

# Homomorphism of an Intuitionistic Fuzzy $\ell$ -ring Ideals

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**Abstract:** In this paper the notion of Intuitionistic fuzzy  $\ell$ -ring ideals (left, right) is introduced and some homomorphism of an intuitionistic fuzzy  $\ell$ -ring ideals (left, right) properties of image have been derived.

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**Keywords:** Fuzzy set, Intuitionistic fuzzy set, Intuitionistic fuzzy subring, Intuitionistic fuzzy sub  $\ell$ -ring, Intuitionistic fuzzy  $\ell$ -ring ideals (left, right), Homomorphism of an Intuitionistic fuzzy  $\ell$ -ring ideal.

## 1. INTRODUCTION

Zadeh in [23], who was the first researcher initiated the theory of fuzzy sets in 1965. In fuzzy set theory the membership of an element to a fuzzy set is a single value between zero and one. Therefore, a generalization of fuzzy sets was considered by Atanassov 1986 as intuitionistic fuzzy sets which incorporate the degree of membership and non-membership degrees respectively [3]. The concept of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. [1] N. Ajmal and K.

V. Thomas discussed the lattice of fuzzy ideals of a ring. [4] K. T. Atanassov proved new operations defined over the intuitionistic fuzzy sets. [5] B. Banerjee and D. K. Basnet introduced the concept of intuitionistic fuzzy subring and ideals. [12] K. Hur, H. W. Kang and H. K. Song developed the concept of intuitionistic fuzzy subgroups and subrings. Also K. Hur, S. Y. Jang and H. W. Kang Intuitionistic fuzzy ideals of a ring [11]. Further, K. Hur, Y. S. Ahn and D. S. Kim [10] extended the lattice of intuitionistic fuzzy ideals of rings. [14] Marashdeh and Salleh they have also introduced and studied the notion of intuitionistic fuzzy rings based on the notion of fuzzy space. K. Meena and K. V. Thomas [13] discussed the idea about intuitionistic  $\ell$ -fuzzy subring.

Lattice theory has been applied to all kinds of fields. [17] R. Natarajan and S. Moganavalli applied the concept of fuzzy sets to lattice theory. [15] M. Mullai B. Chellappa have introduced fuzzy  $\ell$ -filter. [20] Also Double representation for an intuitionistic fuzzy  $\ell$ -filter was introduced by K. R. Sasireka, K. E. Sathappan and B. Chellappa. We also established by the properties of intuitionistic fuzzy sub  $\ell$ -ring. [23] G. J. Wang generalized order-homomorphism on fuzzy. [18] D. M. Olson proved on the homomorphism for hemirings. [19] N. Palaniappan K. Arjunan, introduced and studied the homomorphism, anti-homomorphism of a fuzzy and anti-fuzzy ideals. [7] K. Chandrasekhara Rao and V. Swaminathan studied in detail anti-homomorphism in near-rings. In this paper, we define the notion of the intuitionistic fuzzy  $\ell$ -ring ideals (left, right) and have studied Homomorphism of an intuitionistic fuzzy  $\ell$ -ring ideals.

This paper has been organized as follows: In section 2, some preliminary definitions and examples have been outlined. In section 3, the definition of intuitionistic fuzzy  $\ell$ -ring ideals (left, right) is given and few homomorphism

images have been analysed.

## 2.PRELIMINARIES

In this section, we give some basic definitions. There were two definitions for a lattice, one as a poset and the other as an algebraic structure. A poset  $(L, \leq)$  is said to form a lattice if for any  $a, b \in L$ ,  $\text{Sup}\{a, b\}$  and  $\text{inf}\{a, b\}$  exist in  $L$ . In this case we write  $\text{Sup}\{a, b\} = a \vee b$  and  $\text{inf}\{a, b\} = a \wedge b$ . Throughout this paper we denote a lattice with join ' $\vee$ ' and meet ' $\wedge$ ' by simply  $L$ .

