SUPER MAGIC OF SOME CONNECTED GRAPHS \( P_n \times 2N_6, P_n \times 2N_7, \) AND \( P_n \times 2N_8 \)

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Abstract: Krishnappa, Kishore Kothapalli and Venkaiah [2009] analyzed (1) \( K_n \), n odd, admits a vertex magic total labeling; (2) There is a vertex magic total labeling for \( K_{n+2} \), for all \( n \equiv 2 \pmod{4} \); (3) There is a vertex magic total labeling for \( K_{n+3} \), for all \( n \equiv 0 \pmod{8} \). Subbiaha, and Pandimadevi [2014] found (1) every 2r-regular graph has a 2k-factor for every integer \( k, 0 < k < r \); (2) An even regular graph \( G \) of odd order is not 2-factor super magic decomposable, when the number of factors \( h \) is even; (3) An even regular graph \( G \) of odd order is 2-factor super magic decomposable if and only if \( h \) is odd, where \( h \) is the number of 2-factors of \( G \). Selvaraju, Balaganesan, Renuka [2013] made the contributions (1) \( D_2(P_n) \) is an even sequential harmonious graph; (2) \( D_2(K_{1,n}) \) is an even sequential harmonious graph; \( S_pl(P_n) \) is an even sequential graph; (4) The \( Spl(K_{1,n}) \) is an even sequential harmonious graph; (5) \( Spl(P_n) \) is odd graceful graph; (6) \( D_2(K_{1,n}) \) is felicitous graph.

The aim of the paper is to find graceful labeling for the graphs \( P_n \times 2N_6, P_n \times 2N_7, \) and \( P_n \times 2N_8 \).

Keywords: Magic graph, super magic graph

Section 1: Previous works and related contributions

INTRODUCTION:

Hegde[2003] contributed (1) A (p, q)-graph \( G \) is magic with minimum magic strength \( M(G) \) if and only if it is magic with maximum magic strength \( M(G) = 3(p+q+1) - m(G) \); (2) A path \( P_n \) of \( n \) vertices \((n \geq 1)\), is strong magic if and only if \( n = 2 \). Further for all \( n = 2 \), \( P_n \) is ideal magic; (3) The graph \( bistar B_{n,n} \) is ideal magic for all \( n \geq 1 \); (4) All cycles are ideal-magic; (5) The graph \( W^{(t,3)} \) is weak-magic for all \( t \geq 1 \)

Jaroslav Ivan [2009] gave the results (1) Let \( G \) be a dregular bipartite graph of order 8k. The complement of \( G \) is a supermagic graph if and only if \( d \) is odd; (2) Let \( G \) be a dregular bipartite graph of order 2n, whereon is even and \( d \) is even. The complement of \( G \) is supermagic if and only if \( (n,d) = (3,2) \). (This dot means not equal; (3) Let \( G \) be a dregular bipartite graph of order 2n with parts \( U_1 \) and \( U_2 \). If \( n \geq 5 \) and \( d \) are odd and \( G(U_1,U_2) \) is a Hamiltonian graph, then the complement of \( G \) is a supermagic graph; (4) Let \( G_1, G_2 \) be disjoint dregular Hamiltonian graphs of order \( n \). If \( d \geq 4 \) is even and \( n \) is odd, then the join \( G_1 \oplus G_2 \) is a supermagic graph; (5) Let \( G_i, i \in \{1,2\}, \) be a dregular Hamiltonian graph of order \( n \). If \( 4 \leq d_i \leq 0 \pmod{4} \), \( d_1 = d_2 + 2 \), and \( n \) is odd, then the join \( G_1 \oplus G_2 \) is a supermagic graph.

Section 2 – Super magic labeling

The following definition are stated now:

Definition (Magic labeling) 2.1: A graph \( G(V,E) \) with \( p \) vertices and \( q \) edges is magic if there is a bijection \( f : V \cup E \to \{1,2,...,(p+q)\} \) satisfying the condition that \( f(u) + f(v) + f(uv) \) is a constant, where \( uv \) is an edge joining the vertices \( u \) and \( v \). The constant is called as magic number.

Definition 2.2: A (p,q) graph \( G \) is called super magic if there is a bijective map \( f : V(G) \to \{1,2,...,p\} \) and the same map is a bijective map \( f : E(G) \to \{p+1,p+2,...,p+q\} \) satisfying...
the condition that \( f(u) + f(v) + f(uv) \) is a constant for all edges in the graph \( G \). The constant is called a super magic number of the graph \( G \).

