is oddLet u_1, u_2, ... ,u_nbeverticesofpathP_n. Letv_ibevertexjoinedtou_2i-1andu_2i, for 1 i n-12;w_ibevertexjoinedtov_i, for 1 i n-12andz_ibevertexjoinedtou_ifor 1 i n. Thus,V(G)={u_i,z_i:1 i n} { w_i,v_i:1 i n-12}E(G)={u_iu_i+1:1 i n-1} {u_iz_i:1 i n }

{ u_2iv_i, u_2i-1v_i, v_iw_i :1 i n-12} Then, |V(G)| = p = 3n-1|E(G)| = q = 7n-52Definef:V(G) { 1,3,...,3q-2,3q}asfollows:lll f(u_1) =4

f(u_2) =19

f(u_2i-1) =21(i-1) 2 i n+12

f(u_2i) =21i-3 2 i n-12

f(z_1) =1

f(z_2i-1) =21i-18 2 i n+12

f(z_2i) =21i-6 1 i n-12

f(w_1) =3

f(w_i) =21i-9 2 i n-12

f(v_1) =10

f(v_i) =21i-12 2 i n-12

finducesafunction f^*:E(G) {1,4, ..., 3q-2}definedasfollows:

$$
\begin{aligned}
f^*(u_1z_1) &= 1 \\
f^*(u_{2i}z_{2i}) &= 21i-5 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i-1}z_{2i-1}) &= 21i-20 \quad 2 \le i \le \frac{n+1}{2} \\
f^*(u_1u_2) &= 10 \\
f^*(u_{2i}u_{2i+1}) &= 21i-2 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(u_{2i-1}u_{2i}) &= 21i-14 \quad 2 \le i \le \frac{n-1}{2} \\
f^*(u_1v_1) &= 7 \\
f^*(u_{2i-1}v_i) &= 21i-17 \quad 2 \le i \le \frac{n-1}{2} \\
f^*(u_{2i}v_i) &= 21i-8 \quad 1 \le i \le \frac{n-1}{2} \\
f^*(v_1w_1) &= 4 \\
f^*(v_iw_i) &= 21i-11 \quad 2 \le i \le \frac{n-1}{2}
\end{aligned}
$$

We observe that,

$$|V(G)| = p = 3n-1 |E(G)| = q = \frac{7n-5}{2}$$

Hence f is a one modulo three harmonic mean labeling of G.

Case 2: Thefirsttrianglestartsfromthesecondvertexofthepath

Subcase 21: n is even

Let, $u_{1,}u_2,\dots,u_n$ be vertices of path P_n.Let,

v_i be vertex joined to u_{2i} and u_{2i+1}, for $1 \le i \le \frac{n-2}{2}$;

w_i be vertex joined to v_i, for $1 \le i \le \frac{n-2}{2}$ and

z_i be vertex joined to u_i, for $1 \le i \le n$.

Thus

$$V(G) = \{u_i, z_i : 1 \le i \le n\} \cup \{w_i, v_i : 1 \le i \le \frac{n-2}{2}\}$$

$$E(G) = \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_iz_i : 1 \le i \le n\} \cup \{u_{2i}v_i, \quad u_{2i+1}v_i, v_iw_i : 1 \le i \le \frac{n-2}{2}\}$$

Then

$$|V(G)| = p = 3n-2$$

$|E(G)| = q = \frac{7n-8}{2}$

Define $f: V(G) \to \{1,3,\dots,3q-2,3q\}$as follows:

$$
\begin{aligned}
f(u_1) &= 3\\
f(u_2) &= 12\\
f(u_{2i-1}) &= 21i-18 \quad 2\le i\le \frac{n}{2}\\
f(u_{2i}) &= 21i-15 \quad 2\le i\le \frac{n}{2}\\
f(z_1) &= 1\\
f(z_{2i-1}) &= 21i-21 \quad 2\le i\le \frac{n}{2}\\
f(z_{2i}) &= 21i-12 \quad 1\le i\le \frac{n}{2}\\
f(w_1) &= 4\\
f(w_i) &= 21i-3 \quad 2\le i\le \frac{n-2}{2}\\
f(v_1) &= 16\\
f(v_i) &= 21i-6 \quad 2\le i\le \frac{n-2}{2}
\end{aligned}
$$