**Definition 2.1** [13] Let  $X$  be a non-empty. An intuitionistic fuzzy set  $A$  of  $X$  is an object of the following form  $A = \{\langle X, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ , where  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$ , define the degree of membership and the degree of non-membership of the element  $x \in X$ , respectively and  $\forall x \in X$ ,  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ .

**Definition 2.2** [13] Let  $L$  be a lattice and  $A = \{\langle X, \mu_A(x), \nu_A(x) \rangle / x \in X\}$  be an intuitionistic fuzzy set of  $L$ . Then  $A$  is called an intuitionistic fuzzy sublattice (intuitionistic fuzzy lattice) of  $L$  if the following conditions are satisfied

- (i)  $\mu_A(x \vee y) \geq \min \{ \mu_A(x), \mu_A(y) \}$
- (ii)  $\mu_A(x \wedge y) \geq \min \{ \mu_A(x), \mu_A(y) \}$
- (iii)  $\nu_A(x \vee y) \leq \max \{ \nu_A(x), \nu_A(y) \}$
- (iv)  $\nu_A(x \wedge y) \leq \max \{ \nu_A(x), \nu_A(y) \} \quad \forall x, y \in L$ .

**Definition 2.3** [13] An Intuitionistic fuzzy sublattice  $A$  of  $L$  is called an intuitionistic fuzzy ideal of  $L$  (Intuitionistic fuzzy  $\ell$ -ideal) if the following conditions are satisfied.

- (i)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$       (iii)  $\nu(x \vee y) \leq \max \{ \nu(x) , \nu(y) \}$   
 (ii)  $\mu(x \wedge y) \geq \max \{ \mu(x) , \mu(y) \}$       (iv)  $\nu(x \wedge y) \leq \min \{ \nu(x) , \nu(y) \}$   
 $\forall x, y \in L$ .

**Definition 2.4** [17] A fuzzy subset  $\mu$  of a lattice ordered ring(or  $\ell$ -ring in short)  $R$ , is called fuzzy sub  $\ell$ -ring of  $R$ , if the following conditions are satisfied.

- (i)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$       (iii)  $\mu(x - y) \geq \min \{ \mu(x) , \mu(y) \}$   
 (ii)  $\mu(x \wedge y) \geq \min \{ \mu(x) , \mu(y) \}$       (iv)  $\mu(xy) \geq \min \{ \mu(x) , \mu(y) \}$   
 for all  $x, y \in L$ .

**Definition 2.5** [17] A fuzzy subset  $\mu$  of an  $\ell$ -ring  $R$ , is called a fuzzy  $\ell$ -ring ideal (or) fuzzy  $\ell$ -ideals of  $R$ , if for all  $x, y \in R$  the following conditions are satisfied.

- (i)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$       (iii)  $\mu(x - y) \geq \min \{ \mu(x) , \mu(y) \}$   
 (ii)  $\mu(x \wedge y) \geq \max \{ \mu(x) , \mu(y) \}$       (iv)  $\mu(xy) \geq \max \{ \mu(x) , \mu(y) \}$   
 for all  $x, y \in L$ .

**Definition 2.6** [16] A mapping  $f$  from  $R$  to a ring  $S$  is called an homomorphism,  $\forall x, y \in R$

- (i)  $f(x+y) = f(x) + f(y)$   
 (ii)  $f(xy) = f(x) \cdot f(y)$ .

**Definition 2.7** [17] Let  $R$  and  $S$  be two  $\ell$ -rings. A function  $f : R \rightarrow S$  is called an  $\ell$ -homomorphism if for all  $x, y \in R$ .

- (i)  $f(x \vee y) = f(x) \vee f(y)$   
 (ii)  $f(x \wedge y) = f(x) \wedge f(y)$   
 (iii)  $f(x + y) = f(x) + f(y)$   
 (iv)  $f(xy) = f(x) \cdot f(y)$

**Definition 2.8** [16] Let  $f$  be a mapping from a set  $R$  to a set  $S$  and let  $A$  be a fuzzy subset in  $R$ . Then  $A$  is called  $f$ -invariant if  $f(x) = f(y)$  implies  $A(x) = A(y)$  for all  $x, y \in R$ . Clearly, if  $A$  is  $f$ -invariant, then  $f^{-1}(f(A)) = A$ .