**Definition 2.3:** Let \( P_n \) be a connected graph whose vertex set is \( \{V_1, V_2, \ldots, V_n, U_1, U_2, \ldots, U_{2n}, W_1, W_2, \ldots, W_{2n}, S_1, S_2, \ldots, S_{2n} \} \) and edge set is \( \{V_iV_{i+1} : i = 1 \text{ to } n \} \cup \{V_iU_{i+1}, V_iU_{i+2}: i = 1 \text{ to } n \} \cup \{U_iW_i : i = 1 \text{ to } 2n \} \cup \{W_tS_{i+1} : i = 1 \text{ to } 2n \} \cup \{s_{i+1}t_i : i = 1 \text{ to } 2n \} \cup \{t_{i+1}x_i : i = 1 \text{ to } 2n \} \). Here edge set of \( P_n \) is \( \{V_iV_{i+1} : i = 1 \text{ to } n \} \), and edge set of \( 2nP_6 \) is \( \{V_iU_{i+1} : i = 1 \text{ to } n \} \cup \{V_iU_{i+2}, V_iU_{i+3} : i = 1 \text{ to } n \} \cup \{U_iW_i : i = 1 \text{ to } 2n \} \cup \{W_tS_{i+1} : i = 1 \text{ to } 2n \} \cup \{s_{i+1}t_i : i = 1 \text{ to } 2n \} \cup \{t_{i+1}x_i : i = 1 \text{ to } 2n \} \).

**Theorem 2.4:** The graph \( P_n \) is super magic.

**Proof:** The graph \( P_n \) is given below in the figure 1:

(\text{Vertex rule}): Define a map \( f : V(G) \rightarrow \{0, 1, 2, \ldots, p\} \) by

\[
f(V_i) = \frac{(p+1)i}{2} + 3 \quad \text{or} \quad \frac{p}{2}i + 3 \quad \text{if “p” is odd (or) even respectively}
\]

For \( i \) varying from 1 to \( n \),

\[
f(V_1) = f(V_1) + \frac{11(i-1)}{2} \quad \text{if “i” is odd; } f(V_i) = 9 + \frac{11(i-2)}{2} \quad \text{if “i” is even;}
\]

\[
f(U_1) = 3 ; \quad f(U_2) = 4 ; f(U_3) = f(V_1) + 5 ; \quad f(U_4) = f(V_1) + 6 ;
\]

\[
f(W_1) = f(V_1) - 1 ; \quad f(W_2) = f(V_1) + 1 ; \quad f(W_3) = 8 ; \quad f(W_4) = 10
\]

\[
f(S_1) = 2 ; \quad f(S_2) = 5 ; \quad f(S_3) = f(V_1) + 4 ; \quad f(S_4) = f(V_1) + 7 ;
\]

\[
f(t_1) = f(V_1) - 2 ; \quad f(t_2) = f(V_2) + 2 ; \quad f(t_3) = 7 ; \quad f(t_4) = 11 ;
\]

\[
f(X_1) = 1 ; \quad f(X_2) = 6 ; \quad f(X_3) = f(V_2) + 3 ; \quad f(X_4) = f(V_2) + 8
\]

For \( i \) varying from 1 to \( n \), it finds that

\[
f(U_1) = f(U_1)_i + 11(i-1)/4 ; \quad i \equiv 1 \text{(mod 4)} ; \quad = f(U_2)_i + 11(i-2)/4 ; \quad i \equiv 2 \text{(mod 4)}
\]

\[
= f(U_3)_i + 11(i-3)/4 ; \quad i \equiv 3 \text{(mod 4)} ; \quad = f(U_4)_i + 11(i-4)/4 ; \quad i \equiv 0 \text{(mod 4)}
\]

\[
f(W_1) = f(W_1)_i + 11(i-1)/4 ; \quad i \equiv 1 \text{(mod 4)} ; \quad = f(W_2)_i + 11(i-2)/4 ; \quad i \equiv 2 \text{(mod 4)}
\]

\[
= f(W_3)_i + 11(i-3)/4 ; \quad i \equiv 3 \text{(mod 4)} ; \quad = f(W_4)_i + 11(i-4)/4 ; \quad i \equiv 0 \text{(mod 4)}
\]

\[
f(S_1) = f(S_1)_i + 11(i-1)/4 ; \quad i \equiv 1 \text{(mod 4)} ; \quad = f(S_2)_i + 11(i-2)/4 ; \quad i \equiv 2 \text{(mod 4)}
\]

\[
= f(S_3)_i + 11(i-3)/4 ; \quad i \equiv 3 \text{(mod 4)} ; \quad = f(S_4)_i + 11(i-4)/4 ; \quad i \equiv 0 \text{(mod 4)}
\]