f induces a function $f^*: E(G) \to \{1,4,\dots,3q-2\}$ defined as follows:

$$
\begin{aligned}
f^*(u_2z_2) &= 4\\
f^*(u_{2i}z_{2i}) &= 21i-14 \quad 2\le i\le \frac{n}{2}\\
f^*(u_{2i-1}z_{2i-1}) &= 21i-20 \quad 1\le i\le \frac{n}{2}\\
f^*(u_1u_2) &= 10\\
f^*(u_2u_3) &= 16\\
f^*(u_{2i}u_{2i+1}) &= 21i-8 \quad 2\le i\le \frac{n-2}{2}\\
f^*(u_{2i-1}u_{2i}) &= 21i-17 \quad 2\le i\le \frac{n}{2}\\
f^*(u_2v_1) &= 13\\
f^*(u_3v_1) &= 19\\
f^*(u_{2i+1}v_i) &= 21i-2 \quad 2\le i\le \frac{n-2}{2}\\
f^*(u_{2i}v_i) &= 21i-11 \quad 2\le i\le \frac{n-2}{2}\\
f^*(v_1w_1) &= 7\\
f^*(v_iw_i) &= 21i-5 \quad 2\le i\le \frac{n-2}{2}
\end{aligned}
$$

We observe that,

$$|V(G)| = p = 3n-2$$
$$|E(G)| = q = \frac{7n-8}{2}$$

Hence f is a one modulo three harmonic mean labeling of G.

Subcase 22: n is odd

Let, $u_1, u_2, \dots, u_n$ be vertices of path P_n. Let,

v_i be vertex joined to u_{2i} and u_{2i+1}, for $1 \le i \le \frac{n-1}{2}$;

w_i be vertex joined to v_i, for $1 \le i \le \frac{n-1}{2}$ and

z_i be vertex joined to u_i, for $1 \le i \le n$.

Thus,

$$V(G) = \{u_i, z_i : 1 \le i \le n\} \cup \{w_i, v_i : 1 \le i \le \frac{n-1}{2}\}$$

$$E(G) = \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_iz_i : 1 \le i \le n\} \cup \{u_{2i}v_i, \quad u_{2i+1}v_i, v_iw_i : 1 \le i \le \frac{n-1}{2}\}$$

Then,

$$|V(G)| = p = 3n - 1$$

$|E(G)| = q = \frac{7n-5}{2}$

Define $f: V(G) \rightarrow \{1,3, \dots ,3q - 2,3q\}$ as follows:

$$f(u_1) = 3$$
$$f(u_2) = 12$$
$$f(u_{2i-1}) = 21i - 18 \quad 2 \leq i \leq \frac{n+1}{2}$$
$$f(u_{2i}) = 21i - 15 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f(z_1) = 1$$
$$f(z_{2i-1}) = 21i - 21 \quad 2 \leq i \leq \frac{n+1}{2}$$
$$f(z_{2i}) = 21i - 12 \quad 1 \leq i \leq \frac{n-1}{2}$$
$$f(w_1) = 4$$
$$f(w_i) = 21i - 3 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f(v_1) = 16$$
$$f(v_i) = 21i - 6 \quad 2 \leq i \leq \frac{n-1}{2}$$

f induces a function $f^*: E(G) \rightarrow \{1,4, \dots ,3q - 2\}$ defined as follows:

$$f^*(u_2z_2) = 4$$
$$f^*(u_{2i}z_{2i}) = 21i - 14 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f^*(u_{2i-1}z_{2i-1}) = 21i - 20 \quad 1 \leq i \leq \frac{n+1}{2}$$
$$f^*(u_1u_2) = 10$$
$$f^*(u_2u_3) = 16$$
$$f^*(u_{2i}u_{2i+1}) = 21i - 8 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f^*(u_{2i-1}u_{2i}) = 21i - 17 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f^*(u_2v_1) = 13$$
$$f^*(u_3v_1) = 19$$
$$f^*(u_{2i+1}v_i) = 21i - 2 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f^*(u_{2i}v_i) = 21i - 11 \quad 2 \leq i \leq \frac{n-1}{2}$$
$$f^*(v_1w_1) = 7$$
$$f^*(v_iw_i) = 21i - 5 \quad 2 \leq i \leq \frac{n-1}{2}$$

We observe that,

$$|V(G)| = p = 3n - 1$$
$$|E(G)| = q = \frac{7n-5}{2}$$

Hence f is a one modulo three harmonic mean labeling of G.