**Definition 2.9** [13] An intuitionistic fuzzy set  $A$  is said to have sup-property [inf-property], if for each subset  $T \subseteq A$  there exist  $t_0 \in T$  such that,

$$\begin{aligned} \sup_{t \in T} \{\mu(t)\} &= \mu(t_0) \\ \inf_{t \in T} \{\nu(t)\} &= \nu(t_0) \end{aligned}$$

**Definition 2.10** [13] Let  $f$  be a mapping from a set  $X$  to a set  $Y$  and let  $\mu_A$  and  $\nu_A$  be intuitionistic fuzzy subset in  $X$  and  $Y$  respectively.

(i)  $f(A)$ , the image of  $A$  under  $f$ , is a intuitionistic fuzzy subset in  $Y$  for all  $y \in Y$ . We define,

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi \end{cases}$$

and

$$f(\nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & \text{if } f^{-1}(y) \neq \phi \\ 1 & \text{if } f^{-1}(y) = \phi \end{cases}$$

$f^{-1}(B)$  is the pre-image of  $B$  under  $f$ , is a intuitionistic fuzzy set in  $X$ .

$$\text{ie) } f^{-1}(\mu_A)(x) = \mu_A(f(x)) \quad \text{and}$$

$$f^{-1}(\nu_A)(x) = \nu_A(f(x)), \forall x \in R$$

**Definition 2.11** [20] Let  $R$  be a ring. An Intuitionistic fuzzy set  $A = \{ \langle x, \mu(x), \nu(x) \rangle : x \in R \}$  of  $R$  is said to be intuitionistic fuzzy sub  $\ell$ -ring on  $R$  if for all  $x, y \in R$ .

- (i)  $\mu(x - y) \geq \min \{ \mu(x) , \mu(y) \}$
- (ii)  $\mu(xy) \geq \min \{ \mu(x) , \mu(y) \}$
- (iii)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$
- (iv)  $\mu(x \wedge y) \geq \min \{ \mu(x) , \mu(y) \}$
- (v)  $\nu(x - y) \leq \max \{ \nu(x) , \nu(y) \}$
- (vi)  $\nu(xy) \leq \max \{ \nu(x) , \nu(y) \}$
- (vii)  $\nu(x \vee y) \leq \max \{ \nu(x) , \nu(y) \}$
- (viii)  $\nu(x \wedge y) \leq \max \{ \nu(x) , \nu(y) \}$

**Example 2.12** [17] Now  $(R = a, b, c, d, +, \cdot, \vee, \wedge)$  is an  $\ell$ -ring under the operations  $+, \cdot, \vee$ , and  $\wedge$  defined by the following tables,

|   |   |   |   |   |
|---|---|---|---|---|
| + | a | b | c | d |
| a | a | b | c | d |
| b | b | a | d | c |
| c | c | d | a | b |
| d | d | c | b | a |

|   |   |   |   |   |
|---|---|---|---|---|
| . | a | b | c | d |
| a | a | a | a | a |
| b | a | b | a | b |
| c | a | a | c | c |
| d | a | b | c | d |

|        |   |   |   |   |
|--------|---|---|---|---|
| $\vee$ | a | b | c | d |
| a      | a | b | c | d |
| b      | b | b | d | d |
| c      | c | d | c | d |
| d      | d | d | d | d |

|          |   |   |   |   |
|----------|---|---|---|---|
| $\wedge$ | a | b | c | d |
| a        | a | a | a | a |
| b        | a | b | a | b |
| c        | a | a | c | c |
| d        | a | b | c | d |

### 3. Theorems on Intuitionistic fuzzy $\ell$ -ring ideals.

**Definition 2.13** An intuitionistic fuzzy sub  $\ell$ -ring  $A$  on  $R$  is said to be a intuitionistic fuzzy left  $\ell$ -ideal if for all  $x, y \in R$ .