\[
f(t_1) = f(t_1)_i + 11(i-1)/4 ; \quad i \equiv 1 \text{(mod 4)} ; \quad = f(t_2)_i + 11(i-2)/4 ; \quad i \equiv 2 \text{(mod 4)}
\]

\[
= f(t_3)_i + 11(i-3)/4 ; \quad i \equiv 3 \text{(mod 4)} ; \quad = f(t_4)_i + 11(i-4)/4 ; \quad i \equiv 0 \text{(mod 4)}
\]

\[
f(X_1) = f(X_1)_i + 11(i-1)/4 ; \quad i \equiv 1 \text{(mod 4)} ; \quad = f(X_2)_i + 11(i-2)/4 ; \quad i \equiv 2 \text{(mod 4)}
\]

\[
= f(X_3)_i + 11(i-3)/4 ; \quad i \equiv 3 \text{(mod 4)} ; \quad = f(X_4)_i + 11(i-4)/4 ; \quad i \equiv 0 \text{(mod 4)}
\]

(\text{Edge rule}): Define the map \( f \) on edge set of the graph \( P_n \) as follows:
If \( f(V_iV_{i+1}) = (p+q-10)-(i-1); i=1 \) to \( n \-1 \); For \( i = 1 \) to \( n \), it becomes that
\[
\begin{align*}
f(V_iU_2i+1) &= p+q-4-(i-1); f(V_iU_2i) = (p+q-5)-(i-1) ; \\
f(U_2i+1W_{2i-1}) &= (p+q-3)-(i-1) ; f(U_2iW_{2i}) = (p+q-6)-(i-1) ; \\
f(W_{2i}S_{2i-1}) &= (p+q-2)-(i-1) ; f(W_{2i}S_{2i}) = (p+q-7)-(i-1) ; \\
f(S_{2i-1}t_{2i-1}) &= (p+q-1)-(i-1) ; f(S_{2i-2}t_{2i}) = (p+q-8)-(i-1) ; \\
\end{align*}
\]

The map \( f \) satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph \( (P_n \ast 2nP_6) \) with the magic number \( (28n-2) \). Therefore the graph \( (P_n \ast 2nP_6) \) is super magic.

**Example 2.5:** The graphs \( P_n \ast 2nP_6 \) and \( P_n \ast 2nP_6 \) are super magic graph in the figures 2 & 3.

![Figure 2 – super magic labeling of the graph (P6 * 12P6)](image)

![Figure 3 – super magic labeling of the graph (P6 * 14P6)](image)

**Definition 2.3:** \( P_n \ast 2nP_7 \) is a connected graph whose vertex set is \( \{V_1, V_2, \ldots, V_n, U_1, U_2, \ldots, U_{2n}, W_1, W_2, \ldots, W_{2n}, S_1, S_2, \ldots, S_{2n}, t_1, t_2, \ldots, t_{2n}, x_1, x_2, \ldots, x_{2n}, y_1, y_2, \ldots, y_{2n} \} \) and edge set is \( \{V_iV_{i+1}: i = 1 \) to \( n \} \cup \{V_iU_{2i-1}: i = 1 \) to \( n \} \cup \{U_iW_i: i = 1 \) to \( 2n \} \cup \{w_iS_i: i = 1 \) to \( 2n \} \cup \{s_i t_i: i = 1 \) to \( 2n \} \cup \{t_i x_i: i = 1 \) to \( 2n \} \cup \{x_i y_i: i = 1 \) to \( 2n \} \). Here edge set of \( P_n \) is \( \{V_iV_{i+1}: i = 1 \) to \( n \} \cup \{V_iU_{2i-1}: i = 1 \) to \( n \} \cup \{U_iW_i: i = 1 \) to \( 2n \} \cup \{w_iS_i: i = 1 \) to \( 2n \} \cup \{s_i t_i: i = 1 \) to \( 2n \} \cup \{t_i x_i: i = 1 \) to \( 2n \} \cup \{x_i y_i: i = 1 \) to \( 2n \} \).