Example 1.5 One modulo three harmonic mean labeling of $A(T_9) \odot K_1$, where the first triangle starts from first vertex u_1 is shown in figure 2

Figure 2:

Example 1.6 One modulo three harmonic mean labeling of $A(T_9) \odot K_1$,
where the first triangle starts from second vertex u_2 is shown in figure 3

Figure 3:

Theorem 1.7 The Midddle graph $M(P_n)$of path P_n
admits a one modulo three harmonic mean graph for $n \geq 4$

Proof: Let $G = M(P_n)$ be the graph with p vertices and q edges.Let,

$$V(G) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n-1\}$$

$$E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-2\} \cup \{v_i u_i : 1 \leq i \leq n-1\} \cup \{v_i u_{i-1} : 2 \leq i \leq n\}$$

Then

$$|V(G)| = p = 2n - 1$$
$$|E(G)| = q = 3n - 4$$

Define $f: V(G) \rightarrow \{1,3, \ldots, 3q-2, 3q\}$as follows:

$$f(v_1) = 1$$
$$f(v_2) = 6$$
$$f(v_3) = 15$$
$$f(v_n) = 3$$
$$f(v_i) = 9i - 8 \quad 4 \leq i \leq n-1$$
$$f(u_1) = 10$$
$$f(u_2) = 19$$
$$f(u_i) = 9i - 3 \quad 3 \leq i \leq n-1$$

f induces a function $f^*: E(G) \rightarrow \{1,4, \ldots, 3q-2\}$ defined as follows:

$$f^*(v_1 u_1) = 1$$
$$f^*(v_2 u_2) = 10$$
$$f^*(v_3 u_3) = 19$$
$$f^*(v_i u_i) = 9i - 5 \quad 4 \leq i \leq n-1$$
$$f^*(v_2 u_1) = 7$$
$$f^*(v_{i+1} u_i) = 9i - 2 \quad 2 \leq i \leq n-2$$
$$f^*(v_n u_{n-1}) = 4$$
$$f^*(u_1 u_2) = 13$$
$$f^*(u_2 u_3) = 22$$
$$f^*(u_i u_{i+1}) = 9i + 1 \quad 3 \leq i \leq n-2$$

We observe that,

$$|V(G)| = p = 2n - 1$$
$$|E(G)| = q = 3n - 4$$

Hence we conclude $M(P_n), n \geq 4$, admits one modulo three harmonic mean labeling.

Example 1.8 One modulo three harmonic mean labeling of $M(P_6)$ is shown in figure 4

Figure 4:

Theorem 1.9 Let C_n^{+t} be the graph formed by adding t pendant edges at t adjacent vertices of a cycle C_n, $n \geq 4$. Then C_n^{+t} is one modulo three Harmonic mean labeling if

i) $3(n+t) \leq 102$ for n & t both even

ii) $3(n+t) \leq 99$ otherwise

Proof: Let $G = C_n^{+t}$ be the graph with p vertices and q edges. Let,

$$V(G) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq t\}$$

$$E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq t\} \cup \{u_1 u_n\}$$

Then,

$$|V(G)| = p = n + t$$
$$|E(G)| = q = n + t$$

Case 1: t is odd and $3(n+t) \leq 99$

Define f:V(G) { 1,3,...,3q-2,3q} as follows:lll f(u_1) =13
f(u_2) =15
f(u_2i-1) =12i-6 2 i t+12
f(u_2i) =12i-3 2 i t+12
f(u_i) =3i+27 t+1 i n
f(v_1) =3
f(v_2) =4
f(v_3) =7
f(v_4) =1
f(v_2i+1) =12i+3 2 i t-12
f(v_2i) = 12i 3 i t-12
f induces a function f^*: E(G) {1,4, ..., 3q-2} defined as follows:lll f^*(u_1u_2) = 13
f^*(u_2u_3) = 16
f^*(u_iu_i+1) = 6i+1 3 i t
f^*(u_iu_i+1) = 3i+28 t+1 i n-1
f^*(u_nu_1) = 22
f^*(u_1v_1) = 4
f^*(u_2v_2) = 7
f^*(u_3v_3) = 10
f^*(u_4v_4) = 1
f^*(u_2i+1v_2i+1) = 12i+4 2 i t-12
f^*(u_2iv_2i) = 12i-2 3 i t-12