- (i)  $\mu(x - y) \geq \min \{ \mu(x), \mu(y) \}$
- (ii)  $\mu(xy) \geq \mu(y)$
- (iii)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$
- (iv)  $\mu(x \wedge y) \geq \max \{ \mu(x) , \mu(y) \}$
- (v)  $\nu(x - y) \leq \max \{ \nu(x) , \nu(y) \}$
- (vi)  $\nu(xy) \leq \nu(y)$
- (vii)  $\nu(x \vee y) \leq \max \{ \nu(x) , \nu(y) \}$
- (viii)  $\nu(x \wedge y) \leq \min \{ \nu(x) , \nu(y) \}$

**Definition 2.14** An intuitionistic fuzzy sub  $\ell$ -ring  $A$  on  $R$  is said to be a intuitionistic fuzzy right  $\ell$ -ideal if for all  $x, y \in R$

- (i)  $\mu(x - y) \geq \min \{ \mu(x) , \mu(y) \}$
- (ii)  $\mu(xy) \geq \mu(x)$
- (iii)  $\mu(x \vee y) \geq \min \{ \mu(x) , \mu(y) \}$
- (iv)  $\mu(x \wedge y) \geq \max \{ \mu(x) , \mu(y) \}$

$$\begin{array}{ll}
(v) \nu(x - y) \leq \max \{ \nu(x) , \nu(y) \} & (vi) \nu(xy) \leq \nu(x) \\
(vii) \nu(x \vee y) \leq \max \{ \nu(x) , \nu(y) \} & (viii) \nu(x \wedge y) \leq \min \{ \nu(x) , \nu(y) \}
\end{array}$$

**Definition 2.15** An intuitionistic fuzzy sub  $\ell$ -ring  $A$  on  $R$  is said to be an intuitionistic fuzzy  $\ell$ - ring ideal if it is both an intuitionistic fuzzy left  $\ell$ - ideal and an intuitionistic fuzzy right  $\ell$ - ideal of  $R$ .

**Example 2.16** Consider the Intuitionistic fuzzy  $\ell$ -ring ideal  $R$  defined in example 2.10

$$\mu(x) = \begin{cases} .9 & \text{if } x = a \\ .6 & \text{if } x = b \\ .4 & \text{if } x = c, d. \end{cases}$$

$$\nu(x) = \begin{cases} .1 & \text{if } x = a \\ .3 & \text{if } x = b \\ .5 & \text{if } x = c, d. \end{cases}$$

**Remark 2.17** Every Intuitionistic fuzzy  $\ell$ -ring ideal  $R$  is a Intuitionistic fuzzy sub  $\ell$ -ring of  $R$ . But converse need not be true.

**Proof:** Consider the intuitionistic fuzzy subset  $\mu, \nu$  of  $\ell$ -ring  $(Z, +, \cdot, \vee, \wedge)$

$$\mu_1(x) = \begin{cases} .6 & \text{if } x \in \langle 2 \rangle \\ .3 & \text{otherwise} \end{cases}$$

$$\nu_1(x) = \begin{cases} .4 & \text{if } x \in \langle 2 \rangle \\ .7 & \text{otherwise} \end{cases}$$

**Theorem 2.18** Let  $f$  be a homomorphism from a  $\ell$ -ring  $R$  into  $\ell$ -ring  $S$  and let  $\mu_A, \nu_A$  be an Intuitionistic fuzzy left  $\ell$ -ideal of  $S$ . Then the pre-image  $f^{-1}(\mu_A)$  and  $f^{-1}(\nu_A)$  is a intuitionistic fuzzy left  $\ell$ -ideal of  $R$ .