**Theorem 2.6:** The graphs \( P_n \ast 2nP_7 \) is super magic.

**Proof:** The graph \( P_n \ast 2nP_7 \) is given as follows in the figure 4:
(Vertex set): Define a map \( f: V(G) \rightarrow \{0, 1, 2, \ldots, p\} \) by

\[
 f(V_2) = \left(\frac{p+1}{2}\right) + 10 \text{ (or) } \frac{p}{2} + 10 ; \text{ if } p \text{ is odd (or) even respectively;}
\]

For \( i \) varying from 1 to \( n \),

\[
 f(V_i) = 4 + 13(i-1)/2; \text{ (i is odd)} ; (i=1 \text{ to } n) \quad f(V_i) = f(V_2) + \frac{13(i-2)}{2}; \text{ (i > 2 is even)} ;
\]

For \( i \) varying from 1 to \( n \), it makes that

\[
 f(U_1) = f(U_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(U_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(W_1) = f(W_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(W_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(S_1) = f(S_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(S_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(t_1) = f(t_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(t_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(X_1) = f(X_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(X_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(Y_1) = f(Y_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(Y_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4} \\
 f(V_1) = f(V_2)+13(i-1)/4 ; i \equiv 1 \pmod{4} \quad f(V_2) = 13(i-2)/4 ; i \equiv 2 \pmod{4}
\]

(Edge rule): Define the map \( f \) on edge set of the graph \( P_n \ast 2nP_7 \) as follows:

\[
 f(V_iV_{i+1}) = (p+q-12)-13(i-1) ; i = 1 \text{ to } (n-1);
\]

For \( i = 1 \) to \( n \)

\[
 f(V_1U_{2i-1}) = (p+q-5)-13(i-1) ; f(V_iU_2) = (p+q-6)-13(i-1) \\
 f(U_{2i-1}W_{2i-1}) = (p+q-4)-13(i-1) ; f(U_2W_{2i-1}) = (p+q-7)-13(i-1) \\
 f(W_{2i-1}S_{2i-1}) = (p+q-3)-13(i-1) ; f(W_2S_{2i-1}) = (p+q-8)-13(i-1) \\
 f(S_{2i-1}t_{2i-1}) = (p+q-2)-13(i-1) ; f(S_2t_{2i-1}) = (p+q-9)-13(i-1) \\
 f(t_{2i-1}X_{2i-1}) = (p+q-1)-13(i-1) ; f(t_{2i}X_{2i}) = (p+q-10)-13(i-1) \\
 f(X_{2i-1}Y_{2i-1}) = (p+q)-13(i-1) ; f(X_{2i}Y_{2i}) = (p+q-11)-13(i-1)
\]
The map $f$ satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph $(P_n \ast 2nP_7)$ with the magic number $(33n - 2)$. Therefore the graph $(P_n \ast 2nP_7)$ is super magic.

**Example 2.7:** The graphs $P_6 \ast 12P_7$ and $P_6 \ast 14P_7$ are super magic in the following figures 5 & 6.

**Definition 2.3:** $P_n \ast 2nP_8$ is a connected graph whose vertex set is $\{V_1, V_2, \ldots, V_n, U_1, U_2, \ldots, U_{2n}, W_1, W_2, \ldots, W_{2n}, S_1, S_2, \ldots, S_{2n}, t_1, t_2, \ldots, t_{2n}, x_1, x_2, \ldots, x_{2n}, y_1, y_2, \ldots, y_{2n}, z_1, z_2, \ldots, z_{2n}\}$ and edge set is $\{V_iV_{i+1}: i = 1 \text{ to } n\} \cup \{\{V_{iU_1}: i = 1 \text{ to } n\} \cup \{t_1W_1: i = 1 \text{ to } 2n\} \cup \{s_1W_1: i = 1 \text{ to } 2n\} \cup \{t_2v_1: i = 1 \text{ to } 2n\} \cup \{x_1y_1: i = 1 \text{ to } 2n\} \cup \{y_1z_1: i = 1 \text{ to } 2n\} \cup \{t_2x_1: i = 1 \text{ to } 2n\} \cup \{x_1y_1: i = 1 \text{ to } 2n\} \cup \{y_1z_1: i = 1 \text{ to } 2n\}$.