Weobservethat,|V(G)| = p = n+t|E(G)| = q = n+tThus all edge labels are distinct. Case2: t is even, n is odd and $3(n+t) \leq 99$Define f:V(G)→ $\{1,3,\dots,3q-2,3q\}$ asfollows:lll f(u_1) =13
f(u_2) =15
f(u_2i-1) =12i-6 2 i t+22
f(u_2i) =12i-3 2 i t2
f(u_2i) =6i+3t-3 t+22 i n-32
f(u_2i+1) =6i+3t+6 t+22 i n-32
f(u_n) =3(n+t)
f(u_n-1) =3(n+t)-5
f(v_1) =3
f(v_2) =4
f(v_3) =7
f(v_4) =1
f(v_2i-1) =12i-9 3 i t2
f(v_2i) = 12i 3 i t2
f induces a function $f^*: E(G) \rightarrow \{1,4,\dots,3q-2\}$ defined as follows:lll f^*(u_1u_2) = 13
f^*(u_2u_3) = 16
f^*(u_iu_i+1) = 6i+1 3 i t-1
f^*(u_iu_i+1) = 3i+3t+1 t i n-1
f^*(u_nu_1) = 22
f^*(u_1v_1) = 4
f^*(u_2v_2) = 7
f^*(u_3v_3) = 10
f^*(u_4v_4) = 1
f^*(u_iv_i) = 6i-2 5 i t

We observe that, $|V(G)| = p = n + t$
$|E(G)| = q = n + t$

Thus all edge labels are dictinct.

Case 3: t is even, n is even and $3(n+t) \leq 102$

Define $f: V(G) \rightarrow \{1,3,\ldots,3q-2,3q\}$ as follows:

$$
\begin{array}{lll}
f(u_1) & = 13 & \\
f(u_2) & = 15 & \\
f(u_{2i-1}) & = 12i-6 & 2 \leq i \leq \frac{t+2}{2} \\
f(u_{2i}) & = 12i-3 & 2 \leq i \leq \frac{t}{2} \\
f(u_{2i}) & = 6i+21 & \frac{t+2}{2} \leq i \leq \frac{n}{2} \\
f(u_{2i+1}) & = 6i+30 & \frac{t+2}{2} \leq i \leq \frac{n-2}{2} \\
f(v_1) & = 3 & \\
f(v_2) & = 4 & \\
f(v_3) & = 7 & \\
f(v_4) & = 1 & \\
f(v_{2i-1}) & = 12i-9 & 3 \leq i \leq \frac{t}{2} \\
f(v_{2i}) & = 12i & 3 \leq i \leq \frac{t}{2}
\end{array}
$$

f induces a function $f^*: E(G) \rightarrow \{1,4,\ldots,3q-2\}$ defined as follows:

$$
\begin{array}{lll}
f^*(u_1u_2) & = 13 & \\
f^*(u_2u_3) & = 16 & \\
f^*(u_iu_{i+1}) & = 6i+1 & 3 \leq i \leq t \\
f^*(u_iu_{i+1}) & = 3i+25 & t+1 \leq i \leq n-1 \\
f^*(u_nu_1) & = 22 & \\
f^*(u_1v_1) & = 4 & \\
f^*(u_2v_2) & = 7 & \\
f^*(u_3v_3) & = 10 & \\
f^*(u_4v_4) & = 1 & \\
f^*(u_{2i}v_{2i}) & = 12i-2 & 3 \leq i \leq \frac{t}{2} \\
f^*(u_{2i-1}v_{2i-1}) & = 12i-8 & 3 \leq i \leq \frac{t}{2}
\end{array}
$$

We observe that,

$$|V(G)| = p = n+t$$
$$|E(G)| = q = n+t$$

Thus all edge labels are distinct.

Example 1.10 One modulo three harmonic mean labeling of C_{14}^{+9} where the 9 edges are adjacent is shown in figure 5

Figure 5:

Example 1.11 One modulo three harmonic mean labeling of C_{19}^{+10} where the 10 edges are adjacent is shown in figure 6

Figure 6:

Example 1.12 One modulo three harmonic mean labeling of C_{14}^{+8} where the 8 edges are adjacent is shown in figure 7

Figure 7:

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