**Proof:** Consider a  $\ell$ -ring homomorphism  $f : R \rightarrow S$

Let  $\mu_A, \nu_A$  be a Intuitionistic fuzzy left  $\ell$ -ideal of  $S$ .  $\forall x, y \in R$ .

$$\begin{aligned}
 (i) f^{-1}(\mu_A)(x - y) &= \mu_A f(x - y) \\
 &\geq \min\{\mu_A f(x), \mu_A f(y)\} \\
 f^{-1}(\mu_A)(x - y) &\geq \min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\}. \\
 (ii) f^{-1}(\mu_A)(xy) &= \mu_A f(xy) \\
 &\geq \mu_A f(y) \\
 &\geq f^{-1}(\mu_A)(y) \\
 f^{-1}(\mu_A)(xy) &\geq f^{-1}(\mu_A)(y) \\
 (iii) f^{-1}(\mu_A)(x \vee y) &= \mu_A f(x \vee y) \\
 &\geq \min\{\mu_A f(x), \mu_A f(y)\} \\
 &\geq \min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\} \\
 (iv) f^{-1}(\mu_A)(x \wedge y) &= \mu_A f(x \wedge y) \\
 &\geq \max\{\mu_A f(x), \mu_A f(y)\} \\
 &\geq \max\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\} \\
 (v) f^{-1}(\nu_A)(x - y) &= \nu_A f(x - y) \\
 &\leq \max\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\} \\
 (vi) f^{-1}(\nu_A)(xy) &= \nu_A(f(xy)) \\
 &\leq \nu_A f(y) \\
 &\leq f^{-1}(\nu_A)(y) \\
 (vii) f^{-1}(\nu_A)(x \vee y) &= \nu_A f(x \vee y) \\
 &\leq \max\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\}
 \end{aligned}$$



$$\begin{aligned}
 (viii) f^{-1}(\nu_A)(x \wedge y) &= \nu_A f(x \wedge y) \\
 &\leq \min\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \min\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\}
 \end{aligned}$$

∴  $f^{-1}(\mu_A), f^{-1}(\nu_A)$  is a intuitionistic fuzzy left  $\ell$ -ideal of  $R$ .

**Theorem 2.19** Let  $f : R \rightarrow S$  be an homomorphism from  $\ell$ -ring  $R$  into  $S$ .

(i) If  $\mu_A, \nu_A$  is a Intuitionistic fuzzy right  $\ell$ -ideal of  $S$ , then pre-image  $f^{-1}(\mu_A), f^{-1}(\nu_A)$  is a Intuitionistic fuzzy right  $\ell$ -ideal of  $R$ .

**Proof:** Consider a  $\ell$ -ring homomorphism  $f : R \rightarrow S$ . Let  $\mu_A, \nu_A$  be a Intuitionistic fuzzy right  $\ell$ -ideal of  $S$ , for all  $x, y \in R$ .

$$\begin{aligned}
 (i) f^{-1}(\mu_A)(x - y) &= \mu_A f(x - y) \\
 &\geq \min\{\mu_A f(x), \mu_A f(y)\} \\
 f^{-1}(\mu_A)(x - y) &\geq \min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\}. \\
 (ii) f^{-1}(\mu_A)(xy) &= \mu_A(f(xy)) \\
 &\geq \mu_A(f(x)) \\
 &= f^{-1}(\mu_A)(x) \\
 &\geq f^{-1}((\mu_A)(x)) \\
 (iii) f^{-1}(\mu_A)(x \vee y) &= \mu_A f(x \vee y) \\
 &\geq \min\{\mu_A f(x), \mu_A f(y)\} \\
 &\geq \min\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\} \\
 (iv) f^{-1}(\mu_A)(x \wedge y) &= \mu_A(f(x \wedge y)) \\
 &\geq \max\{\mu_A f(x), \mu_A f(y)\} \\
 &\geq \max\{f^{-1}(\mu_A)(x), f^{-1}(\mu_A)(y)\}
 \end{aligned}$$

$$\begin{aligned}
 (v) f^{-1}(\nu_A)(x - y) &= \nu_A f(x - y) \\
 &\leq \max\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\} \\
 (vi) f^{-1}(\nu_A)(xy) &= \nu_A(f(xy)) \\
 &\leq \nu_A f(x) \\
 &\leq f^{-1}(\nu_A)(x) \\
 (vii) f^{-1}(\nu_A)(x \vee y) &= \nu_A f(x \vee y) \\
 &\leq \max\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \max\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\} \\
 (viii) f^{-1}(\nu_A)(x \wedge y) &= \nu_A f(x \wedge y) \\
 &\leq \min\{\nu_A f(x), \nu_A f(y)\} \\
 &\leq \min\{f^{-1}(\nu_A)(x), f^{-1}(\nu_A)(y)\}
 \end{aligned}$$

$\therefore f^{-1}(\mu_A), f^{-1}(\nu_A)$  is a Intuitionistic fuzzy right  $\ell$ -ideal of  $R$ .

**Theorem 2.20** Let  $R$  and  $S$  be  $\ell$ -rings and  $f : R \rightarrow S$  be an homomorphism from  $\ell$ -ring  $R$  into  $S$ .

(i) If  $\mu_A, \nu_A$  is a Intuitionistic fuzzy  $\ell$ -ideal of  $S$ , then the pre-image  $f^{-1}(\mu_A), f^{-1}(\nu_A)$  is a Intuitionistic fuzzy  $\ell$ -ideal of  $R$ .

**Proof:** It's trivial

**Theorem 2.21** Let  $R$  and  $S$  be  $\ell$ -ring and  $f : R \rightarrow S$  be an homomorphism from  $\ell$ -ring  $R$  into  $S$ .

If  $\mu_A, \nu_A$  is a Intuitionistic fuzzy left  $\ell$ -ideal of a  $\ell$ -ring  $R$  with sup property, then the image  $f(\mu_A), f(\nu_A)$  is a Intuitionistic fuzzy left  $\ell$ -ideal of a  $S$ .

**Proof:** Consider a  $\ell$ -ring homomorphism  $f:R \rightarrow S$ . Let  $\mu_A, \nu_A$  be a Intuitionistic fuzzy left  $\ell$ -ideal of  $R$ . For all  $x, y \in R$ .

$$\begin{aligned}
 (i) f(\mu_A)(f(x) - f(y)) &= f(\mu_A)f(x - y) \\
 &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 &\geq \min\{f(\mu_A)(x), f(\mu_A)(y)\}. \\
 (ii) f(\mu_A)(f(x)f(y)) &= f(\mu_A)(f(xy)) \\
 &\geq \mu_A(y) \\
 &= f(\mu_A)f(y) \\
 &\geq f(\mu_A)(f(y)) \\
 (iii) f(\mu_A)(f(x) \vee f(y)) &= f(\mu_A)f(x \vee y) \\
 &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 &\geq \min\{f(\mu_A)(x), f(\mu_A)(y)\} \\
 (iv) f(\mu_A)(f(x) \wedge f(y)) &= f(\mu_A)(f(x \wedge y)) \\
 &\geq \max\{\mu_A(x), \mu_A(y)\} \\
 &\geq \max\{f(\mu_A)(x), f(\mu_A)(y)\} \\
 (v) f(\nu_A)(f(x) - f(y)) &= f(\nu_A)f(x - y) \\
 &\leq \max\{\nu_A(x), \nu_A(y)\} \\
 &\leq \max\{f(\nu_A)(x), f(\nu_A)(y)\} \\
 (vi) f(\nu_A)(f(x)f(y)) &= f(\nu_A)f(xy) \\
 &\leq \nu_A(y) \\
 &\leq f(\nu_A)(f(y)) \\
 (vii) f(\nu_A)(f(x) \vee f(y)) &= f(\nu_A)f(x \vee y) \\
 &\leq \max\{\nu_A(x), \nu_A(y)\} \\
 &\leq \max\{f(\nu_A)(x), f(\nu_A)(y)\}
 \end{aligned}$$

$$\begin{aligned}
 (viii) f(\nu_A)(f(x) \wedge f(y)) &= f(\nu_A)(f(x \wedge y)) \\
 &\leq \min\{\nu_A(x), \nu_A(y)\} \\
 &\leq \min\{f(\nu_A)(x), f(\nu_A)(y)\}
 \end{aligned}$$

∴  $f(\mu_A), f(\nu_A)$  is a Intuitionistic fuzzy left  $\ell$ -ideal of  $S$ .

**Theorem 2.22** Let  $R$  and  $S$  be  $\ell$ -ring and  $f : R \rightarrow S$  be an homomorphism from  $\ell$ -ring  $R$  into  $S$ .

If  $\mu_A, \nu_A$  is a Intuitionistic fuzzy right  $\ell$ -ideal of  $R$  with sup property, then the image  $f(\mu_A), f(\nu_A)$  is a Intuitionistic fuzzy right  $\ell$ -ideal of  $S$ .

**Proof:** Consider a  $\ell$ -ring homomorphism  $f: R \rightarrow S$ .

Let  $\mu_A, \nu_A$  a Intuitionistic fuzzy right  $\ell$ -ideal of  $R$ . For all  $x, y \in R$ .

$$\begin{aligned}
 (i) f(\mu_A)(f(x) - f(y)) &= f(\mu_A)f(x - y) \\
 &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 &\geq \min\{f(\mu_A)(x), f(\mu_A)(y)\}. \\
 (ii) f(\mu_A)(f(x)f(y)) &= f(\mu_A)(f(xy)) \\
 &\geq \mu_A(x) \\
 &= f(\mu_A)f(x) \\
 &\geq f(\mu_A)(f(x)) \\
 (iii) f(\mu_A)(f(x) \vee f(y)) &= f(\mu_A)f(x \vee y) \\
 &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 &\geq \min\{f(\mu_A)(x), f(\mu_A)(y)\} \\
 (iv) f(\mu_A)(f(x) \wedge f(y)) &= f(\mu_A)(f(x \wedge y)) \\
 &\geq \max\{\mu_A(x), \mu_A(y)\} \\
 &\geq \max\{f(\mu_A)(x), f(\mu_A)(y)\}
 \end{aligned}$$

$$\begin{aligned}
 (v)f(\nu_A)(f(x) - f(y)) &= f(\nu_A)f(x - y) \\
 &\leq \max\{\nu_A(x), \nu_A(y)\} \\
 &\leq \max\{f(\nu_A)(x), f(\nu_A)(y)\} \\
 (vi)f(\nu_A)(f(x)f(y)) &= f(\nu_A)f(xy) \\
 &\leq \nu_A(x) \\
 &\leq \{f(\nu_A)(f(x))\} \\
 (vii)f(\nu_A)(f(x) \vee f(y)) &= f(\nu_A)f(x \vee y) \\
 &\leq \max\{\nu_A(x), \nu_A(y)\} \\
 &\leq \max\{f(\nu_A)(x), f(\nu_A)(y)\} \\
 (viii)f(\nu_A)(f(x) \wedge f(y)) &= f(\nu_A)(f(x \wedge y)) \\
 &\leq \min\{\nu_A(x), \nu_A(y)\} \\
 &\leq \min\{f(\nu_A)(x), f(\nu_A)(y)\}
 \end{aligned}$$

$\therefore f(\mu_A), f(\nu_A)$  is a intuitionistic fuzzy right  $\ell$ -ideals of S.

**Theorem 2.23** *Let  $R$  and  $S$  be a  $\ell$ -ring and  $f: R \rightarrow S$  be an homomorphism from  $\ell$ -ring  $R$  into  $S$ .*

*If  $\mu_A, \nu_A$  is an Intuitionistic fuzzy  $\ell$ -ideal of a  $\ell$ -ring  $R$  with sup property, then the image  $f(\mu_A), f(\nu_A)$  is a fuzzy  $\ell$ -ideal of  $S$ .*

**Proof:** It's trivial.

#### 4.CONCLUSION

In this paper, the concept of Homomorphism on Intuitionistic fuzzy  $\ell$ -ring ideal of images are introduced. To extend this work, one can investigate the other anti-homomorphism properties.

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## REFERENCES

- [1] N. Ajmal and K. V. Thomas, *The Lattice of Fuzzy ideals of a ring*, Fuzzy sets and Systems(1995), 371-379.
- [2] N. Ajmal, *Homomorphism of fuzzy Subgroups*, Correspondents theorem and Fuzzy Sets and Systems 61(1994) 329-339.
- [3] K. T. Atanassov, *Intuitionistic fuzzy sets*. Fuzzy sets and Systems, 20(1) (1986), 87-96.
- [4] K. T. Atanassov, *New operations defined over the intuitionistic fuzzy sets*. Fuzzy sets and Systems 61, (1994), 137-142.
- [5] B. Banerjee and D. K. Basnet, *Intuitionistic Fuzzy Subring and Ideals*.
- [6] G. Birkhoff, *Lattice Theory*, Published by American Mathematical Theory Providence Rhode Island (1967).
- [7] K. Chandrasekharan Rao, and Swaminathan, *Anti-Homomorphism in Near Rings*, Jr of Inst. of maths and computer sciences (Math.Ser) Vol.2 (2006), 83-88.
- [8] Fang Jin-Xuan, *Fuzzy Homomorphism and Fuzzy isomorphism*, Fuzzy Sets and Systems 63(1994) 237-242.

- [9] A. A. M. Hassan, (2003) *On Fuzzy Rings and Fuzzy Homomorphisms*, The Journal of fuzzy Mathematics, Vol.7, No.2, 1999.
- [10] K. Hur, Y. S. Ahn and D.S. Kim *The Lattice of Intuitionistic Fuzzy Ideals of a Ring*, Journal of Appl.math and Computing vol 18 (2005) No, 12pp, 465-486.
- [11] K. Hur, S. Y. Jang and H. W. Kang, *Intuitionistic Fuzzy Ideals of a Ring*, J. Korea Soc Math. Educ Ser. B: Pure Appl.Math., vol. 12,2005, No.3, 193-209.
- [12] K. Hur, H. W. Kang, and H. K. Song, (2003) *Intuitionistic Fuzzy Subgroups and Subrings*, Honam Math J. 25 (1) : 19-41.
- [13] K. Meena, and K. V. Thomas, (2011), *Intuitionistic L-fuzzy Subrings*, International Mathematical Forum, 6(52): 2561-2572.
- [14] M. F. Marshdeh and A. R. Salleh, *Intuitionistic Fuzzy rings*, International Journal of Math Algebra 5(1) (2011) 37-47.
- [15] M. Mullai, B. Chellappa *Fuzzy L-filters*, IOSR Journal of Mathematics, ISSN:2278-5728 vol.issue 3(jul-aug 2012)
- [16] R. Muthuraj and C. Malarselvi, *Homomorphism and Anti-homomorphism of Multi Fuzzy Ideals and Multi-Anti Fuzzy Ideals of a ring*, IOSR Journals of Mathematics, Vol:11 Issue Ver.IV(Nov-Dec 2015), pp 83-94.
- [17] R. Natrajan, S. Moganavalli, *Fuzzy Sublattice Ordered Rings*, International journals of contemporary Mathematical Sciences, Vol. 7, (2012). no.13, 625-630.



- [18] Olson D. M, *On the Homomorphism for Hemirings*, IJMMS, 1(1978), 439-445.
- [19] Palaniappan.N and K.Arjunan, *The Homomorphism , Anti-Homomorphism of a Fuzzy and Anti-Fuzzy ideals*, Varahmihir journal of mathematical sciences, vol.6 no.1(2006), 181-188.
- [20] KR. Sasireka, KE. Sathappan and B. Chellapa *Intuitionistic Fuzzy l-filters*, International Journal of Mathematics and its applications, volume 4, Issue 2c(2016), 171-178.
- [21] A. Sheikabdullah and K. Jeyaraman, *Anti-homomorphism in fuzzy ideals of rings*, Int.J. Contemp. Math. Sciences, Vol. 5, 2010, no. 55, 2717-2721.
- [22] Rajesh Kumar, *A Short Note on Homomorphisms and Fuzzy (Fuzzy Normal) Subgroups*, Fuzzy Sets and Systems 44 (1991) 165-168.
- [23] G. J. Wang, *Order-Homomorphism on fuzzes*, Fuzzy sets and Systems 12 (1984), 281-288.
- [24] LA. Zadeh, *Fuzzy Sets*, Inform and Control, 8(1965), 338-353.