**Theorem 2.9:** The graph $P_n \ast 2nP_8$ is super magic.

**Proof:** $P_n \ast 2nP_8$ is given as follows in the figure 7:

**Figure 7** – one of the arbitrary labeling of the graph $(P_n \ast 2nP_8)$.
**Vertex rule:** Define a map \( f : V(G) \rightarrow \{0, 1, 2, \ldots, p\} \) by
\[
f(V_i) = \begin{cases} \frac{p+1}{2} + 4 \text{ (or) } \frac{p}{2} + 4 & \text{if } p \text{ is odd (or) even respectively;} \\
\end{cases}
\]
For \( i \) varying from 1 to \( n \),
\[
f(V_i) = 4 + 15\left(\frac{i-1}{2}\right); \text{ } i > 1 \text{ is odd}
\]
\[
f(V_i) = 12 + 15\left(\frac{i-2}{2}\right); \text{ } i \text{ is even;}
\]
\[
f(U_1) = 4; \text{ } f(U_2) = 5; \text{ } f(U_3) = f(V_1)+7; \text{ } f(U_4) = f(V_1)+8;
\]
\[
f(W_i) = f(V_i)+1; \text{ } f(W_2) = f(V_1)+1; \text{ } f(W_3) = 11; \text{ } f(W_4) = 13;
\]
\[
f(S_i) = 3; \text{ } f(S_2) = 6; \text{ } f(S_3) = f(V_1)+6; \text{ } f(S_4) = f(V_2)+9;
\]
\[
f(t_i) = f(V_i)-2; \text{ } f(t_2) = f(V_1)+2; \text{ } f(t_3) = 10; \text{ } f(t_4) = 14;
\]
\[
f(X_i) = 2; \text{ } f(X_2) = 7; \text{ } f(X_3) = f(V_1)+5; \text{ } f(X_4) = f(V_1)+10;
\]
\[
f(Y_i) = f(V_i)-3; \text{ } f(Y_2) = f(V_1)+3; \text{ } f(Y_3) = 9; \text{ } f(Y_4) = 15;
\]
\[
f(Z_i) = 1; \text{ } f(Z_2) = 8; \text{ } f(Z_3) = f(V_1)+4; \text{ } f(Z_4) = f(V_1)+11;
\]
For \( i \) varying from 1 to \( n \),
\[
f(U_i) = f(U_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(U_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(U_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(U_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]
\[
f(W_i) = f(W_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(W_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(W_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(W_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]
\[
f(S_i) = f(S_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(S_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(S_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(S_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]
\[
f(X_i) = f(X_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(X_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(X_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(X_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]
\[
f(Y_i) = f(Y_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(Y_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(Y_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(Y_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]
\[
f(Z_i) = f(Z_1)+15\left(i-1\right)/4; \text{ } i \equiv 1 \text{ (mod 4)}; f(Z_2)+15\left(i-2\right)/4; \text{ } i \equiv 2 \text{ (mod 4)}
\]
\[
f(Z_3)+15\left(i-3\right)/4; \text{ } i \equiv 3 \text{ (mod 4)}; f(Z_4)+15\left(i-4\right)/4; \text{ } i \equiv 0 \text{ (mod 4)}
\]

**Edge rule:** Define the map \( f \) on edge set of the graph \( P_n \ast 2nP_8 \)
\[
f(V_1V_{i+1}) = (p+q-14)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
For \( i = 1 \) to \( n \),
\[
f(V_1U_{2i-1}) = (p+q-6)-15\left(i-1\right); \text{ } f(V_1U_{2i}) = (p+q-7)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(U_{2i-1}W_{2i-1}) = (p+q-5)-15\left(i-1\right); f(U_{2i}W_{2i}) = (p+q-8)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(W_{2i-1}S_{2i-1}) = (p+q-4)-15\left(i-1\right); f(W_{2i}S_{2i}) = (p+q-9)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(S_{2i-1}t_{2i-1}) = (p+q-3)-15\left(i-1\right); \text{ } f(S_{2i}t_{2i}) = (p+q-10)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(t_{2i-1}X_{2i-1}) = (p+q-2)-15\left(i-1\right); \text{ } f(t_{2i}X_{2i}) = (p+q-11)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(X_{2i-1}Y_{2i-1}) = (p+q-1)-15\left(i-1\right); \text{ } f(X_{2i}Y_{2i}) = (p+q-12)-15\left(i-1\right); \text{ } \text{for } i = 1 \text{ to } (n-1)
\]
\[
f(X_{2i-1}Z_{2i-1}) = (p+q)-15\left(i-1\right); \text{ } f(X_{2i}Z_{2i}) = (p+q-13)-15\left(i-1\right)
\]

The map \( f \) satisfies the requirements for super magic labeling to both vertex set and edge set of the given graph \( (P_n * 2nP_8) \) with the magic number \( (38n - 2) \). Therefore the graph \( (P_n * 2nP_8) \) is super magic.

**Example 2.10:** \( P_6 \ast 12P_8 \) and \( P_7 \ast 14P_8 \) are super magic graphs.
\[
P = 90; \text{ } q = 89; \text{ } p + q = 179
\]
Figure 8 – super magic labeling of the graph \((P_6 \ast 12P_8)\) with \(s = 226\)

\[P = 105; \quad q = 104; \quad p + q = 209\]

Figure 9 – super magic labeling of the graph \((P_6 \ast 12P_8)\) with \(s = 264\)

REFERENCES: